Journal of the Nigerian Association of Mathematical Physics
***Volume 30,* (*May, 2015*), pp515 – 524**

A Mathematical Model for the Comparative Study of the Blast Response of Aluminium and Steel Panels

M.H. Oladeinde[1], E.Nwankwo[2], S.O.Osuji[3], E.Osamor[4]

[1,4]Department of Production Engineering, University of Benin
[2,3]Department of Civil Engineering, University of Benin.

Abstract

This paper presents a mathematical model of the study of the blast response of aluminium and steel panels. These panels are deemed sufficient to provided protective barrier against explosions, especially from terrorists. With the maximum deflection being the damage criteria used, the behaviour of both panel is studied.

The panels are reduced to single degree of freedom system using assumed mode approach. Corresponding mass, stiffness and effective load form blast are formulated. The corresponding single degree of freedom (SDOF) equation structural dynamics problem is solved using Newmark-β integration scheme.

The results show an increase in blast protection with increasing thickness of panel, and increasing mass. It is also established that boundary conditions have significant effect on the blast response of the panels. This work serves as preliminary guide in designing blast protective panels.

Keywords: Single Degree of Freedom (SDOF), Elastic-plastic, Resistance Function, Numerical Integration.

1.0 Introduction

The terrorist activities and threats to lives, civil, military structures and oil production installations (production platform, pressurized pipelines etc.) have become a growing problem all over the world using explosives. Research in the area of dynamic response of structures subject to such explosive loads is therefore very crucial with a view to improving in their resistance to some of these blast loads, not only saving the human life that inhabit such structures but also protecting the integrity of the physical structure [1].

A bomb explosion within or immediate nearby building can cause catastrophic damage in the buildings' external and internal structural frames, collapsing of walls, blowing out of large expanse of windows and shutting down of critical life-safety systems [2,3].

Blast resistant design should provide a level of safety for persons and equipment in a building that is no less than that for persons outside the building in the event of an explosion. Preventing cascading events due to the loss of control of moving units not involved in an accident is another important objective in blast resistant design, as blast incident in one processing area should not be allowed to affect the safe operation or shutdown of other units or areas [4].

Experiments have shown that unreinforced masonry walls (the most widely used and dominating building material all over) have poor resistance to blast loads as evident in the blast loading curve of resistance masonries. It has been shown that the resistance offered by masonry, comprises of two components; the resistance due to elastic slab action up to and until the point where the mortar on the tension face cracks and the restoring moment due to the self-weight of slab [5-7].

In light of the foregoing, materials with high ductility ratio which have been shown to have high capabilities in absorbing energies was studied in this research [5]. Research has also shown that steel and aluminium metals have sufficient post-yield ductility that can suffice for large displacements caused by blast loads, and were subsequently used in this study [8].

On the general effect of blast loading on structural elements, various researcher have studied the effect of blast on reinforced concrete panels and other sandwiched structures [7, 9-11]. Louca and Boh [12] worked on design and analysis of blast wall and recommended various retrofitting schemes for blast walls subjected to large overpressures.

Corresponding author: M.H. Oladeinde, E-mail: moladeinde@uniben.edu, Tel.: +2348039206421

Magnusson and Hallgre [6] carried out blast analysis of reinforced concrete beams. In a dynamic loading, because the load is temporary and its duration is shot, it is preferable that materials and connection behave in a non-elastic manner so that the structure can tolerate larger deformations and can absorb more energy. Newmark et al [13, 14] asserted that the ratio of the ultimate displacement to displacement of the point of yield is called ductility ration and determine to a very large extent, the degree to which the material can absorb the energy emanating from the blast load.
At this juncture it should be noted that the analytical models are normally validated by experimental models [15]. However, due to the practical limitations of conducting large scale blast load experiments, only the analytical models which is based on well-known principles is presented in this thesis. This work therefore intends to develop adequate blast protective panels, as they would ameliorate the devastating effect of explosions on both structures, lives and properties from terrorists. The simulation of the Newmark –β single step implicit integration was employed in this study. The result shows that both metals have remarkable and effective protective abilities against blast loads; with steel showing exceptional resistance. The thicknesses of each of the metals used in this study are 20mm, 25mm, 40mm and 45mm.The selection of appropriate metal and its thickness that would withstand any anticipated magnitude of blast load and also fits into use for construction of structures considering the fact that light weight structures are desirable in construction can now be done by construction engineers for their design.

2.0 Mathematical Modelling

Most metals generally exhibit elastic-plastic hardening. Since blast loads causes large displacement in structures, thus, causing significant yield in metals. A structure with a stable post-yield behaviour and sufficient ductility renders good blast protective.
Structural dynamics analyses are required to assess certain blast problems, because both load and response vary with time, it is evident that a dynamic problem does not have a single solution as with static problem. Instead, one must establish a response history covering all the relevant phenomena which are of interest in the study.
The response of a SDOF system can be evaluated directly from the solution of a single differential equation of motion, while MDOF system is made complicated due to the necessity of discretization.
SDOF system method is commonly used to assess the blast response in an early design stage, and allow for easy hand calculations. This method is based on simplifying the structural problems into a mass-spring system without damping and includes a load-mass transformation factors to account for various boundary and load conditions [16].
These transformation factors are based on approximations to classical beam and plate theory for deflections in the elastic range and plastic hinge or yield-line theory in the plastic range.
Newmarks –β method of solution [13, 14] was adopted here since we are applying the analytical method in assessing the blast load response of steel and aluminium.
In order to aid the blast assessment of complex structures, such structures are reduced to Single Degree of Freedom (SDOF) i.e. lumped mass system (equivalent system). The general equation for the dynamic response of an SDOF system is:

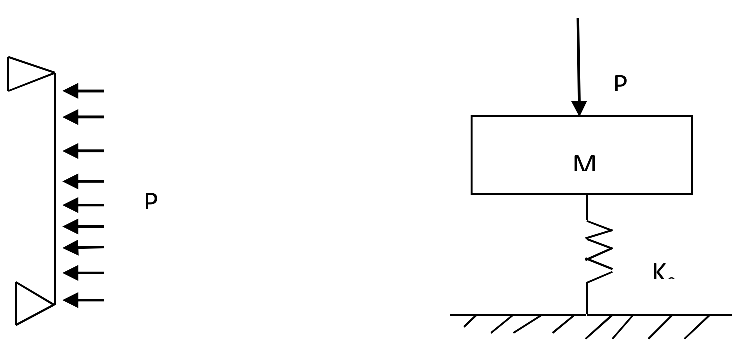
$$\boldsymbol{M}\ddot{x} + \boldsymbol{C}\dot{x} + \boldsymbol{K}x = P(t) \qquad (1)$$

Where $\boldsymbol{M}$ is the mass of the system, $\boldsymbol{C}$ is the damping and $\boldsymbol{K}$ is the stiffness of the system. P (t) is the time varying blast load and x (t) is the displacement with respect of time.
In this research, the damping term $\boldsymbol{C}$ is taken as zero because in one cycle of displacement attenuation is small, thus, ignoring $\boldsymbol{C}$ is a conventional approach in blast design.
Thus, Equation 1 reduces to:

$$\mathbf{M}\ddot{x} + \mathbf{K}x = P(t) \qquad (2)$$

2.1 Determination of Equivalent Parameters – Load Mass Factor

The overall system analysed in this research is shown in Figure 1a. Figure 1b shows the mathematical representation of Figure 1a (i.e. showing equivalent, mass, stiffness and load). The overall problem, thus, falls into a structural dynamics problem.



(a) Real (b) Idealised

Figure 1: Illustration of SDOF idealisation of real

Here, M_e is called the equivalent mass of the SDOF system, which is derived from the actual mass of real system; K_e is called the equivalent stiffness of the equivalent SDOF idealization, this is gotten from the stiffness of the actual system.
Considering Equation 2, it can be shown that

$$\mathrm{K}_M \mathbf{M}\ddot{x} + \mathrm{K}_L \mathbf{K}x = \mathrm{K}_L P(t) \quad (3)$$

$$\mathrm{K}_{LM} \mathbf{M}\ddot{x} + \mathbf{K}x = P(t) \quad (4)$$

Where, K_m is called the mass factor and K_L is called the load factor. K_{Lm} is called the load mass factor.

$$\mathrm{K}_{LM} = \frac{K_m}{K_L} \quad (5)$$

The equation for the deformed shape of the panels is expressed in a mathematical form as:

$$\mathrm{w}(x) = \frac{16}{5L^4}(L^3x - 2Lx^3 - x^4)W_0 \quad (6)$$

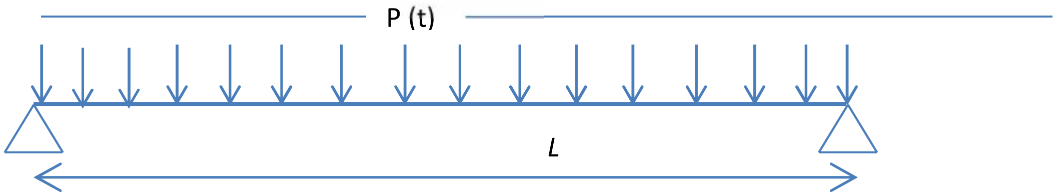


Figure 2: Simply supported beam with uniformly distributed blast load
Where W_o is the maximum displacement at mid-span. The above shape function satisfies all necessary boundary conditions for simply supported panel.

$$w(0) = 0, \quad w(L) = 0, \quad w''(0) = 0, \quad w''(L) = 0$$

Where w (0) =0 and w (L) = 0 represent the displacements at the supports. While $w''(0) = 0, w''(L) = 0$ represent the second derivative of displacements at the supports which is equal to the moments at the supports.
The work done by the beam is:

$$\mathrm{WD} = \int_0^L P(t)w(x)\,\mathrm{d}x = \frac{16P(t)W_0}{25} \quad (7)$$

The strain energy is:

$$U = \int_0^L \frac{M^2}{2EI^2}\mathrm{d}x = \int_0^L \frac{E^2I^2}{2EI}\left(\frac{d^2W}{dx^2}\right)^2 \mathrm{d}x = \frac{8EIW_0{}^2}{5L^3} \quad (8)$$

The kinetic energy,

$$\mathrm{WD} = \frac{1}{2}\int_0^L \rho A\big(w(x)\big)^2 \mathrm{d}x = 0.252\rho AW_0^2 \quad (10)$$

The equivalent system will have the same maximum displacement W_0 and maximum velocity W_o similar to the analysed metallic panels. Thus, the work done,

$$\mathrm{WD} = P_e(\mathrm{t})W_0 \quad (11)$$

$$\mathrm{U} = \frac{\mathrm{K}_e W_0^2}{2} \quad (12)$$

$$\mathrm{KE} = \frac{M_e W_0^2}{2} \quad (13)$$

Equating these quantities to the real system, we obtain:

$$\frac{16W_0P(t)}{25} = \mathrm{P}_e(\mathrm{t})W_0 \quad (14)$$

Thus the load factor, K_L is:

$$K_L = \frac{P_e(t)}{P(t)} = \frac{16}{25} = 0.64 \quad (15)$$

Similarly, the strain energy for the two systems give

$$\frac{25.58\mathrm{EIW}_0^2}{L^3} = \frac{\mathrm{K}_e\mathrm{W}_0^2}{2} \quad (16)$$

Assume,

$$\frac{P}{W_0} = \frac{384EI}{5L^3} = K \text{ for a simply supported beam}$$

$$K_s = \frac{K_e}{K} = 0.64 \quad (17)$$

Equating the KE for the two systems, we have

$$0.252\rho AW_0^2 = \frac{1}{2} M_e W_0^2 \quad (18)$$

Since ρAL is the mass M of the panel and a mass factor K_m can be written as:

$$K_m = \frac{M_e}{M} = 0.504 \quad (19)$$

For the plastic response,

$$K_L = \frac{P_e(t)}{P(t)} = 0.5 \;;\; K_m = \frac{M_e}{M} = 0.333 \quad (20)$$

2.2 Determination of Resistance Function

The simply supported wall is assumed to be elastic-plastic. Figure 3 shows the idealised resistance of the SDOF system used in this work. An elastic perfectly plastic resistance curve is assumed for the SDOF developed in this thesis. F_y represents the yield strength of the SDOF system and x_e represents the displacement at yield.

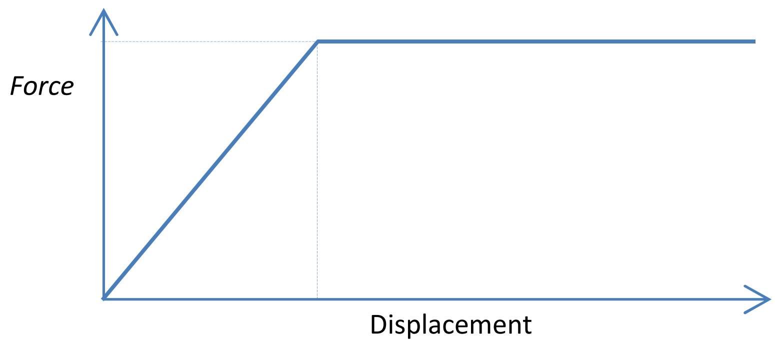


Figure 3: Schematic representation of elastic-plastic resistance curve

The pressure to cause yield,

$$\sigma_e = \frac{8M_{span}}{L^2} \quad (21)$$

Also, the deflection at this stage, x_e, is given in Equation 22.

$$x_e = \frac{\sigma_e L^4}{384EI} \quad (22)$$

NB: This is an analytical method for determining the resistance curve. Finite element methods are usually employed in developing the resistance curve of more complex systems.

2.3 Properties of Section

The maximum response of an SDOF system is dependent on the stiffness (i.e. resistance of the system). In order to evaluate the resistance of the idealised SDOF system, the second moment of area of the real system is obtained. Equations 23-25 show the stiffness, second moment of area and the natural frequency of the panels studied. Tables 1 and 2 show the mechanical panel properties and section properties of the panels used this work.

$$K = \frac{384EI}{5L^3} \quad (23)$$

$$I = \frac{bd^3}{12} \quad (24)$$

$$w = \sqrt{\frac{K}{M}} \quad (25)$$

Table 1: Mechanical Properties of Panels (Steel and Aluminium)

Metal	Density (kg/m^3)	Young's Modulus (GPa)
Aluminium	2700	78
Steel	7800	200

Table 2: Section properties of analysed panels

Panel	Breadth (m)	Length (m)	Thickness (m)	Second moment of area, I(m^4)
A	0.1	0.5	0.02	6.67E-08
B	0.1	0.5	0.025	1.30E-07
C	0.1	0.5	0.03	2.25E-07
D	0.1	0.5	0.035	3.57E-07
E	0.1	0.5	0.04	5.33E-07
F	0.1	0.5	0.045	7.59E-07

2.4 Blast Loads

A Time varying blast load typical of a hydrocarbon explosion is applied to the analysed panels and their maximum deflections obtained. The blast load has duration of 0.007seconds and rises from 0 bar to a peak value of 9 bars in 0.0035seconds. Figure 4 shows the duration of the blast load. Further literature on the mathematical idealisation of blast and impact loads can be seen in [5, 17].

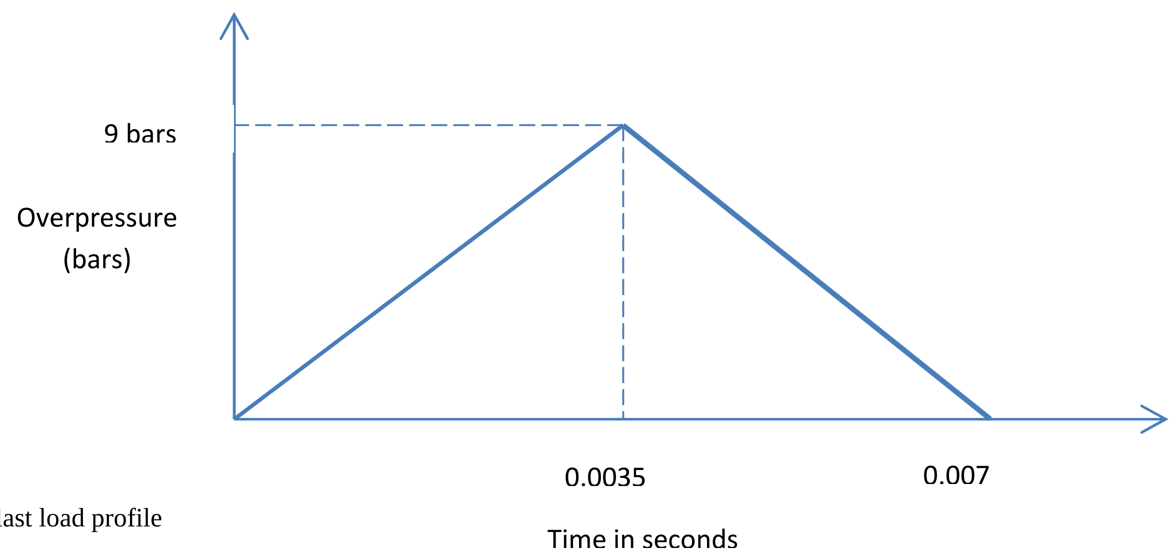


Figure 4: Blast load profile

2.5 Solution Scheme

The most general approach for the response of structural systems to blast loads is the direct numerical integration of the equation of motion. The methodology adopted in this work defines the solution of the system at time zero and attempts to solve the dynamic equation at discrete time intervals.

Generally solution scheme can either be explicit or implicit. Explicit methods do not involve the solution of the equation of motion at each time step (i.e. discrete time step). The explicit method uses the solution of differential equation at time t, to predict the solution at time t + Dt. However the implicit methods attempt to satisfy the differential equation at time 't' after the solution at t-Dt has been obtained. This requires the set of solution of the equation of motion at each time step.

This thesis uses the Newmark family of single-step integration method [13, 14] to solve the equation of motion of the system. The following procedure illustrates the Newmark family of equation developed in a Excel Spread Sheet using Visual Basic Application.

Consider the equation

$$M\ddot{u}_t + K\dot{u}_t = F_t \qquad (26)$$

Using Taylor series expansion we obtain,

$$u_t = u_{t-\Delta t} + \Delta t\dot{u}_{t-\Delta t} + \frac{\Delta t^2}{2}\ddot{u}_{t-\Delta t} + \frac{\Delta t^3}{6}\dddot{u}_{t-\Delta t} + \cdots \ldots \ldots \ldots \qquad (27)$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + \Delta t\ddot{u}_{t-\Delta t} + \frac{\Delta t^2}{2}\ddot{u}_{t-\Delta t} + \cdots \ldots \ldots \qquad (28)$$

Truncating,

$$u_t = u_{t-\Delta t} + \Delta t\dot{u}_{t-\Delta t} + \frac{\Delta t^2}{2}\ddot{u}_{t-\Delta t} + \beta\Delta t^3\dddot{u}_t + \cdots \ldots \ldots \ldots \qquad (29)$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + \Delta t\ddot{u}_{t-\Delta t} + \gamma\Delta t^2\dddot{u}_t + \cdots \ldots \ldots \qquad (30)$$

If the acceleration is assumed to be linear within the time step, we have

$$\dddot{u}_t = \frac{u - u_{t-\Delta t}}{\Delta t} \qquad (31)$$

Substituting the above equation into the truncated form we have:

$$u_t = u_{t-\Delta t} + \Delta t\dot{u}_{t-\Delta t} + \left(\frac{1}{2} - \beta\right)\Delta t^2\ddot{u}_{t-\Delta t} + \beta\Delta t^3\ddot{u}_t + \cdots \qquad (32)$$

$$\dot{u}_t = \dot{u}_{t-\Delta t} + (1-\gamma)\Delta t\ddot{u}_{t-\Delta t} + \gamma\Delta t\ddot{u}_t + \cdots \ldots \ldots \qquad (33)$$

Using the Newmark procedure, an iterative scheme for each step is developed in EXCEL Spreadsheet using visual basic.

Schematic for scheme

a) Determine the stiffness, K and mass, M of the system
b) Specify $\beta = 0.25$ and $\gamma = 0.5$
c) Calculate the constants

$$b_1 = \frac{1}{\beta\Delta t^2}\,; b_2 = \frac{1}{\beta\Delta t}; b_3 = \beta - \frac{1}{2}; b_4 = \gamma\Delta t b_1; b_5 = 1 + \gamma\Delta t b_2;\ \ b_6 = \Delta t(1 + \gamma b_3 - \gamma)$$

d) Form the effective stiffness matrix

$$\bar{K} = K + b_1 M \qquad (34)$$

e) Triangulate effective stiffness matrix

$$\bar{K} = LDL^T \qquad (35)$$

f) Specify the initial conditions
g) For each time step repeat procedure

3.0 Results and Discussion

The analytical model developed is used to analyse 12 set of steel and aluminium panels. Panels of thickness varying from 20 mm to 45mm were analysed. The effect of panel thickness on the maximum displacement of the panel is investigated for aluminium and steel panels under hydrocarbon pulse load. Also, the effect on density on the maximum displacement was investigated by comparing similar dimensions of aluminium and steel. Consequently, the effect of the natural frequency on the maximum displacement was investigated. From the responses of the panels (steel and Aluminium) upon subjection to blast loads.

The panels under investigation were all subjected to a blast load of 9 bars (peak over pressure).The duration of the peak over pressure was 0.007 seconds, with a rise time of 0.0035 seconds. The displacement time histories of the panels studied are as presented in Figures 5 – 16.

Figures 5 – 16 represent the time history curve for the steel panels. The thickness of the panels ranges from 20mm to 45mm, maximum displacement was observed to occur at 0.015seconds, irrespective of panels' thickness.

It can be seen that as the thickness of steel panel increases, the protective ability of the panel to same magnitude of blast load increases, this applies to the aluminium panel as well. Also, for corresponding thickness of steel and aluminium, subjected to same blast load, the protective ability of the steel panel is better at all times.

The uniformity in the time of occurrence of the maximum displacement was due to steels Young's Modulus which is same, since the steel panels have same mechanical properties. After the attainment of this maximum displacement at the occurring time of 0.015seconds, it was observed that an oscillation about a mean permanent position continues and dies out over a short duration of time, the maximum displacement observed for each of the steel panels under investigation decreases with increase in their thickness.

The maximum displacement observed and the corresponding thicknesses of the steel panels are shown in Table 3.

Table 3: Maximum displacement of steel panels

Thickness of steel panel (mm)	Maximum displacement (dm)
20	0.75
25	0.74
30	0.72
35	0.39
40	0.37

45	0.25

The mechanical property for this metal is uniform (same) for all the metals (aluminium) i.e. the density and the young's modulus, but the thickness differs. The results are shown in Table 4.

Table 4: Maximum displacement of aluminium panels

Thickness of the aluminium panel (mm)	Maximum displacement (dm)
20	1.82
25	1.40
30	1.19
35	0.92
40	0.82
45	0.72

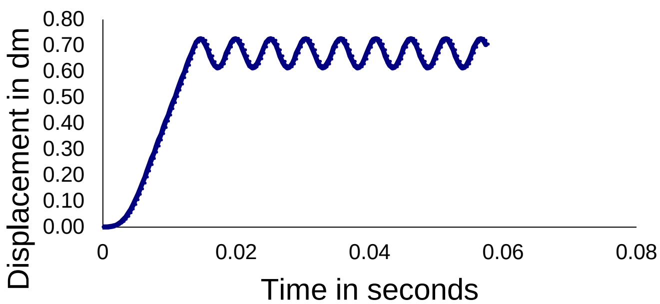


Figure 5: 20mm thick steel panel under applied blast load

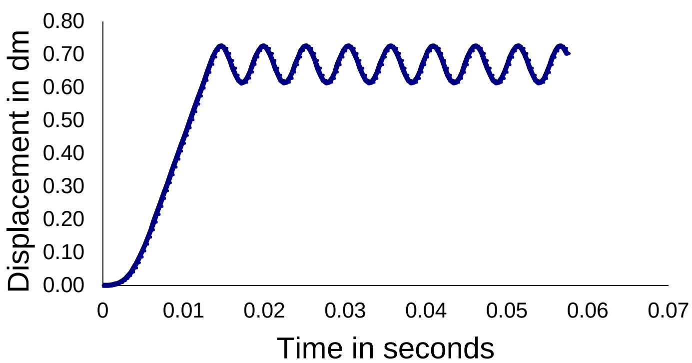


Figure 6: 25 mm thick steel panel under applied blast load

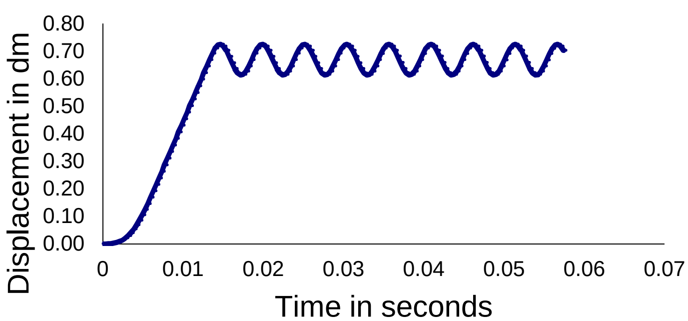


Figure 7: 30 mm thick steel panel under applied blast load

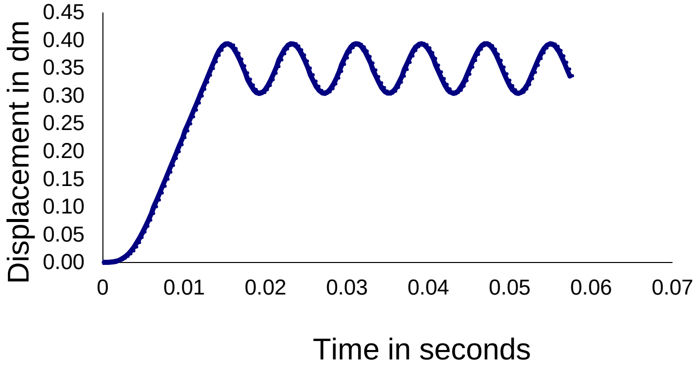


Figure 8: 35 mm thick steel panel under applied blast load

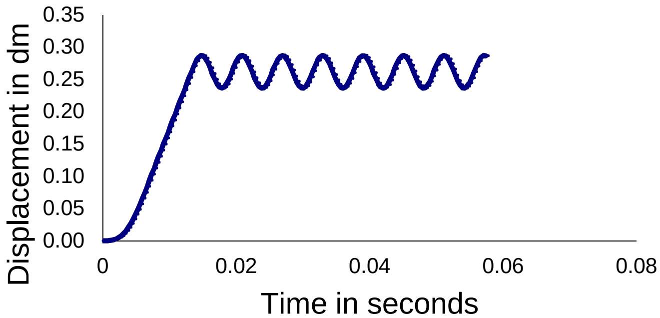


Figure 9: 40 mm thick steel panel under applied blast load

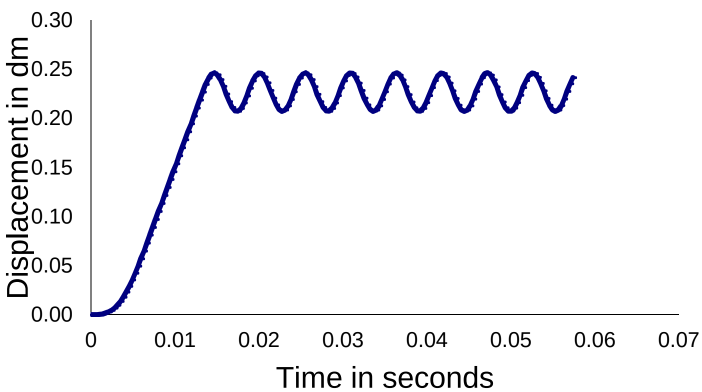


Figure 10: 45 mm thick steel panel under applied blast load

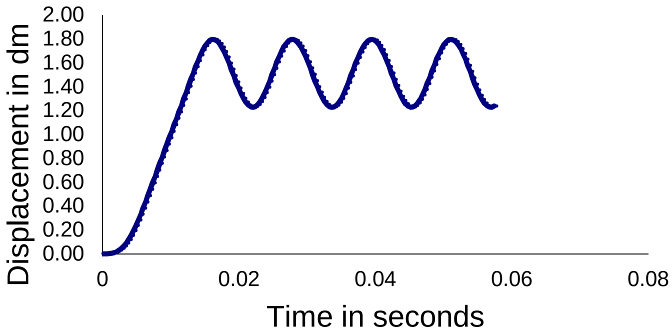


Figure 11: 20 mm thick aluminium panel under applied blast load

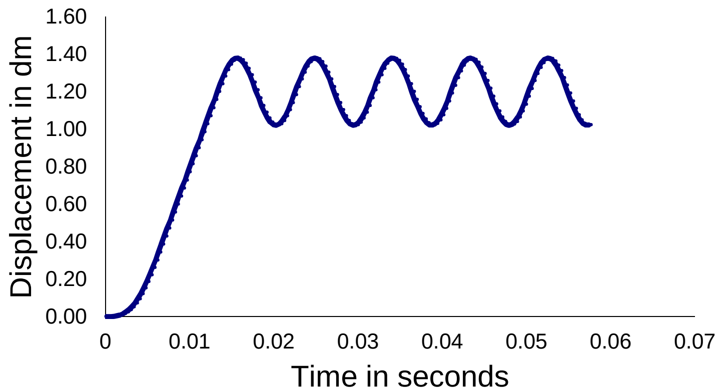


Figure 12: 25 mm thick aluminium panel under applied blast load

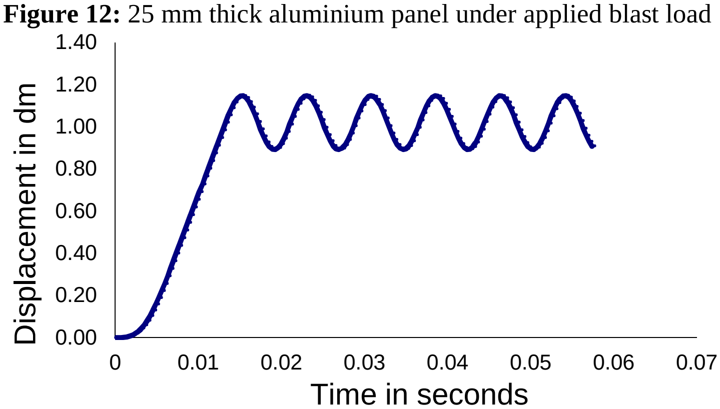


Figure 13: 30 mm thick aluminium panel under applied blast load

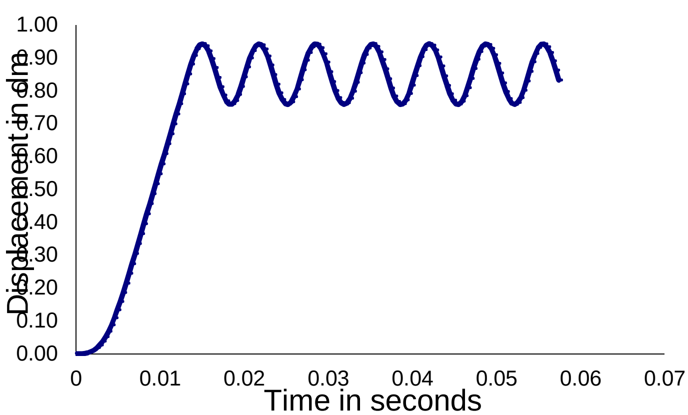


Figure 14: 35 mm thick aluminium panel under applied blast load

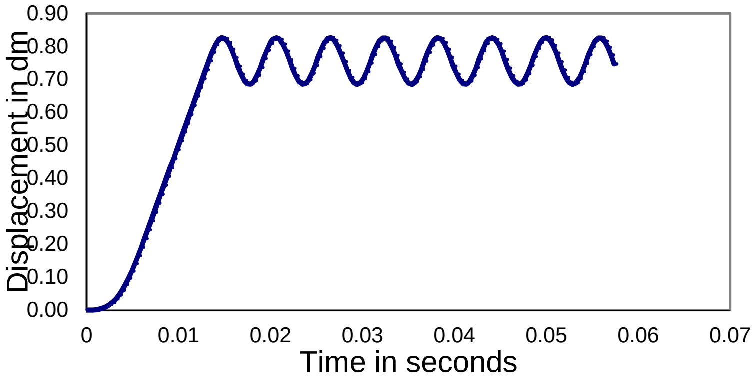


Figure 15: 40 mm thick aluminium panel under applied blast load

Journal of the Nigerian Association of Mathematical Physics Volume 30, (May, 2015), 515 – 524

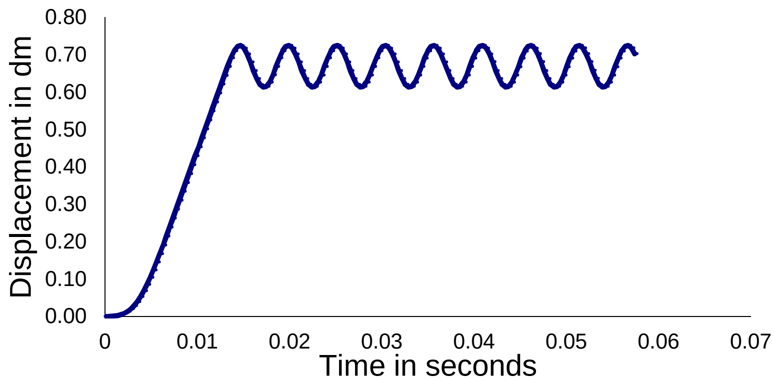


Figure 16: 40 mm thick aluminium panel under applied blast load

4.0 Conclusion

The research work was centred on assessing steel and aluminium metallic panels' resistances to similar hydrocarbon blast load.

The measure of resistance was based on the panel's ability to absorb applied external energy in the form of pressure without being fractured, and this to a very large extent depends on the ductility characteristic of the metals. The more the ductility ratio of a metal, the more effective it will be in blast resistance.

From the investigation conducted on aluminium and steel panels. The following conclusion can be drawn:

1) The larger deflections were observed in the aluminium panels. This implies that the effect of weight is more significant than thickness in providing blast resistant
2) In the steel panels, as the thickness is increased, the maximum deflection is marginally reduced.
3) The implications of large deflections observed in the analytical procedure indicate possible tearing in the aluminium panel. However, a full non-linear finite element analysis is recommended in order to determine precisely the blast loads that result in tearing in the panels
4) In all panels analysed it is observed the maximum response occurred at 0.15 seconds for blast loads of duration 0.007seconds. This suggests an impulsive response of the system. This means that the maximum response occurred much after the blast load has died down.
5) The inference drawn from conclusion (1) is that for blast protective partitions in building and in defence application, a heavy metal panel (i.e. steel panels) is preferable to a lighter metal panel (i.e. aluminium panel) of similar thickness.

References

[1]. Baker, W.E. (1973). Explosions in Air. University of Texas Press.
[2]. Mendis, P and Ngo, T. (2003). Vulnerability assessment of concrete tall buildings subjected to extreme loading conditions. CIB_CTBUH, International Conference on Tall Buildings
[3]. Mendis, A, Ngo T, Gupta and Ramsay, J.(2007). Loading on Structures. EJSE Special Issue
[4]. Hinman, E. (2003). Retrofitting to resist explosive attack. Presentation to the Existing Building Committee of the SEAONVC.
[5]. Hetherington, J.G. and Smith, P.D. (1994). Blast and Ballistic Loading of Structures.Butterworth-Heinemann.
[6]. Magnusson, J. and Hallgre, M (2004). Reinforced high strenght structures and materials. VIS, structures under shocks and impacts VIII, pp 53-62
[7]. Miyamoto, A. and Nakamura. H (2001). Visualization of impact failure behavior for RC slabs. Proceeding of 3rd International Conference on Concrete Severe Conditions. UBC
[8]. Selby, C. and Burgan, B. (1998). Blast and fire engineering for topside structures-phase 2, Final Summary Report. UK
[9]. Luccioni, B. M and Luege, M (2006). Concrete pavement slab underblast loads. International journal of impact engineering, vol. 32, pp 1248-1266
[10]. Lan, S., Lok, T.S. and Heng, L. (2005). Composite structural panels subjected to explosive loading. Construction and building materials, vol. 19, pp. 387 - 395

[11]. Librescu, L. S, and Heng, L. (2004). Linear and non-linear dynamic response of sandwich panels to blast loading. Composites, Part B, vol. 35, pp. 673-683.
[12]. Louca, L.A. and J.W.Boh, J.W. (2004). Analysis and design of profiled blast walls, Imperial College London for the Health and Safety Executive
[13]. Newmark, N. M. (1995). A method of computation for structural dynamics, ASCE Journal of the Engineering mechanics division, vol. 85, No 8 , pp 67-94s
[14]. Newmark, N. M and Hall, W. J.(1982). Earthquake spectra and design. Berkley CA. Earthquake Engineering Research Institute.
[15]. Schleyer G.K., Lowak MJ, Polcyn M.A., Langdon G.S. (2007). Experimental investigation of blast wall panels under shock pressure loading. International Journal of Impact Engineering,34:1095-118
[16]. Merx, W.P.N. (1992). Methods for the determination of possible damage to people and objects resulting from releases of harzadous materials.CPR 16E, TNO, First Edition.
[17]. Khaloo,A.R. and Tariverdilu A.S. (2001). Ductility evaluation of confined high strength concrete flexural members. Iranian Journal of Science and Technology, Transaction B,Engineering, Vol.25, No.B1,129-131.