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Computer Implementation of the Two-Factor DP Model for Manpower Planning

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Abstract

In this paper, a two-factor dynamic programming (DP) model for manpower planning is presented in linear programming (LP) form. It has been observed that practical problems formulated as usually have too many variables and constraints which make their manual solutions cumbersome. The manpower problem presented in this paper has many constraints and variables but is highly sparse having very few non zero coefficients in its matrix. A computer program known as Program Simplex which takes advantage of this sparseness has been applied to obtain an optimal solution to the manpower planning problem presented. It has also been observed that LP models with few nonzero coefficients can easily be solved by using a computer to obtain an optimal solution.

Keywords: Dynamic programming, manpower, wastage, recruitment

1.0 Introduction

Many real life practical problems formulated as linear programming (LP) problem are usually large in size in terms of the number of variables and constraints involved. The solutions are consequently often obtained using digital electronic computers. The first successful solution of a LP problem on a high-speed electronic computer was in January 1952, on the National Bureau of Standards SEAC machine, [1]. Since that time, the simplex algorithm which was developed by George B. Dantzig in 1947 has been coded for most intermediate and large general-purpose electronic computers. As the computer revolution progresses, it has been observed that certain characteristics (structures) of some practical LP problems can be exploited to enhance computer solution approach. One of such features in LP model is contained in [2] in which a block-structured LP model was implemented using a computer.

LP models that are subsequently transformed into dynamic models for manpower planning have been observed to have sparse feature which can be incorporated into computer solution. A LP problem is said to be sparse if it contains very few non-zero elements in its matrix coefficients as contained in [3]. In fact sparseness is one of the major characteristics of practical problems which can be exploited in seeking computer solution to models of such problems. Many researchers have exploited sparseness to solve several LP problems. For example, sparse nonnegative solution of underdetermined linear equations by LP is reported in [4]. Sparse approximation is discussed in [5] and [6]. As remarked in [7], when LP models are applied to real life problems they often result in sparse tableaux.In some cases, a thousand-constraint model may have just 1% non-zero coefficients.

The form a problem may be presented matters a lot to the computer in terms of ease with which to solve such a problem. As contained in [1], large-sized LP problems having many constraints and variables with few zero elements have been found to be difficult for the computer to solve. So sparseness is a property that enhances the mathematical solution of LP problems when using computers.

A practical manpower planning problem formulated as DP model usually have too many constraints, variables and could take up to over twenty tableaux. This makes manual solution cumbersome. Hence thereis the need to critically examine the possibility of its computer implementation for fast and accurate solutions that can enhance the sensitivity analysis of the manpower planning problem.

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The two-factor DP model presented in this paper is based on recruitment and wastage factors. The inclusion of these two-cost factors makes the model an extension of the model in [8].As stated in [9], that 195 papers published between 1952 and 1974 (in leading international journals) applied LP to 12 major areas including agriculture, industries, military, economic analysis etc. Since then scores of new application areas of LP model including manpower planning are being added annually thereby making LP one of the most applicable techniques of Operations Research.

2.0 The Two-Factor DP Manpower Planning Model

The following are the mathematical notations associated with the model formulation.

$x_j =$ Number of staff that are onwastage in period j.

$y_j =$ Number of staff that are recruited in period j.

$c_j =$ Average accrued revenue to the organization from each wastage staff in period j by virtue of their exit from the system.

$c'_j =$ Average salary per recruited staff in period j.

$h =$ Initial number of staff on ground in the organization at the beginning of the time horizon.

$H =$ total number of staff at the end of the time horizon under consideration.

The problem of the manpower planning is to maximize the periodic additional revenue accruable to the organization from the wastage staff Wage bill less the periodic salary of recruited staff i.e. $\sum_{j=1}^{n}\left(c_j x_j - c'_j y_j\right)$.

The objective function can be written as:

$$Maximize \;\; z = \sum_{j=1}^{n}\left(c_j x_j - c'_j y_j\right) \tag{1}$$

There are two sets of staffing constraints and two sets of non-negativity constraints in this manpower planning problem.

(i) The overstaffing constraints:

The constraints of overstaffing state that the total number of overstaffing staff of the first i periods should not exceed the available vacancies $(H-h)$ in the establishment, i.e.

$$\sum_{j=1}^{i}\left(y_j - x_j\right) = -\sum_{j=1}^{i} x_j + \sum_{j=1}^{i} y_j \le H - h, \quad i = 1(1)n \tag{2}$$

Where $\left(y_j - x_j\right) > 0$ is the number of staff by which the organization is overstaffed in period j

The LHS of equation (2) can also be called the net increase in manpower in the first i periods.

(ii) The understaffing constraints:

The constraints of understaffing represent the number of staff by which the organization is understaffed for the first $(i-1)$ periods plus wastages at period i and this should not exceed h the number of staff originally in the organization. If it does, it means the organization has only material resources which is not the case in practical situation as existence of an organization is based on the contribution of human and material resources. Mathematically this is expressed as:

$$\sum_{j=1}^{i-1}\left(x_j - y_j\right) + x_i = \sum_{j=1}^{i} x_j - \sum_{j=1}^{i-1} y_j \le h, \quad i = 1(1)n \tag{3}$$

Where $\left(x_j - y_j\right) > 0$ is the number of staff by which the organization is understaffed in period j

The L.H.S of equations (3) can also be called the net increase in manpower subtracted from wastage staff in the first $(i-1)$ periods plus the wastage manpower in period i.

Note that the second summation in equation (3) does not exist for $i = 1$.

(iii) Nonnegativity constraints: The nonnegativity constraints are

$$x_j,\; y_j \ge 0, \quad j = 1(1)n \tag{4}$$

Equation (1) stated above constitutes the total manpower planning cost from all the n periods while equations (1)-(4) constitute a DP problem which is stated thus:

Primal DP Problem

$$\left.\begin{aligned} &Maximize \quad z=\sum_{j=1}^{n}\left(c_j x_j - c'_j y_j\right)\\ &s.t.\\ &\quad -\sum_{j=1}^{i} x_j + \sum_{j=1}^{i} y_j \le H-h, \quad i=1(1)n\\ &and \quad \sum_{j=1}^{i} x_j - \sum_{j=1}^{i-1} y_j \le h, \quad i=1(1)n\\ &\quad x_j\,,\ y_j \ge 0\ ,\quad j=1(1)n \end{aligned}\right\} \qquad (5)$$

The system (5) is the DP model of the manpower planning problem which makes use of both recruitment and wastage factors. The DP model in system (5) has 2n linear constraints, 2n nonnegativity constraints in 2n variables.
Further simplification of (5) yields the system in (6).

$$\left.\begin{aligned} &Max \ \ z = c_1x_1 + c_2x_2 + \ldots + c_nx_n - c'_1y_1 - c'_2y_2 - \ldots - c'_ny_n\\ &s.t.\\ &-x_1 \qquad\qquad\qquad +y_1 \qquad\qquad\qquad \le H-h\\ &-x_1-x_2 \qquad\qquad +y_1+y_2 \qquad\qquad \le H-h\\ &-x_1-x_2-x_3 \qquad +y_1+y_2+y_3 \qquad \le H-h\\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ &-x_1-x_2-x_3-\ldots-x_n+y_1+y_2+y_3+\ldots+y_n \le H-h\\ &x_1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad \le h\\ &x_1+x_2 \qquad\qquad -y_1 \qquad\qquad\qquad \le h\\ &x_1+x_2+x_3 \qquad -y_1-y_2 \qquad\qquad \le h\\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ &x_1+x_2+x_3+\ldots+x_n-y_1-y_2-y_3-\ldots-y_{n-1} \ \le \ h\\ &x_j,\ y_j \ge 0, \ \ j=1(1)n \end{aligned}\right\} \qquad (6)$$

The matrix skeleton of the system (6) is shown in Fig. 1

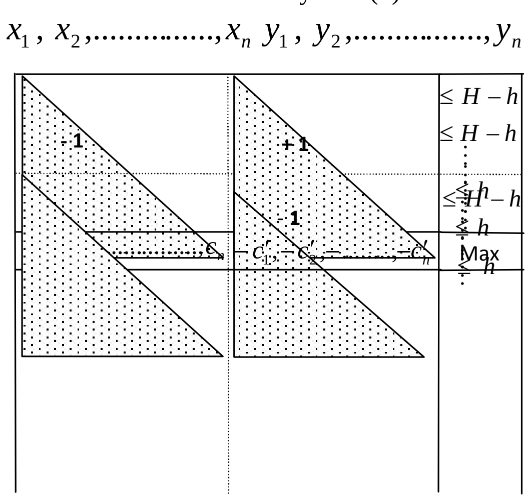

Fig. 1: Matrix skeleton of the primal DP model coefficient arrays.
The matrix array consists of triangular blocks, which are typical of dynamic situations. The coefficients within the blocks are either $+1 \ or \ -1$ depending on the block. The triangular block in the lower right-hand corner is smaller by one row and one column than the other three.
The DP model of this section for manpower planning based on recruitment and wastage factors is sparse and consequently has the advantage of less computational time when using the computer.

For the illustration of computer implementation solution to the two-factor DP model presented in section 2, we obtain data of junior staff monthly salaries from XYZ College of Education in Nigeria.

3.0 Numerical Illustration

The data in Table 1 show (a) the average monthly salary (c_j) of junior staff on wastage for the year 2001 to 2012 and (b) the average monthly salary (c'_j) of recruited junior staff for the year 2001 to 2012.

Table 1: Average monthly salary for **junior staff** on wastage and recruitment

Year	2001 (1)	2002 (2)	2003 (3)	2004 (4)	2005 (5)	2006 (6)	2007 (7)	2008 (8)	2009 (9)	2010 (10)	2011 (11)	2012 (12)
c_j	33286	32045	35770	35918	36637	37552	38437	39126	33065	32281	38084	40124
c'_j	30148	32281	33665	34305	37545	34305	37894	36157	32981	30467	37688	36645

We considered average monthly salary for a period of up to 12 years so that our results can give good estimates of staff wastage (x_j) and recruitment (y_j) when initial number of junior staff (h) and future capacity staff strength (H) are known. In 2012, XYZ College of Education had 162 junior staff. Based on the present salary trend, we want to determine the optimal annual number of staff on wastage and recruitment that will maximize total accruable revenue to the institution in the next 12 years (i.e. by the year 2024) when the junior staff strength is planned to be 393.

Solution by Linear Programming (LP) Approach

The LP model of the manpower planning problem in this section is highly sparse with triangular block structures. The advantage of the sparseness property (in terms of quicker solution) can now be utilized to obtain the computer solution to the LP model of the manpower planning problem. The primal LP model based on wastage and recruitment factors (for junior staff) is given below:

$$Max\ z = 33286x_1 + 32045x_2 + 35770x_3 + 35918x_4 + 36637x_5 + 37552x_6 + 38437x_7 + 39126x_8 + 33065x_9 + 32281x_{10} + 38084x_{11} + 40124x_{12}$$
$$-30148x_{13} - 32281x_{14} - 33665x_{15} - 34305x_{16} - 37545x_{17} - 34305x_{18} - 37894x_{19} - 36157x_{20} - 32981x_{21} - 30467x_{22} - 37688x_{23} - 36645x_{24}$$

$s.t.$

$$-x_1 + x_{13} \le 231$$
$$-x_1 - x_2 + x_{13} + x_{14} \le 231$$
$$-x_1 - x_2 - x_3 + x_{13} + x_{14} + x_{15} \le 231$$
$$-x_1 - x_2 - x_3 - x_4 + x_{13} + x_{14} + x_{15} + x_{16} \le 231$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \le 231$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} \le 231$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \le 231$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} \le 231$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} \le 231$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 - x_{10} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} \le 231$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 - x_{10} - x_{11} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} \le 231$$
$$-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 - x_{10} - x_{11} - x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} + x_{24} \le 231$$
$$x_1 \le 162$$
$$x_1 + x_2 - x_{13} \le 162$$
$$x_1 + x_2 + x_3 - x_{13} - x_{14} \le 162$$
$$x_1 + x_2 + x_3 + x_4 - x_{13} - x_{14} - x_{15} \le 162$$
$$x_1 + x_2 + x_3 + x_4 + x_5 - x_{13} - x_{14} - x_{15} - x_{16} \le 162$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} \le 162$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} \le 162$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} \le 162$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} \le 162$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} - x_{21} \le 162$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} - x_{21} - x_{22} \le 162$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} - x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} - x_{21} - x_{22} - x_{23} \le 162$$
$$x_1, x_2, \cdots, x_{24} \ge 0$$

Solution Process
We use the Program Simplex in PASCAL.
Syntax of Input Data for Program Simplex
How to input Data (Note All constraints are of the type '≤')
m is number of linear constraints (or rows)
n is number of columns i.e. number of variables as in initial tableau
Key in the data as in **initial tableau** after initial computational form in this **order:**

(a) All the entries row after row (including R.H.S. values i.e. b_i)

(b) Objective function coefficients when in the form:

$$z - c_1 x_1 - c_2 x_2 - \cdots - c_k x_k + 0x_{k+1} + 0x_{k+2} + \cdots + 0x_n = 0$$

i.e. key in $-c_1 - c_2 - \cdots - c_k \quad 0 \; 0 \cdots\cdots 0 \quad \underbrace{0 \leftarrow R.H.S}$

m zeros

The m zeros are coefficients of the slack variables.
Note $n = k + m$, where k is the no. of decision variables which are originally present
(c) Subscripts of initial basic variables

$$k+1 \quad k+2 \quad k\underbrace{+3 \cdots\cdots n}$$

m of them

Proceed to File and Save As, Compile and Run to obtain the following output in Fig.2 which contains the initial and optimal tableau.

InitialTableau

I SIMPLEX METHOD

ITERATION

0

BASE VAR	VALUE	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	X21	X22	X23	X24
X25	231.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X26	231.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X27	231.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X28	231.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X29	231.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X30	231.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
X31	231.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
X32	231.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
X33	231.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00
X34	231.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00
X35	231.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
X36	231.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
X37	162.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X38	162.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X39	162.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X40	162.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X41	162.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X42	162.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

BASE VAR	X25	X26	X27	X28	X29	X30	X31	X32	X33	X34	X35	X36	X37	X38	X39	X40	X41	X42	X43	X44	X45	X46	X47	X48
X25	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X26	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X27	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X28	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X29	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X30	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X31	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X35	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X38	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X41	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00

X43 162.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00

X44 162.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00

X45 162.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00

X46 162.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00

X47 162.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00
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X48 162.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00

z 0.0030148.0032281.0033665.0034305.0037545.0034305.0037894.0036157.0032981.0030467.0037688.0036645.00-33286.00-32045.00-35770.00-35918.00-36637.00-375
39126.00-33065.00-32281.00-38084.00-40124.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

ITERATION 19

Optimal Tableau

BASE VAR VALUE X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X15 X16 X17 X18 X19 X20 X21
X29 X30 X31 X32 X33 X34 X35 X36 X37 X38 X39 X40 X41 X42 X43 X44 X45 X46 X47 X48

X13 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X26 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X15 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X28 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

X17 393.00 0.00 0.00 0.00 0.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X30 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00
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X19 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00
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X20 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00
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X21 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00

X34 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00
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X23 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00
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X24 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00
0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00

X1 162.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

X2 393.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00
0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

X3 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 -1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

X4 393.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

X41 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 -1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

X6 393.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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X7 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 -1.00 1.00 0.00 0.00 0.00 0.00 0.00

X8 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00
X9 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00
X10 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00
X11 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 1.00 0.00
X12 393.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00
z 17669508.00 0.00 0.00 0.00 0.00 908.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1620.00 0.00 719.00 0.00 342.00 0.00 0.00 0.00 5407.00 0.00
3138.00 1384.00 2105.00 2332.00 0.00 3589.00
543.00 2969.00 84.00 7221.00 396.00 3479.00
MINIMUM
AT z= -
17669508.00

From the optimal tableau (19th iteration), the optimal solution based on only the decision variables is given as: $x_1 = 162,\ x_2 = 393,\ x_3 = 0,\ \ x_4 = 393,\ x_6 = 393,\ x_7 = 0,\ x_8 = 393,\ x_9 = 393,\ x_{10} = 393,\ x_{11} = 0,\ x_{12} = 393,$
$y_1 (i.e.\ x_{13}) = 393,$
$y_3 (i.e.\ x_{15}) = 393,\ y_5 (i.e.\ x_{17}) = 393,\ y_7 (i.e.\ x_{19}) = 393,\ y_8 (i.e.\ x_{20}) = 393,\ \ y_9 (i.e.\ x_{21}) = 393,$
$y_{11} (i.e.\ x_{23}) = 393,\ y_{12} (i.e. x_{24}) = 393$ and z = ₦17,669,508. This is also the recruitment/wastage policy **cost** of the manpower planning problem using Program Simplex.

3.0 Conclusion

In this paper we have presented a two-factor DP model for manpower planning. The LP model of the manpower planning problem is highly sparse (i.e its matrix coefficient are predominantly zeros). The sparseness property in term of quick solution has aided the use of computer in obtaining the optimal solution of the manpower planning problem in LP form. The recruitment/wastage policy cost is obtained at the 19th iteration by using program simplex. The total recruitment/wastage cost is found to be N17, 669,508.

4.0 References

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