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Incorporating Spatial Structures in the Analysis of Two Sets of Experimental Data

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Abstract

Spatial variability, concerned with variation among observations in space, is usually ignored in the analysis of field experiments. Inclusion of significant random spatial effects enhances the efficiency of estimation of fixed effects. Mixed modelling provides the opportunity to perform such analysis. Two datasets were investigated for the existence of spatial patterns and, where appropriate, the incorporation of such existence in the analysis of the data sets. The study indicated that spatial patterns varied over different datasets and that these patterns could be modelled using appropriate spatial variance and covariance structures. The approach is recommended as a standard practice in the analysis of agronomic and spatial-temporal related trials.

1.0 Introduction

Statistics, the science of uncertainty, attempts to model order in disorder [1]. It is not surprising that most people find the subject enigmatic. However, as life experiences and scientific experiences accumulate, statistics has come to be recognized as an extremely powerful research tool. Even when the disorder is discovered to have a perfectly rational explanation at one scale, where the data do not fit the theory exactly and when the need arises to investigate the new residual uncertainty, scientists and engineers have attempted to measure the level of disorder using entropy [1].

All data have precise spatial and temporal labels. Generally, data that are close together in space (and time) are often more alike than those that are far apart. A spatial model incorporates this spatial variation into the generating mechanism in contrast to a non-spatial model [2]. Whether one chooses to model the spatial variation through the non-stochastic mean structure (also called large-scale variation) or the stochastic-dependence structure (also called small-scale variation) depending on the underlying scientific problem, there can sometimes be simply a trade-off between model fit and parsimony of model description. Allowing for explanatory variables, models with spatial dependence typically have a more parsimonious description than classical trend-surface models [3]. They also have more stable spatial extrapolation properties and hence yield more efficient estimators of explanatory - variable effects.

Statistical methods for spatial correlation can be divided into 2 basic groups, characterization and adjustment. Characterization involves estimating covariance parameters and making spatial maps. Adjustment involves removing the effects of spatial correlation to obtain more accurate and more precise estimates of, for example, treatment means or differences. Some statistical computer packages including PROC MIXED in SAS [4] and REML in GENSTAT [5] for mixed model analysis are particularly well suited for these adjustments.

The subject matter of this paper is simply based on the fact that methods need to be developed to ensure as much closeness as possible to realities of the empirical data and of the substantive field base of the data. The objective of this study is to demonstrate the existence and subsequent isolation of spatial pattern in experimental data and then

(i) to demonstrate how the spatial distance and direction from a particular focal point affect the response, and
(ii) to investigate the spatial variance structures most appropriate for rectifying the observed anomalies

2.0 Methodology

Models for spatial correlation have their origin in statistical methods developed in geology for application in mining industry. Owing to this heritage, many important spatial models are called ‘geostatistical” models [1,6].

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We describe the approach used in SAS [4]. In the simplest sense, we have a set of observations whose physical locations and responses are known. Our primary objective is to estimate spatial correlation.
The model we are considering is a stationary model

$$Y_i = \mu + e_i \qquad (1)$$

where Y_i is the i^{th} observation on Y and e_i is the corresponding error. Let s_i denote the physical location of Y_i where s_i is specified by two coordinates. For example, the coordinates could be latitude and longitude. Alternatively, the coordinates might be indices on a grid, such as north-south and east-west, or row-column dimensions respectively. In this particular work, we refer to the coordinates of s_i as the "row" and "column" coordinates.
In general, spatial correlation models can be defined by letting

$$\text{Var}(e_i) = \sigma^2_{ii} \qquad (2)$$

$$\text{Cov}(e_i, e_j) = \sigma_{ij} \qquad (3)$$

Typically, the covariance is assumed to be a function of the distance between the locations s_i and s_j. Let d_{ij} be distance between s_i and s_j. The resulting models have the general form:

$$\text{Cov}(e_i, e_j) = \sigma^2 \, [f(d_{ij})] \qquad (4)$$

Models for which $f(d_{ij})$ is the same for all pairs of equally distant locations in a given direction, for example, along the row, along the column, or otherwise are called "stationary" models. If, in addition, $f(d_{ij})$ does not depend on the direction, then the model is said to be "isotropic" [7,8].
SAS allows one to work with the following models which are available in PROC MIXED: -

(1) Spherical

$$f(d_{ij}) = [1 - 1.5\,(d_{ij}/\rho) + 0.5\,(d_{ij}/\rho)^3]\ \ell\,(d_{ij} < \rho) \qquad (5)$$

(2) Experimental

$$f(d_{ij}) = [\exp(-d_{ij}/\rho)] \qquad (6)$$

(3) Gaussian

$$f(d_{ij}) = [\exp(-d_{ij}^2/\rho^2)] \qquad (7)$$

(4) Linear

$$f(d_{ij}) = (1 - \rho d_{ij})\,\ell\,(\rho d_{ij} < 2) \qquad (8)$$

(5) Linear log

$$f(d_{ij}) = [1 - \rho \log(d_{ij})][\rho \log\ \ell\ (d_{ij} < 2)] \qquad (9)$$

(6) Power

$$f(d_{ij}) = \rho^{dij} \qquad (10)$$

The power function is a reparameterization of the exponential covariance model. The function $\ell\,(d_{ij} < \rho)$, used in the spherical model, equals 1 when $d_{ij} < \rho$, and equals 0 otherwise. Similarly $1(\bullet)$ functions used for the linear and linear log models equal 1 when the condition within the parenthesis holds and equals 0 otherwise [7].
In some applications, the covariance models given above may not adequately account for abrupt changes over relatively small distances. These cases can be modelled by adding an additional parameter. The resulting covariance models are

$$\text{Var}(e_i) = \sigma^2 + \sigma^2_1 \qquad (11)$$

$$\text{Cov}(e_i, e_j) = \sigma^2\,[f(d_{ij})] \qquad (12)$$

where the $f(d_{ij})$ can be any of the six models given above. For these models, the parameters σ^2_1, $\sigma^2 + \sigma^2_1$, and ρ correspond to the geostatistics parameters nugget, sill and range, respectively. Using geostatistics terminology, models with $\text{Var}(e_i) = \sigma^2 + \sigma^2_1$, are called models with a nugget effect whereas models with $\text{Var}(e_i) = \sigma^2$ are called no nugget models [9].

In this work, two case studies were considered.

CASE STUDY 1

A brief description of the experiment and treatment design and the analysis of the split-plot experiment which formed our case study one is given below.
The experiment was carried out at the International Institute for Tropical Agriculture (IITA) main station in Ibadan, Nigeria. The site had been uncropped for at least 25 years. There were four fallow management systems namely Bush fallow regrowth, *Pueraria phaseoloides* live mulch, Alley cropping with *Leucaena leucocephala* and forest were implemented. The cropping cycles adopted were continuous cropping, 3-year-cycle and two 4-year-cycles. In each year, one of the cropping cycles was cropped, the others remained or returned into undisturbed fallow for soil fertility restoration.
The experiment had a split-plot complete block design with four replicates, where fallow management system was the mainplot and cropping frequency the subplot. Plots measured 12×24m. The experiment was on a contiguous plot of land. After clearing in 1989. *L. leucocephala* hedgerows were planted at 4m interrow distance and *P. phaseoloides* was planted at 1.0×0.25m spacing in all phases of all cropping frequency treatments. After three years of fallow, all biomass of the 25 percent cropping frequency treatment of Bush fallow regrowth (BFR) and Phaseoloides Live Mulch (PLM) was slashed and burned. In the alley cropping, five distances from the hedge rows called position were incorporated.

0 - 10, 10 - 30, 30 - 60, 60 - 100, 100 – 200 cm respectively. Two factors, the X and the Y coordinates of the plots are used to identify the positions of the plots. The blocking effect was intuitively taken care of by the coordinates.

The coordinates are shown in Table 1.

Table 1: Co-ordinates for the layout of the split-plot experiment

Rep1	**Fallow and Cropping systems**	**Rep2**	**Fallow and Cropping systems**	**Rep3**	**Fallow and Cropping systems**	**Rep4**	**Fallow and Cropping systems**
1,5	**C6**	1,3	B10	1,4	C1	1,4	B7
2,3	C7	1,4	**B6**	1,5	A6	1,5	C7
2,5	C1	1,5	C10	2,1	B1	2,1	A6
3,4	C10	2,2	A6	2,4	C6	2,3	B1
4,4	A10	2,3	B1	2,5	A1	3,1	A1
5,4	A7	2,5	C7	3,1	B7	3,3	B6
5,5	B6	3,6	C1	4,1	B10	3,5	C10
5,6	B7	4,1	A7	4,2	B6	3,6	C6
6,3	A6	4,2	A10	4,5	A10	4,2	A10
6,5	B1	4,3	B7	4,6	A7	4,5	C1
7,5	B10	5,1	A1	5,3	C10	5,1	A7
8,4	A1	5,5	C6	5,4	C7	5,3	B10

Keys to Table 1

Fallow System Codes (letters)

A = Bush Fallow
B = Pueraria live mulch
C = Alley Cropping
D = Forest

Cropping Cycle Codes(numbers)

1 = Continuous cropping
2 = 3 year-cycle (6)
3 = 4 year-cycle (7)
4 = 4 year-cycle (10)

(For example, C6 from Table 1 represents 'Alley Cropping' for a 3 year-cycle, A1 represents 'Bush Fallow' system on continuous cropping).

The data were collected for six months at three or four days interval. The analyses carried out on the data include Traditional split-plot design analysis (with and without repeated observations over time and mixed modelling with several variance structures using SAS version 9.3 [4].

CASE STUDY 2 – Simulated data

These data were simulated [10,11], purposely to ensure the presence of trend in the spatial pattern. Three cultivars (CULT) of winter wheat were randomly assigned to rectangular plots within each of three blocks. The nine plots were located side-by-side, and a line-source sprinkler was placed through the middle. Each plot was subdivided into twelve subplots, six to the north of the line-source and six to the south (DIR). The two plots closest to the line-source represented maximum irrigation level, the two next-closest plots represent the next-highest level, and so forth.

Analyses:

Two alternative analytical procedures namely general linear model (GLM) and the mixed model (MIXED) procedures in SAS software were used [4] .

3.0 Results

The Split Plot, Repeated Measure Data of Case Study One

A summary of the analysis of the split-plot trial where the observations were made repeatedly over six times are presented in Tables 2 to 6.

Table 2: Analysis Of Variance Over The Six Time

SOURCE	DF	TIME ONE MS	TIME ONE p-value	TIME TWO MS	TIME TWO p-value	TIME THREE MS	TIME THREE p-value	TIME FOUR MS	TIME FOUR p-value	TIME FIVE MS	TIME FIVE p-value	TIME SIX MS	TIME SIX p-value
REP	3	4.2201	0.9150	31.7453	0.2897	438.8596	0.0001	699.0849	0.0001	742.6047	0.0002	10.5394	0.0812
	2	166.1016	0.0663	565.1909	0.0357	1026.8738	0.0029	993.6523	0.0117	1206.4577	0.0039	26.2194	0.0139
Wholeplot Error	6	37.6360	0.1818	92.5249	0.5317	56.7956	0.0340	97.4185	0.0809	75.3492	0.5910	2.7683	0.7167
CC	3	129.6999	0.0026	37.8872	0.7887	181.7753	0.0002	72.6404	0.0001	1217.8839	0.0001	24.1066	0.0023
FS × CC	6	50.7040	0.0697	93.7807	0.5233	136.9440	0.0001	289.7880	0.0001	723.2052	0.0001	14.2669	0.0087
Subplot Error	27	20.5824	0.6878	85.9840	0.7401	149.8113	0.0001	206.5434	0.0001	436.9068	0.0001	19.3737	0.0001
Residual	64	24.5532		108.0277		23.1792		49.0934		96.9811		4.4942	

Keys to Table 2: DF – degree of freedom, MS-Mean Square

Table 3: Tests of hypothesis for between subject effects and Univariate test of hypothesis for within Subject effects

Between Subjects				Within Subjects					
Source	**DF**	**MS**	**p-value**	**Source**	**DF**	**MS**	**p-value**	**Adj G-G**	**H-F**
Rep	3	1170.2958	0.0003	**Time**	5	2236.72	0.0001	0.0001	0.0001
FS	2	3177.2602	0.0001	Time× Rep	15	151.35	0.0001	0.0006	0.0001
Wholeplot Error	6	119.9129	0.6301	Time × FS	10	161.44	0.0001	0.0001	0.0001
CC	3	1060.6436	0.0007	Time × FS × Rep	30	48.52	0.0129	0.0371	0.0129
FS*CC	6	881.7623	0.0002	Time × CC	15	220.67	0.0001	0.0001	0.0001
Subplot Error	27	572.4096	0.0001	Time × FS × CC	30	85.38	0.0001	0.0001	0.0001
Residual	64	165.1135		Time × FS× CC× Rep	90	85.25	0.0001	0.0001	0.0001

Keys to Table 3: DF – degree of freedom, MS-Mean Square

In table 3, two types of adjustments to p-values are presented, the G-G, for Greenhouse - Geisser and the H-F, for Huynh-Feldt.

Table 4: Polynomial and Profile Options in Analysis of Time Effects

Profile Analysis				Polynomial Effects Analysis			
Contrast	**DF**	**MS**	**p-value**	**Effect**	**DF**	**MS**	**p-value**
Time 1 vs Time 2	1	2908.9524	0.0001	Linear	1	2908.9524	0.0001
Time 2 vs Time 3	1	400.6186	0.0440	Quadratic	1	400.6186	0.0440
Time 3 vs Time 4	1	339.0123	0.0020	Cubic	1	339.0123	0.0020
Time 4 vs Time 5	1	58.3878	0.1696	Quartic	1	58.3878	0.1696
Time 5 vs Time 6	1	12486.3536	0.0001	Quintic	1	12486.3530	0.0001

Keys to Table 4: DF – degree of freedom, MS-Mean Square

Mixed Model Procedure.

Here the total worm cast during the whole period of the experiment is the dependent variable, while new variables like time and the co-ordinates X and Y are included in the analysis as explanatory factors.

Table 5: Covariance Parameter Estimates

	REML							
Covariance Parameter	**Estimate**	**Standard Error**	**Ƶ**	**p-value**	**Estimate**	**Standard Error**	**Ƶ**	**p-value**
Rep	10.8373	11.8762	0.91	0.3615	10.8487	11.8609	0.91	0.3604
FS x Rep	0.000	-	-	-	0.00	-	-	-
FS x CC	28.6892	8.9161	3.22	0.0013	29.0454	8.8306	3.29	0.0010
Residual	91.1871	5.1569	17.68	0.0001	79.0706	4.6856	16.88	0.0001

*REML – Restricted Maximum Likelihood

Table 6: Tests of Fixed Effects

Source	NDF	DDF	Type III F	**p-value**
FS	2	6	7.56	0.0217
CC	3	25	3.80	0.0225
FSxCC	6	25	3.42	0.0132
TIME	5	564	28.09	0.0001**
FSxTIME	10	564	1.95	0.0370
CCxTIME	15	564	3.29	0.0001**
FSxCCxTIME	30	564	1.19	0.2222
X	1	564	0.03	0.8667
Y	1	564	0.01	0.9373

** significant at 5%

In Table 6, NDF stands for Numerators degree of freedom, DDF stands for Denominators degree of freedom. In this table, the co-ordinates do not show any significance which suggests that the co-ordinates do not affect the spatial pattern of the data. But time is very significant

ANALYSIS OF DATA 2

Table 7: GLM Analysis of Data Two (ANOVA)

Source	**MS**	p-value
BLK	221.0803	0.0001
DIR	105.0694	0.0001
IRRIG	90.7844	0.0001
DIRxIRRIG	2.8414	0.7772
CULT	3.0024	0.6667
DIRxCULT	8.5658	0.2246
IRRIGxCULT	2.4611	0.9633
DIRxIRRIGxCULT	4.8938	0.6140

Table 7 shows the regular analysis of variance table for data 2, where the direction (DIR) and the irrigation method (IRRIG) are highly significant.

Table 8: Covariance Parameter Estimates (REML) {Mixed Model}

Covariance Parameter	**Subject**	**Estimate**	**Std Error**	**Z**	**Pr>|Z|**
BLK		5.8553	6.4725	0.90	0.3657
BLK X DIR		0.7649	1.5369	0.50	0.6187
BLK X IRRIG		0.7199	0.7231	1.00	0.3194
TOEP (2)	BLK x CULT	-0.7660	0.5827	-1.31	0.1887
TOEP (3)	BLK x CULT	1.2835	0.4242	3.03	0.0025
TOEP (4)	BLK x CULT	-1.9564	0.7299	-2.68	0.0074
Residual		4.6810	1.0318	4.54	0.0001

Table 8 lists the covariance parameters for the data. The first three rows are the variance components and the final four make up the Toeplitz structure. Here, the estimated range or ρ for the different factors are given in the 'Estimate' column and their various Standard Errors in column 4. The estimated sill or σ^2 is seen as the residual with a standard error of 1.0318.

The model fitting Information or fit statistics for the covariance parameter estimates gave -186.31 for the residual log likelihood, -193.31 for the Akaike's Information Criterion,(AIC) -201.43 for the Schwarz's Bayesian Criterion (BIC) and 372.63 for the -2 Res Log Likelihood. These values are used to determine how good your model is, the smaller, the better your model.

Table 9: Tests Of Fixed Effects

Source	NDF	DDF				
			Type III F	**Pr>F**	**Type III F**	**Pr>F**
CULT	3	61	0.92	0.4388	2.38	0.0782
DIR	1	2	4.28	0.1744	4.38	0.1713
CULT x DIR	3	61	0.92	0.4376	0.84	0.4786
IRRIG	5	10	11.32	0.0007	11.42	0.0007
CULT x IRRIG	15	61	0.74	0.7342	0.97	0.4920
DIR x IRRIG	5	61	0.77	0.5765	0.59	0.7055

In Table 9, only IRRIG is significant at 5% level. discuss

Estimates and tests of significance of a cropping system contrast, linear effect of IRRIG and the interaction of the two effects are presented in Table 10.

Table 10: Cropping System, Irrigation and Interaction Effects

Effect	**Estimate**	**Std Error**	**DF**	**T**	Pr>|T|
C1 vs C2	-0.1521	0.3659	61	-0.42	0.6790
Linear IRRIG	42.0572	5.8772	10	7.16	0.0001
C1 vs C2 x Linear IRRIG	1.7783	9.0260	61	0.20	0.8445

Table 11: Estimates of Covariance models:

Cov Parm	Subject	**Compound Symmetry (CS) Covariance**	**Autoregressive Order 1 [AR(1)] Covariance**			
		Estimate	**Estimate**	**Std Error**	**Z**	**Pr>|Z|**
BLK		8.3094	5.5417	6.1743	0.90	0.3694
BLK x DIR		1.4118	0.7078	1.5920	0.44	0.6566
BLK x IRRIG		0.2415	1.1216	0.6657	1.68	0.0920
AR (1)	BLK x CULT	-1.9668	-0.5171	0.1199	-4.31	0.0001
RESIDUAL		4.9227	4.1951	0.9180	4.57	0.0001

In Table 11, the fit statistics for the estimates of covariance models gave -188.31 for the residual log likelihood, -193.47 for the Akaike's Information Criterion,(AIC) -199.27 for the Schwarz's Bayesian Criterion (BIC) and 376.95 for the -2 Res Log Likelihood. These values are just slightly different from what was observed in Table 8 model.

4.0 Discussion

From the first data, it was discovered in the traditional GLM analysis (Table 2) that the Cropping Cycle (CC) is very significant in the 3rd, 4th, and 5th months while it is not significant in the 2nd month; TA represents the 1st month of the experiment, TB the 2nd and so on.

The Fallow System (FS) is significant throughout, It was noted that there is a very significant interaction between the FS and CC in the 3rd month to the 6th month of the experiment (Table 2) while it is slightly significant in the first month and definitely not significant in the 2nd month. From this, we can see that the time has a great effect on the experiment.

From the tests of hypothesis for between subject effects (Table 3), it's noted that every other effect except FS x Rep was significant, this indicates that there's a strong relationship between subject effects in the model.

In the Univariate test of hypothesis for within subject effects (Table 3), we have the F-test for the within subject effects with time and adjustments to the p-values according to the assessed degree of failure for the Huynh-Feldt conditions to be met. Two types of adjustments to p-values are presented in this table, the G-G, for Greenhouse - Geisser and the H-F, for Huynh-Feldt. Each of these adjustments is obtained by discounting the degree of freedom by a factor of "epsilon". Epsilon is a quantity whose value equals 1 if the H-F conditions is not met, and has a value, which decreases with increasing failure for the H-F conditions to be met. These two adjustments differ in the manner by which epsilon is estimated.

This could make the result rather inconsistent so it's advisable to use PROC MIXED in their stead.

For these data, G-G Epsilon = 0.6181 and H-F Epsilon = 1.1300.

In the ANOVA of contrast variables (Table 4), we have the ANOVA tables for the variables computed sequentially as the regression coefficients for polynomial models. It was noted that Time 1, which is the linear coefficient in the linear model, is very significant. Time 2, the quadratic coefficient in a quadratic model is not significant in the profile option while the story is different in the polynomial option. Here, it is noted that there is no linear trend but there are quadratic, cubic and quartic trends in the data.

Using the Mixed Model approach, with co-ordinates and time as additional factors (Table 6), we discover that the co-ordinates are not significant but the time is with covariance parameter estimates (Table 5), we have the estimated range ρ, for the effects listed under the estimate column with their various standard errors and the estimated sill, σ^2 i.e. the variance as the residual to be 79.0706 with a standard error of 4.6856 and is highly significant.
From the second data, it was observed that using the GLM approach (Table 7), the Block, DIR and irrigation levels are highly significant. On the other hand, in the Mixed Model approach (Table 9), it was discovered that only the Irrigation levels are significant, i.e. the block does not have any effect contrary to what was seen in the GLM.
Modelling the covariance structure (Table 8), it was discovered that the covariance structure follows the Toeplitz with four (4) bands. From Table 10, it was observed that C1 is not significantly different from C2, the interaction of the two is not significant and IRRIG possesses a strong linear component.
The data were tested using three covariance structures, the result is presented in table 11. It is noted here that the Unstructured and the Compound Symmetry have infinite likelihood and unable to make hessian positive definite respectively and therefore model fitting diagnosis are not available. On the other hand, the Toeplitz criterion estimate (-0.5171+ 0.1199) in AR(1) model suggests that the data have Auto Regressive order (1) structure.

5.0 Conclusion

After modelling with two different datasets, a conclusion could be reached that the spatial patterns in different datasets differ and that the pattern could be modeled using various covariance structures and spatial covariance structures. Our data suggested the Toeplitz with four (4) bands covariance structure and Autoregressive order (1) structure.

6.0 References

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