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A Survey of the Mathematical Equations of Coronal Mass Ejections (CMEs)

Said R.S, Okoli N. A. and Ekwubiri E.C

Department of Physics, Bayero University, Kano.

Abstract

Coronal Mass Ejections (CMEs) are transient events in which large amounts of plasma are ejected from the solar corona. Coronal mass ejections release huge quantities of electromagnetic radiations into space above the Sun's surface, either near the corona or farther into the planet system. The ejected material is plasma consisting primarily of electrons and protons but may contain small quantities of heavier element such as helium, oxygen and even iron. It is associated with enormous changes and disturbances in the coronal magnetic field. The usual method of observations of Coronal Mass Ejections (CMEs) is in visible light using coronagraphs. Since CMEs are composed primarily of plasma, they therefore contain large amount of free electrons and the light observed are scattered and bounce off these electrons through the Thomson Scattering Process (TSP). In this paper, the dynamics of Coronal Mass Ejections (CMEs) are surveyed through the Thomson Scattering theory by presenting the mathematical equations governing the observation of this phenomenon.

Keywords: Coronal Mass Ejections (CMEs), Thomson Scattering, Plasma, Solar Corona.

1.0 Introduction

Coronal Mass Ejections (CMEs) are instantaneous burst of energetic materials that can be observed from the Sun's corona. They occur on a time scale of a few minutes to several hours and involve the appearance and outward motion of a new, discrete, and bright, white-light feature in the coronagraph field of view [1-3]. Typical velocities of CMEs range between 20km/s to 3200km/s with an average of 489km/s. CMEs are massive bursts of **solar wind** and magnetic fields, which are released into space. CMEs are usually associated with other forms of solar activity, particularly **solar flares**. However, a causal relationship is yet to be established between them. Most CMEs originate from the active region on the sun surface such as groups of **Sunspots**. Near **solar maxima**, the Sun produces about three CMEs every day, while at **solar minima**, CMEs occur only once in about five days. The first coronal mass ejections were observed in 1971 [4]. It was recorded by the OSO-7 orbiting coronagraph. There was only a record of small number of events [5]. The second observation was the skylab observation, which studied CME properties [6]. Many thousands of observations later, in which events were recorded from space-borne coronagraphs [7-9] and many significant properties of CMEs have been established.

2.0 Definition of Terms

Solar Wind is a stream of plasma (charged particles) released from the upper atmosphere of the sun. It mainly consists of electrons and protons with energies mostly (1.5 and 10) keV. The stream of particles varies in density, temperature and speed over time and over solar longitude. These particles can escape the Sun's gravity due to their high kinetic energy and the high temperature of the corona.

Solar Flare Solar flares are sudden release of magnetic energy stored in the corona, where the intense magnetic field penetrate the photosphere to link the corona to the solar interior

Suns pot Are temporary phenomena on the photosphere of the sun that appear visibly as dark spots compared to surrounding regions. They usually appear as pair, with each sunspot having the opposite magnetic pole to the other.

Corresponding author: Said R.S, E-mail: rssaidu.phy@buk.edu.ng, Tel.: +2348033976393

3.0 The Thomson Scattering Process and Equations

The equations help to identify how the appearance of the CME images change due to the Physics of the surroundings. The knowledge of how light reaches the instruments help in identifying the physical characteristics of the CMEs. This therefore underscores the importance of the equations.

Thomson Scattering is a special case of the theory of scattering of electromagnetic radiation by charged particles [10]. When electromagnetic radiation is incident on a free particle that carries a charge (e) and has a mass m, the particle will be accelerated. As the particle accelerates, it emits radiation. In Thomson Scattering, the momentum transfer from photon to the electron is ignored, so the frequency of the scattered radiation is the same as the incident radiation. It is this Thomson radiation scattered in all direction.

The Thomson scattering process follows as outlined: If a particle with charge *e* and mass $m_e$moves with speeds that are small compared with the speed of light *c*, then the acceleration or radiation field $\mathbf{E}_a$is

$$E_a = \frac{e}{4\pi\varepsilon_0 c}\left[\frac{\hat{n}\times\left(\hat{n}\left(\frac{a}{c}\right)\right)}{R}\right] \quad (1)$$

where ε_0 is the permittivity of free space (we are assuming the charge is moving in a vacuum or near vacuum), *R* is the distance traveled by the particle in a given time,$\hat{\boldsymbol{n}}$is a unit vector in the direction of *R* and **a** is the acceleration vector at the same time. The energy flux at this time is the Poynting vector defined by

$$\vec{S} = E \times H = \frac{1}{\mu_0} E \times B \qquad (2)$$

Since the speed of light is $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, Eand B are related such that

$$E \times B = \left(\frac{1}{c}\right)|E^2|\hat{n} \qquad (3)$$

Therefore equation(2) can be written as

$$\vec{S} = \varepsilon_0 c^2\left(\frac{1}{c}|E|^2\hat{n}\right) = \varepsilon_0 c|E|^2\hat{n} \qquad (4)$$

But

$$\frac{dp}{d\omega} = \varepsilon_0 c|RE_a|^2 \quad = \varepsilon_0 c\left(\frac{e}{4\pi\varepsilon_0 c}\right)^2\left|\hat{n}\times\left(\hat{n}\left(\frac{a}{c}\right)\right)\right|^2 = \frac{\varepsilon_0}{c}\left(\left(\frac{e}{4\pi\varepsilon_0 c}\right)^2\left|\hat{n}\times\left(\hat{n}\left(\frac{a}{c}\right)\right)\right|^2\right) \qquad (5a)$$

$$\frac{dp}{d\omega} = \frac{\varepsilon_0}{c}\left(\frac{e}{4\pi\varepsilon_0 c}\right)^2|\epsilon^*.a|^2 \qquad (5b)$$

$$E(x,t) = \epsilon_0 E_0 \mathrm{e}^{(k_0.x-\omega_0^t)i} \qquad (6)$$

$$E(x,t) = \epsilon_0\frac{e}{m_e}E_0\mathrm{e}^{(k_0.x-\omega_0^t)i} \qquad (7)$$

$$\overline{\frac{dp}{d\omega}} = \frac{1}{2}\frac{e^2}{m_e^2}E_0\frac{\varepsilon_0}{c}\left(\frac{e}{4\pi\varepsilon_0 c}\right)^2|\epsilon^*.\epsilon_0|^2 \qquad (8)$$

$$\frac{d\sigma}{d\omega} = \frac{\text{Energy radiated/unit time/unit solid angle}}{\text{Incident energy flux/unit area/unit time}} \qquad (9)$$

$$\text{Incident energy flux} = \frac{1}{2}\varepsilon_0 cE_0^2 \qquad (10)$$

So equation (8) becomes;

$$\frac{d\sigma}{d\omega} = \frac{e^2}{m_e^2c^2}\left(\frac{e}{4\pi\varepsilon_0 c}\right)^2|\epsilon^*.\epsilon|^2$$

$$= \left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2|\epsilon^*.\epsilon|^2 \qquad (11)$$

Using the geometry in equation 2, we can write;

$$\epsilon_1 = (cos\chi cos\psi, cos\chi sin\psi, -sin\chi) \qquad (12)$$

$$\epsilon_2 = (-sin\psi, cos\psi, 0) \qquad (13)$$

$$|\epsilon^*.\epsilon_0|_x^2 = |(\epsilon_1^2+\epsilon_2^2).(\epsilon_{0x}^2)|$$

$$= |((cos\chi cos\psi)^2+(-sin\psi)^2, (cos\chi sin\psi)^2+(cos\psi)^2, (-sin\chi)^2).(1.0,0)|$$

$$= cos^2\chi\, cos^2\psi + sin^2\psi \qquad (14)$$

And for the y component we have;

$$|\epsilon^*.\epsilon_0|_y^2 = |(\epsilon_1^2+\epsilon_2^2).(0,1,0)|$$

$$= cos^2\chi\, sin^2\psi + cos^2\psi \qquad (15)$$

$$|\epsilon^*.\epsilon_0|^2 = \frac{1}{2}\left(|\epsilon^*.\epsilon_0|_x^2 + |\epsilon^*.\epsilon_0|_y^2\right)$$

$$= \frac{1}{2}(cos^2\chi\, cos^2\psi + sin^2\psi + cos^2\chi\, sin^2\psi + cos^2\psi)$$

$$= \frac{1}{2}\left((sin^2\psi + cos^2\psi) + cos^2\chi(cos^2\psi + sin^2\psi)\right)$$

$$= \frac{1}{2}(1 + cos^2\chi) \quad (16)$$

So the scattering cross-section (11) becomes;

$$\frac{d\sigma}{d\omega} = \frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 (1 + cos^2\chi) \quad (17)$$

Now, by integrating over all solid angles we can then derive the total cross-section σ_T as

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 = \frac{8\pi}{3}r_e^2 = 6.65 \times 10^{-29}m^2 \quad (18)$$

Where r_e is the classical electron radius. A useful alternative value is the differential cross-section for perpendicular scattering (σ_e)

$$\sigma_e = \frac{e^4}{(4\pi\varepsilon_0)^2 m_e^2 c^4} = r_e^2 = 7.95 \times 10^{-30}m^2 sr^{-1} \quad (19)$$

4.0 Application to the Solar Corona

Equations (1)-(19) are applicable for an assumption of scattering of radiation from the solar corona as a point source by a single electron in the plasma stream. However, the solar corona is not a point source. This necessitates integration of the scattering over radiation from the visible disk of the Sun [10-12].

Howard [10]surmises that the difficult aspect of the integration over the visible photosphere is the expression of the polarization components from an element of the photosphere in terms of a common coordinate system. Billings [13], simplified the integration by resolving the radius joining the center of the Sun into parallel and perpendicular components, while dealing with each component separately. The process is to select a point on the photosphere and angles ω and θ such that the radiation emitted from the element of photosphere is un-polarized. Then we obtain vector components as in equations (20)-(22).

$$q_y = q_n sin^2\omega \quad (20)$$

$$q_h = q_n cos^2\omega$$

$$q_{hx} = q_h cos^2\theta = q_n cos^2\omega\, cos^2\theta$$

$$q_{hz} = q_h sin^2\theta = q_n cos^2\omega\, sin^2\theta \quad (21)$$

And; $q_{tx} = q_t sin^2\theta$

$$q_{tz} = q_t cos^2\theta \quad (22)$$

Using the geometry of Equations(20)-(22) we may resolve the *y* and *x* components q_y and q_h into their *r* and *p* components:

$$q_{yr} = q_y sin^2\chi = q_n sin^2\omega\, sin^2\chi$$

$$q_{yp} = q_y cos^2\chi = q_n sin^2\omega\, cos^2\chi$$

$$q_{hxr} = q_{hx} cos^2\chi = q_n cos^2\omega\, cos^2\theta\, cos^2\chi$$

$$q_{hxp} = q_{hx} sin^2\chi = q_n cos^2\omega\, cos^2\theta\, sin^2\chi \quad (24)$$

$$q_{txr} = q_{tx} cos^2\chi = q_t sin^2\theta\, cos^2\chi$$

$$q_{txp} = q_{tx} sin^2\chi = q_t sin^2\theta\, sin^2\chi \quad (25)$$

$$q_T = q_{hz} + q_{tz} = q(cos^2\theta + cos^2\omega\, sin^2\theta) \quad (26)$$

$$q_R = q_{yr} + q_{hxr} + q_{txr} = q(sin^2\omega\, sin^2\chi + cos^2\omega\, sin^2\theta\, cos^2\chi + cos^2\theta\, cos^2\chi) \quad (27)$$

5.0 Incident and Scattered Intensity

With the incident and emergent radiation terms resolved into components in the xyz-plane, the received and scattered radiations are then considered. The intensity received at an observation point of radiation emergent from an element of photosphere is given by

$$q = I\, sin\theta d\theta d\omega = -I d\theta d(\cos\omega) \quad (28)$$

Where, I is the emitted intensity from the photosphere in units of power per unit area per unit solid angle. The tangential I_T and radial I_R intensities are given by equations (29) and (30)

$$I_T = \frac{\sigma_e}{2Z^2}\int_{cos\Omega}^{1}\int_0^{2\pi} 1(cos^2\theta + cos^2\omega\, sin^2\theta)d\theta d(cos\,\omega) \quad (29)$$

And

$$I_R = \frac{\sigma_e}{2Z^2}\int_{cos\Omega}^{1}\int_0^{2\pi} 1(sin^2\omega + sin^2\chi + cos^2\omega\, sin^2\theta cos^2\chi + cos^2\theta cos^2\chi)d\theta d(cos\,\omega) \quad (30)$$

The total scattered intensity is then given by equation (31)

$$I_T = \frac{\pi\sigma_e}{2Z^2}\int_{cos\Omega}^{1} I(1 + \cos^2\omega)d(cos\omega) \quad (31)$$

This total intensity is the equation necessary to determine the scattered light from the solar corona. The total scattered intensity is governed by the scattering efficiency that is minimized on the Thomson surface and the incident intensity that is also maximized on the Thomson surface.

6.0 Conclusion

The equations governing the dynamics of coronal mass ejections have been surveyed through the Thomson scattering process. It is shown that the total intensity equation is necessary to determine the scattered radiation from the solar corona. It is also noted that the total scattering intensity is governed by the scattering efficiency and the incident intensity, which are both maximized on the Thomson surface. Finally, it is shown that the Thomson scattering equations form the theoretical basis for how CMEs are detected in white light by coronagraphs.

7.0 References

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