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Modelling Permeability with Porosity and Grain Size Diameter

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Abstract

Sand sample of different porosities from riverbed were used as porous media. A modeled experiment was set up to determine the volume of water flowing across a unit cross sectional area per unit time in these saturated sand samples packed in a vertical transparent cylindrical tube of radius $1.85x10^{-2}$m. Values of volume flux rate were determined for hydraulic gradient between 1.875 and 30.000 by using vertical flow form of Darcy's equation. The plot of hydraulic conductivity (K) against porosity ϕ and that of permeability (k) against porosity ϕ give relation $K = 0.0342e^{10.113\phi}$ ($R^2 = 0.925$) and $k = 0.0351e^{10.122\phi}$ ($R^2= 0.925$) respectively. While the plot of hydraulic conductivity (K) against grain size diameter (d_s) and permeability (k) against grain size diameter (d_s) give relation $K = 0.395e^{0.049ds}$ ($R^2 = 0.96$) and $k =0.4027e^{0.0049ds}$ ($R^2 = 0.96$) respectively. From the K-ϕ, k-ϕ, K-d_s and k-d_s curves, it shows that there is a strong correlation between grain size and hydraulic conductivity vis-à-vis permeability relatively to porosity and that irrespective of the grain size diameter or porosity, the value of permeability and hydraulic conductivity can never be zero.

Keywords: Volume flux rate, porous media, permeability, porosity, hydraulic gradient.

1.0 Introduction

The expression for flows in porous medium is known as Darcy's law. It states that the velocity of the flow is proportional to the hydraulic gradient. It holds when the water particle moves in a smooth, orderly procession in the direction of flow that is laminar flow [1].In this study, the fluid is assumed to be Newtonian and behaves a continuum while the flow is laminar steady, fully developed and incompressible. It has been found that the presence of appreciable clay in a porous medium appears to be associated with deviation from Darcy's law [2]. He later made a generalized conclusion that percentage of a surface-active material such as clay in porous media determines the extent of this deviation.

The purpose of this research is to consider a vital common property of soils which can be easily determined in the laboratory and from which this measure of permeability can be related with. Thus, porosity and grain size were chosen. This is necessary because other porous media apart from clay with smaller grain sizes can also be a surface-active material. It has been found that permeability of porous media especially clay soils decreases with increase in time [3]. The permeability – time (k-t) curve for three different samples of clay with porosity 0.25, 0.36 and 0.46 given the relationship to be $k = 0.9216t^{-0.164}$, $k = 7.8053t^{-0.097}$ and $k = 9.7788t^{-0.228}$ respectively [3].

The determination of seepage velocity of fluid in the porous media, which is one of the major parameter in application of Darcy's Law in solving environmental problems depend strongly on porosity media [1, 4]. Therefore there is a need to look at effect of porosity on the deviation from Darcy's Law.

A porous medium is any material with interconnected pores, which allows the passage of fluid such as water. The rate at which fluid can flow in a porous medium (or a material) depends on the material's porosity. Porosity can be defined as the ratio of void space (or pores) in a material to the bulk volume of the material. Porosity is a fraction from 0 and 1, although it may also be expressed in percent by multiplying the fraction by 100. It depends on particle size, size distribution, packing configuration, shapes, continuity and tortuosity of pores [5].

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A porous medium is said to be saturated if all the pores are completely filled with water under hydrostatic pressure. If the pores become filled with air instead of water, the saturation decreases. A porous medium is said to be homogenous if the permeability in a given direction is the same from point to point, while it is heterogeneous if the permeability varies from point to point. The term anisotropy is used to describe a material where permeability or conductivity is direction dependent but when permeability is the same in all directions, the material is isotopic. Isotropy and homogeneity are often assumed in the analysis of groundwater problems.

The purpose of this research is to determine a model for permeability – porosity and permeability-grain size. This is necessary in order to obtain a simple model to determine the permeability at any time for a particular porosity.

Theory: hydraulic conductivity k, is the specific discharge per unit hydraulic gradient. It expresses the ease with which a fluid is transported through a void space. It depends on the solid matrix and fluid properties .the permeability k, of a porous medium is its fluid capacity for transmitting a fluid under the influence of a hydraulic gradient [6]. It depends solely on the geometrical structure of the material (that is, porosity, grain size distribution, tortuosity and connectivity).

The simplified form of fluid flow through a porous material of a permeability k, by Darcy's law is [7]

$$v_s = -\frac{k}{\mu}\nabla(P - \rho g z) \tag{1}$$

The negative shows that the suction is towards the porous medium. Eq. (1) can be written as

$$v_s = -\frac{k}{\mu}\left(\frac{dp}{ds} - \rho g\frac{dz}{ds}\right) \tag{2}$$

$s =$ Distance in the direction of flow, always positive;

v_s = Volume flux across a unit area of the porous medium in unit time;

z = Vertical coordinate, considered downward;

ρ = Density of the fluid;

g = Acceleration of gravity;

$\frac{dp}{ds}$ = pressure gradient along s at the point to point to what v_s refers;

μ = viscosity of the fluid;

k = permeability of the medium;

From $\frac{v\mu}{k} = \frac{p}{s} + \rho g\sin\theta$

Then for vertical flow, we have

$$v_s = \frac{\rho g}{\mu}k\left(\frac{h}{L}+1\right) \tag{3}$$

By using Hubert King relation [8]

$$K = \frac{\rho g}{\mu}k \tag{4}$$

Where

K= hydraulic conductivity (ms^{-1})

k = permeability (m^2)

μ = viscosity of fluid (Nsm^{-2})

ρ = density of fluid (kgm^{-3})

$\frac{\mu}{\rho}$ = kinematic viscosity (for water) = $1x10^{-6}m^2s^{-1}$

The hydraulic conductivity contains properties of both medium and fluid with unit ms^{-1} and characterizes the capacity of a medium to transmit water, whereas the permeability with unit m^2 characterizes the capacity of the medium to transmit any fluid. The two properties are related by Eq. (4).

By using Eq. (4) in (3), Eq. (3) becomes

$$v_s = k\left(\frac{h}{L}+1\right) \tag{5}$$

$$v_s = q = \frac{Q}{A} = K\left(\frac{h}{L}+1\right) \tag{6}$$

Where,

Q= Volumetric flow rate (m^1s^{-1})

A= average cross-sectional area perpendicular to the line of flow (m^2)

q = Volume flux (ms^{-1})

h= Head constant (or hydraulic head) (m)

L= Flow path length of the sample (m)

Thus, Eq. (6) can be written in simple form as [9]

$$q = k\left(\frac{h}{L}+1\right) \tag{7}$$

$$q = Ki \tag{8}$$

Where

$$i = \left(\frac{h}{L}+1\right) = \text{ Hydraulic gradient}$$

While the volume flux q, has the units of the velocity, it is not the velocity of the water in the pores. The matrix takes up some of the flow area. The average pore water velocity is termed the seepage velocity, v and this can be written as [10]

$$v = \frac{Q}{A\phi} = \frac{q}{\phi} \tag{9}$$

v= seepage velocity (ms^{-1})

q=Volume flux (ms^{-1})

ϕ= porosity of the media

The maximum pore velocity is a function of the pore geometry and cannot be easily predicted except for simple shapes. However, it can be determined in the laboratory by volumetric method.

2.0 Material and Methods

Sand samples were collected from the riverbed of river. Sizeable quantities of these samples were washed and rinsed in order to remove organic particles and unwanted grains brought to the laboratory. Thereafter, the sand samples sun dried and later placed in an oven. After, the samples were allowed to cool down, stony particle and pepples were removed. Five different sieves were used to sieve the available and samples in order to obtain samples of different grain sizes. The porosity ϕ of each sample was determined by volumetric approach in the laboratory. The porosity of each sample was determined by volumetric approach. In the laboratory.

In an experiment set up in the laboratory, volume flux rate q, for each sample at different hydraulic gradients, a saturated sand sample was transferred to the transparent cylindrical tube of cross-sectional area 2.69 x $10^{-4}m^2$. To ascertain uniform compaction throughout the sample, the screened end was blocked so as to prevent the water passing through when the sample was being transferred. A continuous steady supply of water was fed through the sand samples of length L and at height h a hole was drilled; this enabled the height to be maintained, as excess water got drained through an overflow arrangement. The volume of water discharged, Q through the sample for a period of 60 sec after steady state has been attained at constant head was measured by measuring cylinder. It must be noted that the length L is varied in order to obtain different hydraulic gradient, i. measurement were made at hydraulic gradients of i= 1.875, 3.750, 7.500, 15.000 and 30.000; for each samples

Table1: Experimental determined values of volume of discharge Q for samples at various hydraulic gradients for 60 sec

(Hydraulic gradient)	Discharge volume $Q*10^6(m^2)A$	Discharge volume $Q*10^6(m^2)B$	Discharge volume $Q*10^6(m^2)C$	Discharge volume $Q*10^6(m^2)D$	Discharge volume $Q*10^6(m^2)E$
1.875	1.79±0.02	3.16±0.02	3.60±0.04	4.74±0.06	9.42±0.07
3.750	3.19±0.03	6.19±0.04	7.10±0.03	9.71±0.05	18.99±0.09
7.500	4.19±0.07	12.00±0.03	13.20±0.05	19.68±0.12	38.13±0.06
15.000	8.20±0.11	23.40±0.10	25.60±0.04	39.47±0.04	76.39±0.10
30.000	22.80±0.14	50.41±0.12	55.60±0.14	79.15±0.13	152.93±0.15

Table 2: Experimental determined values of volume flux rate q for samples at various hydraulic gradients

hydraulic gradient	Volume flux rate $q*10^{-4}(ms^{-1})A$	Volume flux rate $q*10^{-4}(ms^{-1}B$	Volume flux rate $q*10^{-4}(ms^{-1})C$	Volume flux rate $q*10^{-4}(ms^{-1})D$	Volume flux rate $q*10^{-4}(ms^{-1})E$
1.875	1.11±0.02	1.98±0.02	2.23±0.04	2.94±0.06	5.84±0.07
3.750	1.98±0.03	3.84±0.04	4.40±0.03	6.02±0.05	11.77±0.09
7.500	2.60±0.07	7.44±0.03	8.18±0.05	12.20±0.12	23.64±0.06
15.000	5.08±0.11	14.51±0.10	15.87±0.04	24.47±0.04	47.36±0.10
30.000	14.14±0.14	31.25±0.12	34.47±0.14	49.07±0.13	94.81±0.15

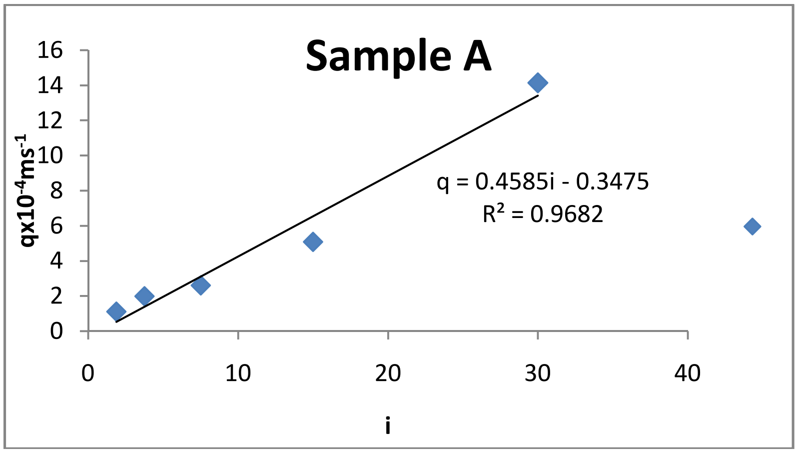

Fig 1: Graph of q against i

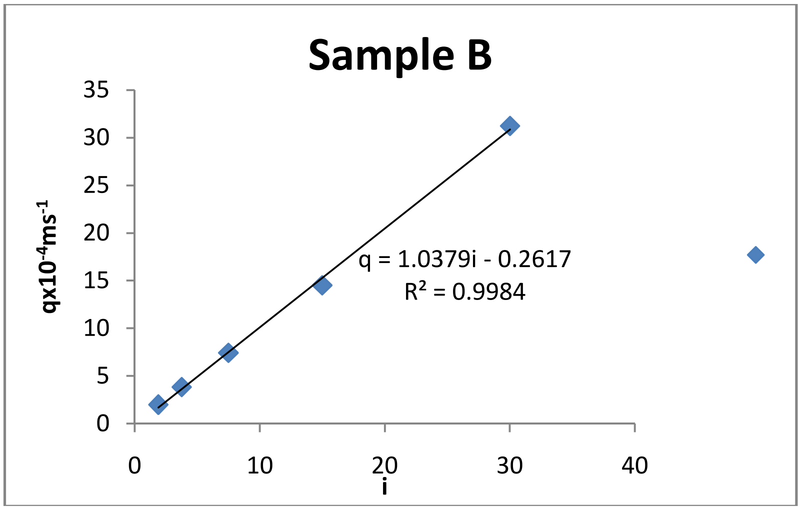

Fig 2: Graph of q against i

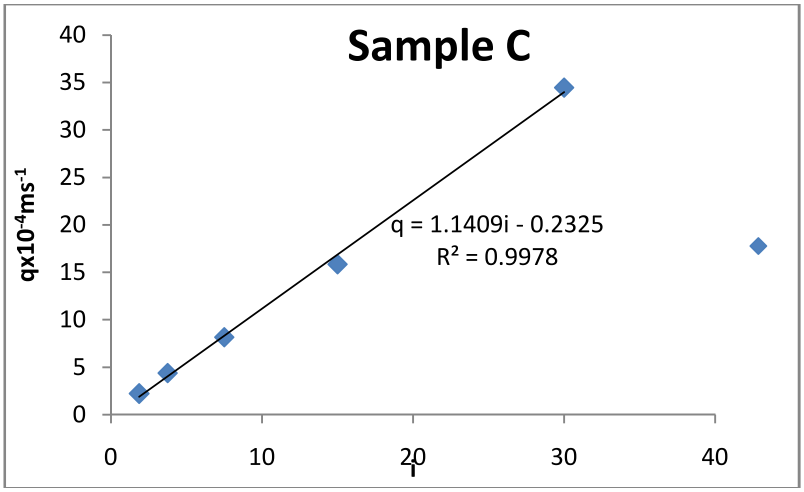

Fig 3: Graph of q against i

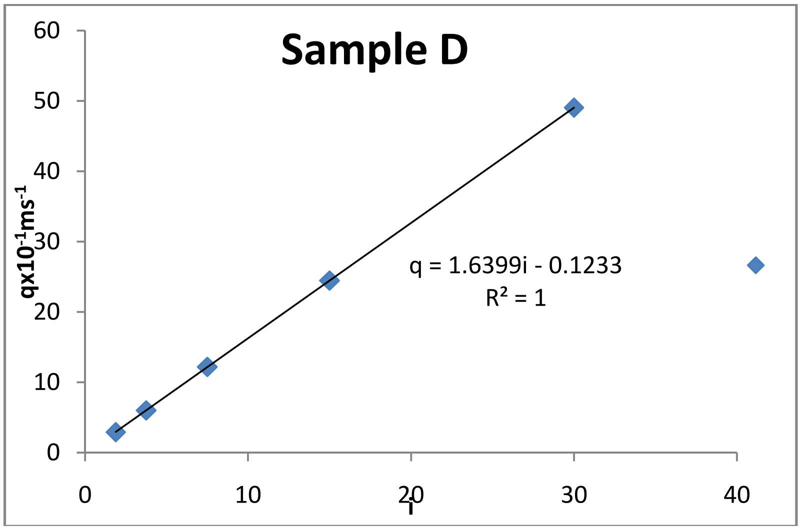

Fig 4: Graph of q against i

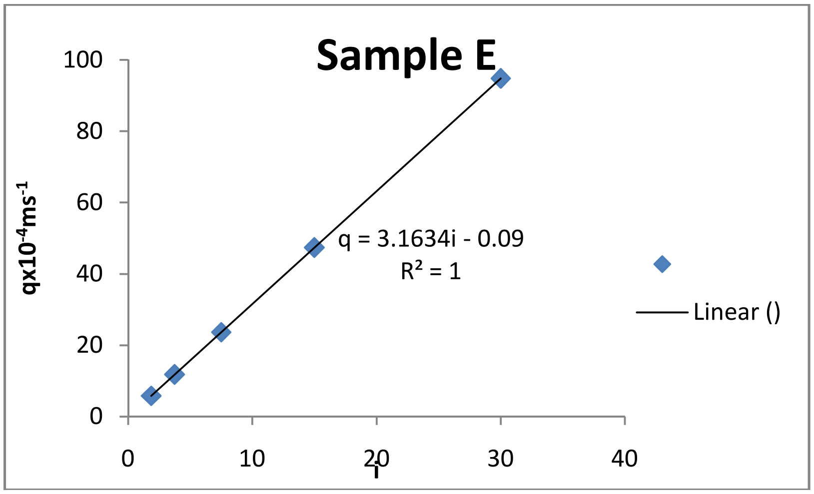

Fig 5: Graph of q against i

Table 3: Values of porosity, hydraulic conductivity and permeability for various samples

Sample	grain size (µm)	porosity	Hydraulic Conductivity (ms^{-1})	Permeability (m^2)
A	63	0.250±0.010	0.46×10^{-4}	0.47×10^{-11}
B	150	0.333±0.002	1.04×10^{-4}	1.06×10^{-11}
C	212	0.364±0.001	1.14×10^{-4}	1.16×10^{-11}
D	300	0.400±0.001	1.63×10^{-4}	1.67×10^{-11}
E	425	0.420±0.010	3.16×10^{-4}	3.23×10^{-11}

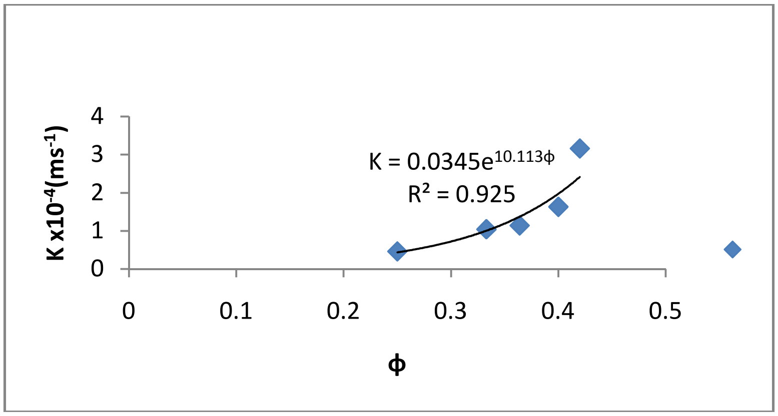

Fig 6: Graph of hydraulic conductivity (K) against porosity (ϕ)

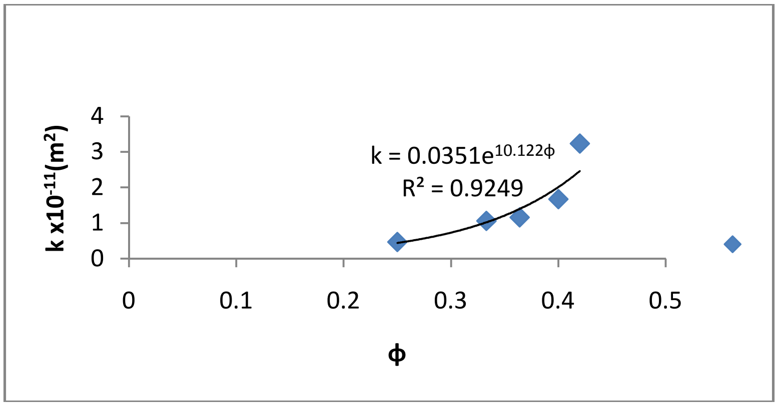

Fig 7: Graph of k against ϕ

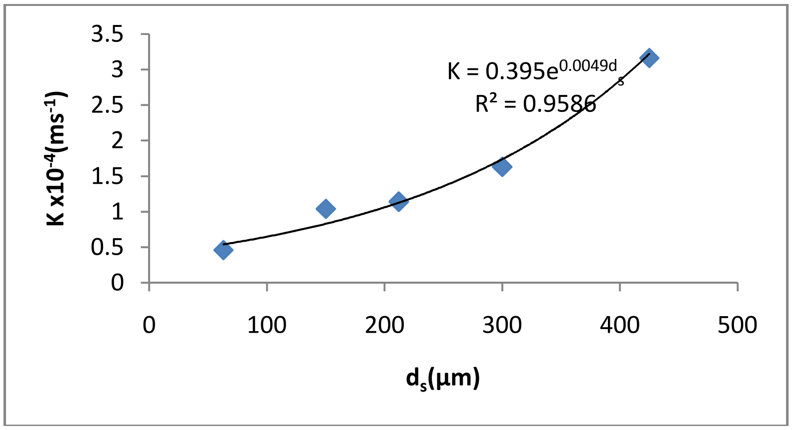

Fig 8: Graph of K against d_s

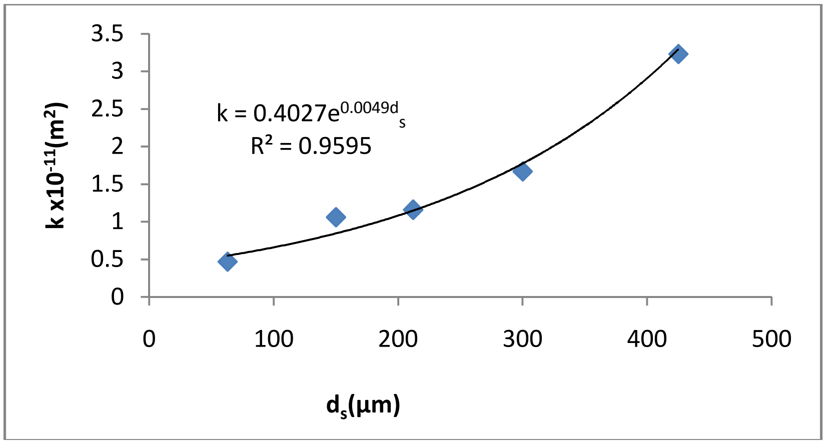

Fig 9: Graph of k against d_s

3.0 Results and Discussion

Table 1 shows the values of discharge volume, Q at each hydraulic gradient for each sample A-E. Table 2 shows the values of volume flux rate, q at each hydraulic gradient computed from the values of discharge volume, Q by using Eq. (6), the volume flux rate, q was determined knowing that q= Q/A, where $A=\pi\ (d/2)^2$ and d is the diameter of the cylindrical tube used given as $1.85\text{x}10^{-2}$m.

Using Eq. (7), the slope of the graph of volume flux rate, q against hydraulic gradient *i* equals hydraulic conductivity, K. Thus from the equation of the volume flux rate-gradient curve for each sample (Figs. 1-5), the value of hydraulic conductivity is determined and presented in Table 3. Permeability was determined from the respective values of hydraulic conductivity by using Eq. (4).

A further analysis was done on this work to determine models for permeability – porosity and permeability – grain size diameter relationships. This is necessary in order to obtain a simple model to determine the permeability at any time for a particular porosity.

The plot of hydraulic conductivity *(K)* against porosity ϕ and that of permeability *(k)* against porosity ϕ give relation $K = 0.0342e^{10.113\phi}$ (R^2 = 0.925) and $k = 0.0351e^{10.122\phi}$ (R^2= 0.925) respectively. While the plot of hydraulic conductivity (K) against grain size diameter (*ds)* and permeability (k) and grain size diameter (*ds)* give relation $K = 0.395e^{0.049ds}$ (R^2 = 0.96) and $k = 0.4027e^{0.0049ds}$ (R^2 = 0.96) respectively.

4.0 Conclusion

At the end of the study it was found that there is a strong correlation between grain size and hydraulic conductivity vis-à-vis permeability relatively to porosity. This is very obvious from the K-ϕ, k-ϕ, K-d_s *and* k-d_s curves (Figs. 6-9). Furthermore, these equations show that irrespective of the grain size diameter or porosity, the value of permeability and hydraulic conductivity can never be zero.

5.0 References

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