Journal of the Nigerian Association of Mathematical Physics
***Volume 30, (May, 2015*), pp505 – 514**

Heat Transfer to Pulsatile Slip Flow in a Porous Channel Filled With Porous Media

Usman M.A, Hammed F.A and Onitilo S.A

Department of Mathematical Sciences, Olabisi Onabanjo University, Ago Iwoye.

Abstract

This paper investigate the effect of slip on the hydromagnetic pulsatile flow through a porous channel filled with saturated porous medium with time dependent boundary condition on the heated wall. Based on the pulsatile flow nature, the dimensionless flow governing equations are resolved to harmonic and non-harmonic parts. Exact solutions are obtained for the temperature and velocity fields. Parametric study of the solutions are conducted and discussed.

Keywords: Pulsatile flow, Navier slip, porous medium, heat transfer, magnetic field, thermal radiation.

Nomenclature

t' - time,	u' - axial velocity,
° - fluid density,	p' - fluid pressure,
v - kinematic viscosity,	k - porous permeability,
$\pm_e$ - electrical conductivity,	B_0 - magnetic field intensity,
g - gravitational acceleration,	- volumetric expansion,
C_p - is the specific heat at constant pressure	~ - is the term due to thermal radiation,
k - is the thermal conductivity,	T' - fluid temperature,
T_0 - referenced fluid temperature,	¤ - is the slip parameter due to the porous medium,
v_0 - is the suction/injection term,	Re - is the Reynolds' number,
H^2- is Hartmann`s number,	Gr - is the Grashof number,
Pe - is the Peclet number,	N - is the thermal radiation parameter,
¥ - is the cold wall slip parameter,	s - is the suction/injection parameter.

1.0 Introduction

In many branches of engineering science, the flow of an electrically conducting fluid has important applications. These branches of engineering include magneto hydrodynamics {MHD} generators, nuclear reactor, plasma studies, geothermal energy extraction electromagnetic propulsion, the boundary layer control in field of aerodynamics and so on.

The oscillatory flow and heat transfer in a rigid tube of vary cross-section with permeable wall The flow across any representative section f the tube is viewed as composed of a pulsatile part superimposed on a steady part and the effect of fluid absorption through the permeable wall is accounted by prescribing flux by arbitrary function of axial distance and time. The flow downstream of this section is investigated by expanding stream-function, vorticity and fluid temperature in asymptotic series about a small parameter, e, characterizing the tube aspect ration. Numerical results computed for wavy wall tube and a tapering tube are presented graphically and discussed quantitatively [1-3].

The entrained flow and heat transfer of a non-Newtonian third grade fluid due to a linearly stretching surface with partial slip. The partial slip is controlled by a dimensionless slip factor, which varies between zero (total adhesion) and infinity (full slip). Suitable similarity transformations are used to reduce the resulting highly nonlinear partial differential equations into ordinary differential equations. The issue of paucity of boundary conditions is addressed and an effective second order numerical scheme has been adopted to solve the obtained differential even without augmenting any extra boundary conditions.

Corresponding author: Usman M.A, E-mail:-, Tel.:-

The important finding in this communication is the combined effects of the partial slip and the third grade fluid parameter on the velocity, skin-friction coefficient and the temperature field. It is interesting to find that the slip and the third grade parameter have opposite effects on the velocity and the thermal boundary layers [4-7].
The effect of radiative heat transfer to oscillatory hydromagnetic non-Newton couple stress fluid flow through a porous channel with non-uniform wall temperature due to periodic heat input at the heated wall. Based on some simplifying assumptions, the dimensionless partial differential equations are transformed into a set of ordinary differential equations and then solved using the Adomian decomposition method. The effects of the flow parameters on temperature and velocity profiles are shown graphically and discussed [7]

1.1 Mathematical Formulation

Consider the unsteady laminar slip flow of an incompressible, viscous and electrically conducting fluid through a channel with non-uniform wall temperature. The fluid is assumed to be under the influence of an external magnetic field applied across the channel. We choose a Cartesian coordinate system (x’, y’) where x’ lies along the centre of the channel, y’ is the distance measured in the normal section such that y’ = a is the channel’s half width as shown in the figure(1.1).

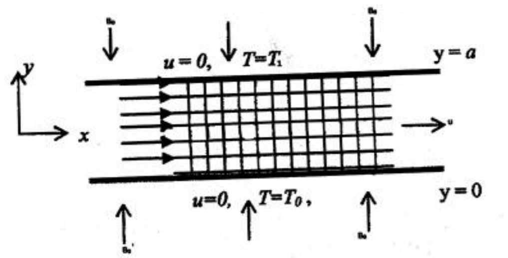

Figure 1: Flow Geometry

Under the usual Bousinesq approximation, the governing equations of the flow are:

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = -\frac{1}{\circ}\frac{dP'}{dx'} + v\frac{\partial^2 u'}{\partial y'^2} - \frac{v}{K}u' - \frac{\pm_e B_0^2}{\circ}u' + g\ \left(T'-T_0\right) \quad (1)$$

$$\frac{\partial T'}{\partial t'} - v_0\frac{\partial T'}{\partial y'} = \frac{k_f}{\circ C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{4\sim^2}{\circ C_p}\left(T-T_0\right) \quad (2)$$

with the boundary conditions;

$$u' = \text{¤}_1\frac{du'}{dy'}, T = T_0\ , \qquad y' = 0 \quad (3)$$

$$u' = 0\ , T = T_1\ , \qquad y' = a \quad (4)$$

Introducing the dimensionless parameters and variables given by

$$(x,y) = \frac{(x',y')}{h}, u = \frac{hu'}{v},, t = \frac{vt'}{h^2}, p = \frac{h^2 p'}{\circ v^2}, Gr = \frac{g\ \left(T_1-T_0\right)h^3}{v^2}, \Pr = \frac{\circ C_p v}{k}, \text{¥} = \frac{\text{¤}_1}{h}$$

$$\text{¯} = \frac{T-T_0}{T_1-T_0}, \text{¢} = \frac{4\sim^2 h^2}{\circ C_p v}, s = \frac{v_0}{v}, H^2 = \frac{\pm_e B_0^2 h^2}{\circ v}, Da = \frac{K}{h^2}\ s^2 = \frac{1}{Da}, \text{ª} = -\frac{dP}{dx}.$$

we obtain the dimensionless equation

$$\mathrm{Re}\left(\frac{\partial u}{\partial t} + s\frac{\partial u}{\partial y}\right) = \text{ª} + \frac{\partial^2 u}{\partial y^2} - \left(H^2 + s^2\right)u + Gr\text{¯}\ , \quad (5)$$

$$Pe\left(\frac{\partial \text{¯}}{\partial t} + s\frac{\partial \text{¯}}{\partial y}\right) = \frac{\partial^2 \text{¯}}{\partial y^2} + N^2\text{¯}\ , \quad (6)$$

with the appropriate boundary conditions

$$u = \text{¥}\frac{du}{dy}, \text{¯} = 0 \quad and \quad y = 0\ , \quad (7)$$

$$u = 0, \ \text{¯} = 1\ \ and\ \ y = 1 \quad (8)$$

For pulsatile flow, we assume a solution of the form,

$$\text{ª} = \text{ª}_0 + \text{ª}_1 e^{i\acute{}t}\ , u(t,y) = A(y) = B(y)e^{i\acute{}t}\ , \text{¯}(t,y) = E(y) + F(y)e^{i\acute{}t} \quad (9)$$

Substituting (9) in (5) to (8), we obtain the following ordinary differential equations

$$A''(y)+sA'(y)-\left(H^2+\ {}^2\right)A(y)=-GrE(y)-a_0 \tag{10}$$

$$B''(y)+sB'(y)-\left(H^2+\ {}^2+\mathrm{Re}\,i\acute{}\ \right)B(y)=-GrF(y)-a_1 \tag{11}$$

$$E''(y)+sE'(s)+N^2\ E(y)=0 \tag{12}$$

$$F''(y)+sF'(y)-\left(N^2-i\acute{}\,Pe\right)F(y)=0, \tag{13}$$

subject to the following boundary conditions

$$A(0)=¥A'(0),\ A(1)=0, \tag{14}$$

$$B(0)=¥B'(0),\ B(1)=0, \tag{15}$$

$$F(0)=0,\ F(1)=1, \tag{16}$$

$$E(0)=0,\ E(1)=1, \tag{17}$$

Solving (10) – (12) together with (14) – (17), we get

$$E(y)=C_1e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)y}+C_2e^{\frac{1}{2}\left(-s+\sqrt{-4N^2+s^2}\right)y}$$

$$F(y)=C_3e^{\frac{1}{2}y\left(-s-\sqrt{-4N^2+s^2+4iPe\acute{}}\right)y}+C_4e^{\frac{1}{2}y\left(-s+\sqrt{-4N^2+s^2+4iPe\acute{}}\right)}$$

$$A(y)=C_5e^{\frac{1}{2}y\left(-s-\sqrt{4H^2+s^2+4\ {}^2}\right)}+C_6e^{\frac{1}{2}y\left(-s+\sqrt{4H^2+s^2+4\ {}^2}\right)}+\left(4e^{-\frac{1}{2}\left(s+\sqrt{4N^2+s^2}\right)(-1+y)}\ \left(-GrH^2\right.\right.$$

$$+e^{\sqrt{-4N^2+s^2y}}\ GrH^2-Gr\ {}^2+e^{-\sqrt{4N^2+s^2y}}\ Gr\ {}^2$$

$$+e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\left(s+\sqrt{-4N^2+s^2}\right)(-1+y)}\ H^2a_0-e^{\frac{1}{2}\left(s+\sqrt{-4N^2+s^2}\right)(-1+y)}\ H^2a_0$$

$$+e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\left(s+\sqrt{-4N^2+s^2}\right)(-1+y)}\ N^2a_0-e^{\frac{1}{2}\left(s+\sqrt{-4N^2+s^2}\right)(-1+y)}\ N^2a_0$$

$$+e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\left(s+\sqrt{-4N^2+s^2}\right)(-1+y)}\ {}^2a_0-e^{\frac{1}{2}\left(s+\sqrt{-4N^2+s^2}\right)(-1+y)}\ {}^2a_0\left.\right)\left.\right)/\left(\left(-1\right.\right.$$

$$\left.+e^{\sqrt{-4N^2+s^2}}\left(H^2+N^2+\ {}^2\right)\left(-s+\sqrt{4H^2+s^2+4\ {}^2}\right)\left(s+\sqrt{4H^2+s^2+4\ {}^2}\right)\right)$$

$$B(y)=C_7e^{\frac{1}{2}y\left(-s-\sqrt{4H^2+s^2+4\ {}^2+4iR\mu}\right)}+C_8e^{\frac{1}{2}y\left(-s+\sqrt{4H^2+s^2+4\ {}^2+4iR\mu}\right)}$$

$$+\left(4e^{-\frac{1}{2}(-1+y)\left(s+\sqrt{4N^2+s^2+4iPe\mu}\right)}\ \left(-GrH^2+e^{y\sqrt{-4N^2+s^2+41Pe\mu}}\ GrH^2-Gr\ {}^2\right.\right.$$

$$+e^{y\sqrt{-4N^2+s^2+4iPe\mu}}\ Gr\ {}^2-iGrR\mu+ie^{y\sqrt{-4N^2+s^2+4iPe\mu}}\ GrR\mu$$

$$-e^{\frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ H^2a_1$$

$$+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ H^2a_1$$

$$-e^{\frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ N^2a_1$$

$$+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ N^2a_1$$

$$-e^{\frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ {}^2a_1$$

$$+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ {}^2a_1$$

$$
\begin{aligned}
&+ie^{\frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ Pe\mu^{a}{}_{1} \\
&-ie^{\sqrt{-4N^2+s^2+4iPe\mu}\ \frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ Pe\mu^{a}{}_{1} \\
&-ie^{\frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ R\mu^{a}{}_{1} \\
&+ie^{\sqrt{-4N^2+s^2+41Pe\mu}+\frac{1}{2}(-1+y)\left(s+\sqrt{-4N^2+s^2+41Pe\mu}\right)}\ R\mu^{a}{}_{1}))/((-1 \\
&+e^{\sqrt{-4N^2+s^2+41Pe\mu}\,)\left(H^2+N^2+\ \ {}^2-iPe\mu+iR\mu\right)-(-s} \\
&+\sqrt{-4H^2+s^2+4\ \ {}^2+41R\mu}\,\Big)\Big(s+\sqrt{4H^2+s^2+4\ \ {}^2+41R\mu}\,\Big)\Big)
\end{aligned}
$$

1.2 Results and Discussion

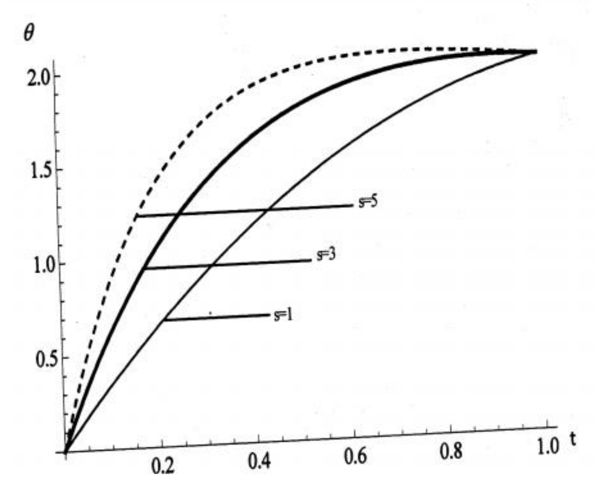

Figure 2: Effect of the suction/injection parameters S on temperature θ.

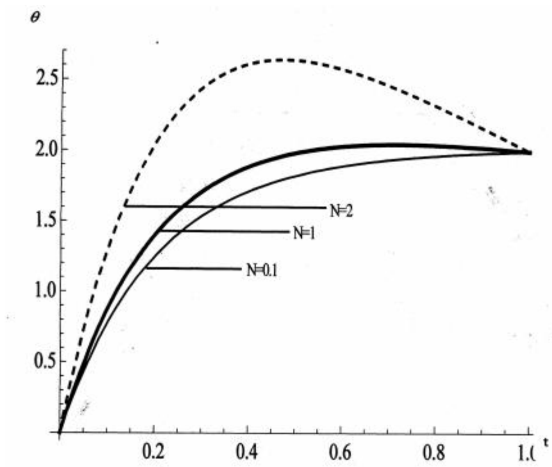

Figure 3: Effect of Thermal radiation parameter N on the temperature θ

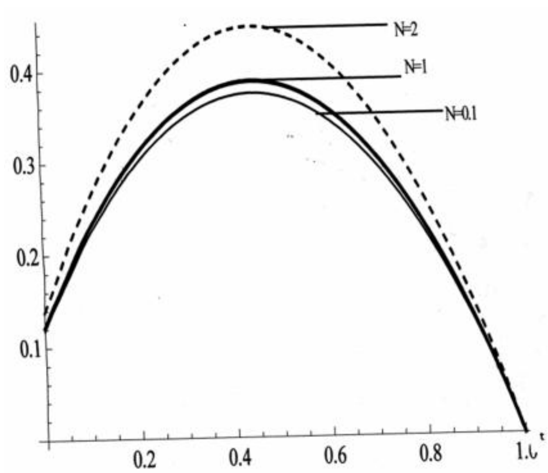

Figure 4: Effect of Thermal radiation N on velocity U

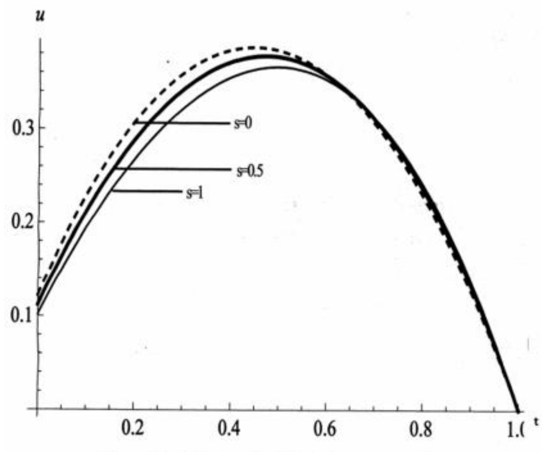

Figure 5: Effect of suction/injection parameter S on velocity U

From Figure 2, it is observed that the suction/injection parameter S increases as the temperature increases. This means that Vo the suction/injection term increases.

From Figure 3, it is observed. that the heat absorption is more toward the lower plate/wall than the upper plate as N increases.

Figure 4, shows the oscillatory behaviour of u with t with the value of N observed to be increasing as velocity increases.

From Figure 5, the graph of u vs t shows the oscillatory behaviour. As s decreases, u increases. This means that the suction/injection term also decreases.

The goal of this paper is to investigate the effect of wall slip on the pulsatile flow through a channel with non-uniform wall temperature that is filled with a porous medium. Exact solutions of the velocity and temperature fields are obtained. There is a good agreement between the limiting case of the present result and the purely oscillatory case obtained by Makinde and Mhone [1] when σ = 0 = γ and that obtained by Mahmood and Ali [2] when σ = 0. However, for the pulsatile case our result showed that, Nusselt number increases at the cold plate but decreases at the heated plate with an Peclet number. Increasing heated wall slip causes a flow reversal towards the heated plate while both slip parameters encourages skin friction at both plates.

Appendix

$$C_1 = \frac{1}{e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)} - e^{\frac{1}{2}\left(-s+\sqrt{-4N^2+s^2}\right)}}$$

$$C_2 = \frac{1}{e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)} - e^{\frac{1}{2}\left(-s+\sqrt{-4N^2+s^2}\right)}}$$

$$C_3 = \frac{1}{e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)} - e^{\frac{1}{2}\left(-s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}}$$

$$C_4 = \frac{1}{e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)} - e^{\frac{1}{2}\left(-s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}}$$

$$\begin{aligned}
C_5 = -\Big(4\Big(&-2e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\quad ^2}}\, GrH^2 + 2e^{\frac{s}{2}+\sqrt{-4H^2+s^2}+\frac{s}{2}\sqrt{+4H^2+s^2+4\quad ^2}}\, GrH^2 \\
&- 2e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\quad ^2}}\, Gr\quad ^2 + 2e^{\frac{s}{2}+\sqrt{-4H^2+s^2}+\frac{s}{2}\sqrt{+4H^2+s^2+4\quad ^2}}\, Gr\quad ^2 \\
&- e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\quad ^2}}\, GrH^2 s\text{¥} + 2e^{\frac{s}{2}+\sqrt{-4N^2+s^2}+\frac{s}{2}\sqrt{+4H^2+s^2+4\quad ^2}}\, GrH^2 s\text{¥} \\
&+ 2e^{\frac{s}{2}+\frac{1}{2}\sqrt{4N^2+s^2}}\, GrH^2 \sqrt{-4N^2+s^2\text{¥}}\; e^{\frac{s}{2}+\frac{1}{2}\sqrt{+4H^2+s^2+4\quad ^2}} Grs\quad ^2\text{¥} \\
&+ e^{\frac{s}{2}+\sqrt{-4N^2+s^2+4\quad ^2}\frac{1}{2}+\sqrt{4H^2+s^2+4\quad ^2}}\, Grs\quad ^2\text{¥} + 2e^{\frac{s}{2}+\frac{1}{2}\sqrt{-4N^2+s^2}Gr\sqrt{-4N^2+s^2}}\quad ^2\text{¥} \\
&- e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\quad ^2}}\, GrH^2\sqrt{4H^2+s^2+4\quad ^2}\text{¥} \\
&+ e^{\frac{s}{2}+\sqrt{-4N^2+s^2}+\frac{1}{2}\sqrt{4H^2+s^2+4\quad ^2}}\, GrH^2\sqrt{4H^2+s^2+4\quad ^2}\text{¥} \\
&- e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\quad ^2}}\, Gr\quad ^2\sqrt{4H^2+s^2+4\quad ^2}\text{¥}
\end{aligned}$$

$$+e^{\frac{s}{2}+\sqrt{-4N^2+s^2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,Gr\ ^2\sqrt{4H^2+s^2+4\ ^2}¥+2H^2a_0$$
$$-2e^{\sqrt{-4N^2+s^2}}\,H^2a_0-2e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,H^2a_0-2e^{\frac{s}{2}+\sqrt{-4N^2+s^2}-+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,H^2a_0$$
$$+2N^2a_0-2e^{\sqrt{-4N^2+s^2}}\,N^2a_0-2e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,N^2a_0$$
$$+2e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,N^2a_0+2\ ^2a_0-2e^{\sqrt{-4N^2+s^2}}\ ^2a_0$$
$$-2e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\ ^2a_0+2e^{\frac{s}{2}+\sqrt{-4N^2+s^2+4\ ^2}+\frac{s}{2}\sqrt{4H^2+s^2+4\ ^2}}\ ^2a_0$$
$$-e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,H^2s¥a_0+e^{\frac{s}{2}+\sqrt{-4N^2+s^2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,H^2s¥a_0$$
$$-e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,N^2s¥a_0+e^{\frac{s}{2}+\sqrt{-4N^2+s^2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,N^2s¥a_0$$
$$-e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,s\ ^2¥a_0+e^{\frac{s}{2}+\sqrt{-4N^2+s^2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,s\ ^2¥a_0$$
$$-e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,H^2\sqrt{4H^2+s^2+4\ ^2}¥a_0$$
$$+e^{\frac{s}{2}+\sqrt{-4N^2+s^2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,H^2\sqrt{4H^2+s^2+4\ ^2}¥a_0$$
$$-e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,N^2\sqrt{4H^2+s^2+4\ ^2}¥a_0$$
$$+e^{\frac{s}{2}+\sqrt{-4N^2+s^2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\,N^2\sqrt{4H^2+s^2+4\ ^2}¥a_0$$
$$-e^{\frac{s}{2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\ ^2\sqrt{4H^2+s^2+4\ ^2}¥a_0$$
$$+e^{\frac{s}{2}+\sqrt{-4N^2+s^2}+\frac{1}{2}\sqrt{4H^2+s^2+4\ ^2}}\ ^2\sqrt{4H^2+s^2+4\ ^2}¥a_0))/\left(\left(-1+e^{\sqrt{-4N^2+s^2}}\right)(H^2\right.$$
$$\left.+N^2+\ ^2)\left(-s+\sqrt{4H^2+s^2+4\ ^2}\right)\left(s+\sqrt{4H^2+s^2+4\ ^2}\right)(-2\right.$$
$$+2e^{\sqrt{4H^2+s^2+4\ ^2}}-s¥+e^{\sqrt{4H^2+s^2+4\ ^2}}\,s¥+\sqrt{4H^2+s^2+4\ ^2}¥$$
$$\left.+e^{\sqrt{4H^2+s^2+4\ ^2}}\sqrt{4H^2+s^2+4\ ^2}¥)\right)$$

$$C_6=-\frac{4\left(1-\frac{1}{2}\left(-s+\left(\sqrt{4H^2+s^2+4\ ^2}\right)¥\right)\right)\left(-GrH^2+e^{\sqrt{-4N^2+s^2}}\,GrH^2-Gr\ ^2+e^{\sqrt{4H^2+s^2}}\,Gr\ ^2-H^2a_0+e^{\sqrt{-4}}\right.}{\left(-1+e\sqrt{-4N^2+s^2}\right)\left(H^2+N^2+\ ^2\right)\left(-s+\sqrt{4H^2+s^2+4\ ^2}\right)(s}$$
$$+e^{\frac{1}{2}\left(-s+\sqrt{4H^2+s^2+4\ ^2}\right)}\left(\left(4e^{\frac{s}{2}\left(s+\sqrt{-4N^2+s^2}\right)}\left(-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)}\right)H^2a_0+e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)}H^2a_0\right.\right.$$
$$-e^{\frac{1}{2}\left(\sqrt{4H^2+s^2}\right)}\,N^2a_0+e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)}\,N^2a_0+e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)}\ ^2a_0$$
$$+e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\left(-s-\sqrt{4N^2+s^2}\right)}\ ^2a_0))/\left(\left(-1+\ e^{\sqrt{-4N^2+s^2¥}}\right)\left(H^2+N^2\ ^2\right)(-s\right.$$
$$\left.+\sqrt{4H^2+s^2+4\ ^2}\right)\left(s+\sqrt{4H^2+s^2+4\ ^2}\right)\Big)-\left(2e^{\frac{1}{2}\left(s+\sqrt{-4N^2+s^2+4\ ^2}\right)}\right)(-s$$
$$\left.-\sqrt{-4H^2+s^2}\right)¥\left(-e^{\frac{1}{2}\left(-s-\sqrt{4H^2+s^2+4\ ^2}\right)}\,H^2a_0\,e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)}\,H^2a_0\right.$$
$$-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)}\,N^2a_0+e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)}\,N^2a_0+e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2}\right)}\ ^2a_0$$
$$+e^{\sqrt{-4N^2+s^2}+\frac{1}{2}\left(-s-\sqrt{4N^2+s^2}\right)}\ ^2a_0))/\left(\left(-1+\ e^{\sqrt{-4N^2+s^2¥}}\right)\left(H^2+N^2\ ^2\right)(-s\right.$$

$$
+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}}\Big)\Big(s+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}}\Big)\Big)\Big)-\Big(4e^{\frac{1}{2}\left(s+\sqrt{4N^{2}+s^{2}}\right)}\yen\ \Big(GrH^{2}\sqrt{-4N^{2}+s^{2}}
$$

$$
+Gr\sqrt{-4N^{2}+s^{2}+\ \ ^{2}}-\tfrac{1}{2}e^{\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}}\right)}H^{2}\Big(s+\sqrt{-4N^{2}+s^{2}}\Big)\alpha_{0}
$$

$$
+\tfrac{1}{2}e^{\sqrt{-4N^{2}+s^{2}}+\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}}\right)}H^{2}\Big(s+\sqrt{-4N^{2}+s^{2}}\Big)\alpha_{0}+\tfrac{1}{2}e^{\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}}\right)}N^{2}\Big(s
$$

$$
+\sqrt{-4N^{2}+s^{2}}\Big)\alpha_{0}+\tfrac{1}{2}e^{\sqrt{-4N^{2}+s^{2}}+\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}}\right)}N^{2}\Big(s+\sqrt{-4N^{2}+s^{2}}\Big)\alpha_{0}+\tfrac{1}{2}e^{\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}}\right)}\Big(s
$$

$$
+\sqrt{-4N^{2}+s^{2}}\Big)\ ^{2}\alpha_{0}+\tfrac{1}{2}e^{\sqrt{-4N^{2}+s^{2}}+\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}}\right)}\Big(s+\sqrt{-4N^{2}+s^{2}}\Big)\ ^{2}\alpha_{0}\Big)\Big)/\Big(\Big(-1
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}}}\Big)\Big(H^{2}+N^{2}+\ \ ^{2}\Big)\Big(-s+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}}\Big)\Big(s
$$

$$
+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}}\Big)\Big)\Big)\Big)\Big/\Big(e^{\frac{1}{2}\left(-s+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}}\right)}\Big(1-\tfrac{1}{2}\Big(-s-\sqrt{4H^{2}+s^{2}+4\ \ ^{2}}\Big)\yen\Big)
$$

$$
-e^{\frac{1}{2}\left(-s-\sqrt{4H^{2}+s^{2}+4\ \ ^{2}}\right)}\Big(1-\tfrac{1}{2}\Big(-s+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}}\Big)\yen\Big)\Big)
$$

$$
C_{7}=-\Big(4e^{\frac{1}{2}\left(+s-\left(\sqrt{4H^{2}+s^{2}+4\ \ ^{2}+4iR\mu}\right)\right)}\Big)\Big(-GrH^{2}+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}GrH^{2}-Gr\ \ ^{2}
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}Gr\ \ ^{2}-iGrR\mu+ie^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}GrR\mu-H^{2}\alpha_{1}
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}H^{2}\alpha_{1}-N^{2}\alpha_{1}+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}N^{2}\alpha_{1}-\ \ ^{2}\alpha_{1}
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}\ \ ^{2}\alpha_{1}+iPe\mu\alpha_{1}-ie^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}Pe\mu\alpha_{1}-iR\mu\alpha_{1}
$$

$$
+ie^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}R\mu\alpha_{1}\Big)\Big)/\Big(\Big(-1+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}\Big(H^{2}+N^{2}+\ \ ^{2}-iPe\mu
$$

$$
+iR\mu\Big)\Big(-s+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}+4iR\mu}\Big)\Big(s+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}+4iR\mu}\Big)\Big)
$$

$$
+\Big(e^{\frac{1}{2}\left(-s-\sqrt{-4H^{2}+s^{2}+4\ \ ^{2}+4iR\mu}\right)+\frac{1}{2}\left(s-\sqrt{+4N^{2}+s^{2}+4\ \ ^{2}+4iR\mu}\right)}\Big(-\Big(4\Big(1-\tfrac{1}{2}\yen\Big(-s
$$

$$
+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}+4iR\mu}\Big)\Big)\Big(-GrH^{2}+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}GrH^{2}-Gr\ \ ^{2}
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}Gr\ \ ^{2}-iGrR\mu+ie^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}GrR\mu-H^{2}\alpha_{1}
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}H^{2}\alpha_{1}-N^{2}\alpha_{1}+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}N^{2}\alpha_{1}-\ \ ^{2}\alpha_{1}
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}\ \ ^{2}\alpha_{1}+iPe\mu\alpha_{1}-ie^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}Pe\mu\alpha_{1}-iR\mu\alpha_{1}
$$

$$
+ie^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}R\mu\alpha_{1}\Big)\Big)/\Big(\Big(-1+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}}\Big(H^{2}+N^{2}+\ \ ^{2}-iPe\mu
$$

$$
+iR\mu\Big)\Big(-s+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}+4iR\mu}\Big)\Big(s+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}+4iR\mu}\Big)\Big)
$$

$$
+e^{\frac{1}{2}\left(-s+\sqrt{4H^{2}+s^{2}+4\ \ ^{2}+4iPe\mu}\right)}\Big(\Big(\Big(4e^{\frac{1}{2}\left(s+\sqrt{-4N^{2}+s^{2}+4iPe\mu}\right)}\Big(-e^{\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}+4iPe\mu}\right)}H^{2}\alpha_{1}
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}+4iPe\mu}\right)}H^{2}\alpha_{1}-e^{\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}+4iPe\mu}\right)}N^{2}\alpha_{1}
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}+4iPe\mu}\right)}N^{2}\alpha_{1}-e^{\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}+4iPe\mu}\right)}\ \ ^{2}\alpha_{1}
$$

$$
+e^{\sqrt{-4N^{2}+s^{2}+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}+4iPe\mu}\right)}\ \ ^{2}\alpha_{1}+ie^{\frac{1}{2}\left(-s-\sqrt{-4N^{2}+s^{2}+4iPe\mu}\right)}Pe\mu\alpha_{1}
$$

$$-ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ Pe\mu a_1-ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ R\mu a_1$$
$$+ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ R\mu a_1))/((-1+e^{\sqrt{-4N^2+s^2+4iPe\mu}}\ (H^2$$
$$+N^2+\quad{}^2-iPe\mu+iR\mu)\left(-s+\sqrt{4H^2+s^2+4\quad{}^2+4iR\mu}\right)(s$$
$$+\sqrt{-4N^2+s^2+4\quad{}^2+4iPe\mu}))-\left(2e^{\frac{1}{2}\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ ¥(s\right.$$
$$-\sqrt{-4N^2+s^2+4iPe\mu})\left(-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ H^2a_1\right.$$
$$+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ H^2a_1-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ N^2a_1$$
$$+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ N^2a_1-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\quad{}^2a_1$$
$$+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\quad{}^2a_1+ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ Pe\mu a_1$$
$$-ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ Pe\mu a_1-ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ R\mu a_1$$
$$+ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ R\mu a_1))/((-1+e^{\sqrt{-4N^2+s^2+4iPe\mu}}\ (H^2$$
$$+N^2+\quad{}^2-iPe\mu+iR\mu)\left(-s+\sqrt{4H^2+s^2+4\quad{}^2+4iR\mu}\right)(s$$
$$+\sqrt{4H^2+N^2+\quad{}^2+4iPe\mu}))$$
$$-\left(4e^{\frac{1}{2}\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}¥\ (GrH^2\sqrt{-4N^2+s^2+4iPe\mu}\right.$$
$$+Gr\quad{}^2\sqrt{-4N^2+s^2+4iPe\mu}+iGrR\mu\sqrt{-4N^2+s^2+4iPe\mu}$$
$$-\frac{1}{2}e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ H^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1$$
$$+\frac{1}{2}e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ H^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1$$
$$-\frac{1}{2}e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ N^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1$$
$$+\frac{1}{2}e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ N^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1$$
$$-\frac{1}{2}e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\quad{}^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1$$
$$+\frac{1}{2}e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\quad{}^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1$$
$$-\frac{1}{2}ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ Pe\mu\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1$$
$$+\frac{1}{2}ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ Pe\mu\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1$$
$$-\frac{1}{2}ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ R\mu\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1$$
$$+\frac{1}{2}ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ R\mu(s$$

$$
\begin{aligned}
&+\sqrt{-4N^2+s^2+4iPe\mu}\Big)a_1))/((-1+e^{\sqrt{-4N^2+s^2+4iPe\mu}}\left(H^2+N^2+\ \ ^2-iPe\mu\right.\\
&\left.+iR\mu\right)\left(-s+\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\right)\Big(s\\
&+\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\Big)\Big)\Big)\Big)\Big)\Big/\Bigg(e^{\frac{1}{2}\left(-s+\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\right)}\left(1-\frac{1}{2}\yen\,\Big(-s\right.\\
&-\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\Big)\Big)\Big)\Big/e^{\frac{1}{2}\left(-s-\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\right)}\left(1-\frac{1}{2}\yen\,\Big(-s\right.\\
&+\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\Big)\Big)\Big)
\end{aligned}
$$

$$
\begin{aligned}
C_8=-\Bigg(-\Bigg(4\Bigg(1-\frac{1}{2}\yen\,\Big(-s+\Big(\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\Big)\Big)\Big)\Big(-GrH^2+e^{\sqrt{-4N^2+s^2+4\ \ ^2+4iR\mu}}\,GrH^2-Gr\ \ ^2\\
&+e^{\sqrt{-4N^2+s^2+4\ \ ^2+4iPe\mu}}\,Gr\ \ ^2-iGrR\mu+ie^{\sqrt{-4N^2+s^2+4\ \ ^2+4iPe\mu}}\,GrR\mu-H^2a_1\\
&+e^{\sqrt{-4N^2+s^2+4\ \ ^2+4iPe\mu}}\,H^2a_1-N^2a_1+e^{\sqrt{-4N^2+s^2+4\ \ ^2+4iPe\mu}}\,N^2a_1-\ \ ^2a_1\\
&+e^{\sqrt{-4N^2+s^2+4\ \ ^2+4iPe\mu}}\ \ ^2a_1+iPe\mu a_1-ie^{\sqrt{-4N^2+s^2+4\ \ ^2+4iPe\mu}}\,Pe\mu a_1-iR\mu a_1\\
&+ie^{\sqrt{-4N^2+s^2+4\ \ ^2+4iPe\mu}}\,R\mu a_1))/((-1+e^{\sqrt{-4N^2+s^2+4\ \ ^2+4iPe\mu}}\left(H^2+N^2+\ \ ^2-iPe\mu\right.\\
&\left.+iR\mu\right)\left(-s+\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\right)\left(s+\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\right)\Big)\\
&+e^{\frac{1}{2}\left(-s+\sqrt{-4N^2+s^2+4\ \ ^2+4iR\mu}\right)}\Bigg(\Bigg(\Bigg(4e^{+\frac{1}{2}\left(s-\sqrt{-4N^2+s^2+4\ \ ^2+4iR\mu}\right)}\Bigg(-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4\ \ ^2+4iR\mu}\right)}H^2a_1\\
&+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}H^2a_1-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}N^2a_1\\
&+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}N^2a_1-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ \ ^2a_1\\
&+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ \ ^2a_1+ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}Pe\mu a_1\\
&-ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}Pe\mu a_1+ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}R\mu a_1\\
&+ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}R\mu a_1))/((-1+e^{\sqrt{-4N^2+s^2+4iPe\mu}}\Big(H^2\\
&+N^2+\ \ ^2-iPe\mu+iR\mu\Big)\left(-s+\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\right)\Big(s\\
&+\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\Big)\Big)-\Bigg(2e^{\frac{1}{2}\left(s+\sqrt{-4N^2+s^2+4\ \ ^2+4iPe\mu}\right)}\yen\,\Big(-s\\
&-\sqrt{-4N^2+s^2+4iPe\mu}\Big)\Bigg(-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}H^2a_1\\
&+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}H^2a_1-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}N^2a_1\\
&+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}N^2a_1-e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ \ ^2a_1\\
&+e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ \ ^2a_1+ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}Pe\mu a_1\\
&-ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}Pe\mu a_1-ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}R\mu a_1\\
&+ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}R\mu a_1))/((-1+e^{\sqrt{-4N^2+s^2+4iPe\mu}}\Big(H^2\\
&+N^2+\ \ ^2-iPe\mu+iR\mu\Big)\left(-s+\sqrt{4H^2+s^2+4\ \ ^2+4iR\mu}\right)\Big(s
\end{aligned}
$$

$$
\begin{aligned}
&+\sqrt{4H^2+s^2+\ ^2+4iR\mu}\Big)\Big)\\
&-\Big(4e^{\frac{1}{2}\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)}¥\ \big(GrH^2\sqrt{-4N^2+s^2+4iPe\mu}\\
&+Gr\ ^2\sqrt{-4N^2+s^2+4iPe\mu}+iGrR\mu\sqrt{-4N^2+s^2+4iPe\mu}\\
&-\frac{1}{2}e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}H^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1\\
&+\frac{1}{2}e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}H^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1\\
&-\frac{1}{2}e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}N^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1\\
&+\frac{1}{2}e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}N^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1\\
&-\frac{1}{2}e^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ ^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1\\
&-\frac{1}{2}e^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}\ ^2\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1\\
&-\frac{1}{2}ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}Pe\mu\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1\\
&+\frac{1}{2}ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}Pe\mu\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1\\
&-\frac{1}{2}ie^{\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}R\mu\left(s+\sqrt{-4N^2+s^2+4iPe\mu}\right)a_1\\
&+\frac{1}{2}ie^{\sqrt{-4N^2+s^2+4iPe\mu}+\frac{1}{2}\left(-s-\sqrt{-4N^2+s^2+4iPe\mu}\right)}R\mu\big(s\\
&+\sqrt{-4N^2+s^2+4iPe\mu}\big)a_1\big)\big)/\Big(\Big(-1+e^{\sqrt{-4N^2+s^2+4iPe\mu}}\big(H^2+N^2+\ ^2-iPe\mu\\
&+iR\mu\big)\left(-s+\sqrt{4H^2+s^2+4\ ^2+4iR\mu}\right)\big(s\\
&+\sqrt{4H^2+s^2+4\ ^2+4iR\mu}\big)\Big)\Big)\Big)\Big/\Big(e^{\frac{1}{2}\left(-s+\sqrt{4H^2+s^2+4\ ^2+4iR\mu}\right)}\Big(1-\frac{1}{2}¥\ \big(-s\\
&-\sqrt{4H^2+s^2+4\ ^2+4iR\mu}\big)\Big)\Big)\Big/e^{\frac{1}{2}\left(-s-\sqrt{4H^2+s^2+4\ ^2+4iR\mu}\right)}\Big(1-\frac{1}{2}¥\ \big(-s\\
&+\sqrt{4H^2+s^2+4\ ^2+4iR\mu}\big)\Big)\Big)
\end{aligned}
$$

References

[1] O. D. Makinde (2OO Asymptotic approximations for oscillatory flow in a tube of varying cross-section with permeable isotherm wall, Rom. Journ. Phys., Vol. 52, Nos. 1-2, pp. 61-72, Bucharest.

[2] Basant K. Jha, Abiodun O. Ajibade (2012): Effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary conditions, Commun. Non-linear Sci Numer. Simulat. 17: pp. 1576-1587

[3] Samuel O. Adesanya (2012): Steady Magneto-hydrodynamic Visco-Elastjc Heat generating/absorbing Slip Flow with Thermal Radiation through a Porous Medium, International Journal of Heat and Technology, vol. 30, No 1, pp 69 — 74.

[4] Olumide Adesanya and Jacob A. Gbadeyan (2010): Adomian Decomposition Approach to Steady Visco-elastic Fluid Flow with Slip through a Planer channel, International Journal of Nonlinear Science, Vol. 9 No. 2, pp. 86—94

[5] O. D. Makinde and P. Y. Mhone (2005): Heat transfer to MHD oscillatory flow in a channel filled with porous medium, Rom. Journ. Phys., Vol. 50, Nos. 9-10, pp. 931 — 938, Bucharest.

[6] M. Subhas Abel, N. Mahesha (2008): Heat transfer in MHD Visco-elastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation, Applied Mathematical Modelling 32, 1965 — 1983..

[7] O. D. Makinde and R. J. Moitsheki (2008): Non-perturbative techniques for Thermal Radiation Effect on Natural Convection past a Vertical Plate Embedded in a Saturated Porous Medium, Mathematical Problems in Engineering Vol. 13 Article ID 689074, pp.107 17.