Modelling Monthly Mental Sickness Cases Using Principal Component Regression Method

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Abstract

This study was carried out to solve the problem of inadequate data information on the monthly mental sickness cases at the Federal Neuropsychiatric Hospital, Kaduna . This research tackled this problem by deriving a model from the data obtained that can be used to predict Monthly Total Observation of mental illness that will enhance effective mental health management, logistic planning and assist in decision making process. The methodology was principal component analysis (PCA) using data obtained from the hospital to estimate regression coefficients and parameters. It was found that the principal component regression model that was derived was good predictive tool. The principal component regression model obtained was okay and this was corroborated by large coefficient of determination (R^2) , predictive power and forecast results.

Keywords: Principal component analysis, mental illness, factor loading, eigenvalue, eigevector regression, forecasting, variance inflation factor

1.0 Introduction

Principal component analysis (PCA) was invented in 1901 by Karl Pearson. It is mostly used as a tool in explanatory data analysis and for making predictive models. Depending on the field of application, it is also named the discrete Karhunen Loeve-Transform (KLT), the Hotelling transform or proper orthogonal decomposition (POD).

PCA is the simplest of the true eigenvector-based multivariate analyses. Often, its operation can be thought of revealing the internal structure of the data in a way which best explains the variance in the data.

Principal component regression model (PCRM) is a regression model that use principal component analysis (PCA) to estimate regression coefficients and parameters. It is a procedure used to overcome problems which arises when explanatory variables are close to being collinear

The use of principal component analysis method in analyzing explanatory data have attracted the attention of several researchers who have employed PCA to estimate the regression coefficients [1-4].

However, some researchers have shown that principal component regression model outperformed all other predictive models because it takes into account not only the errors in both independent variables and dependent variables but also the latent structure from the augmented subspace of independent and dependent variables [5-7].

This research concentrates on the principal component analysis method to obtain the regression coefficients and parameters from the data. The statistical package for social sciences version 15 (SPSS-15), has an inbuilt mechanism for obtaining principal component analyses [12].

1.1 Problem Statement

There has been need for a model that can provide an adequate data information on the monthly sickness cases at Federal Neuro-psychiatric Hospital, Kaduna and also be able to predict monthly total observation of mental illness at the hospital.

1.2 Purpose of the Study

The main purpose of this research is to develop an optimal model that can be used to predict (forecast) monthly total observation of mental illness considering the number of people coming to the hospital for treatment from time to time so that accurate forecast can be made about the number of people that may report sick in any given month. Consequently, the aim of

this research work is to enhance the performance of the Federal Neuro-Psychiatric Hospital through a better demand for service at the Hospital. The model will enhance effective planning and budgeting for efficient operation of the Hospital.

2.0 Methodology

2.1. Hypothesis of the Research

Three hypotheses of the research are summarized as follows:

- H_0 : $S_1 = S_2 = S_3 = \dots = S_{10} = 0$: All the model parameters are not statistically significant. (1) H_1 : $S_1 \neq S_2 \neq S_3 \neq \dots \neq S_{10} \neq 0$: All the model parameters are statistically significant.
- $H_0: ... = 0$: The 's are not auto correlated with the first order scheme. (2) $H_1: ... \neq 0$: The 's are auto correlated with the first order scheme.
- $H_0: b_1 = b_2 = b_3 = \dots = b_{10} = 0$: All the model parameters are not statistically Significant. $H_1: b_1 \neq b_2 \neq b_3 \neq \dots \neq b_{10} \neq 0$: All the model parameters are statistically Significant.

Principal Component Analysis

The aim of the method of principal components is the construction out of a set of variables, x, s (i= 1,2, - --- k) of the new variables (p) called principal components, which are linear combination of the x's:

$$P_{1} = a_{11} x_{1} + a_{12} x_{2} + a_{13} x_{3} + \dots + a_{1k} x_{k}$$

$$P_{2} = a_{21} x_{1} + a_{22} x_{2} + a_{23} x_{3} + \dots + a_{2k} x_{k}$$

$$P_{3} = a_{31} x_{1} + a_{32} x_{2} + a_{33} x_{3} + \dots + a_{3k} x_{k}$$

$$P_{m} = a_{m1} x_{1} + a_{m2} x_{2} + a_{m3} x_{3} + \dots + a_{mk} x_{k}$$
We have retained m (where m = k) for the principal components . m = 10 and k = 10, so that

$$P_{1} = a_{11} z_{1} + a_{12} z_{2} + a_{13} z_{3} + \dots + a_{110} z_{10}$$

$$P_{2} = a_{21} z_{1} + a_{22} z_{2} + a_{23} z_{3} + \dots + a_{210} z_{10}$$

$$P_{10} = a_{101} z_{1} + a_{102} z_{2} + a_{103} z_{3} + \dots + a_{1010} z_{10}$$

$$(2)$$

Given the a^{-s} and the γ 's we transform back from the γ 's to obtain estimates of the b^{-s} , the coefficients of the standardized $\chi^{,s}$ in the original model.

The original model is:

$$Y = b_1 x_1 + b_2 x_2 + b_3 x_3 + \dots + b_{10} x_{10}$$
 The principal component model is

$$\dot{\mathbf{Y}} = \chi_{1} p_{1} + \chi_{2} p_{2} + \chi_{3} p_{3} + \dots + \chi_{10} p_{10} + u \dots (4)$$

Substituting equation (2) into equation (4), we obtained equation (5)

$$Y = \chi_{1}(\hat{a}_{11} Z_{1} + \hat{a}_{21} Z_{2} + \dots + \hat{a}_{101} Z_{10}) + \chi_{2}(\hat{a}_{12} Z_{1} + \hat{a}_{22} Z_{2} + \dots + \hat{a}_{102} Z_{10}) + \dots + \chi_{10}(\hat{a}_{101} Z_{1} + \hat{a}_{102} Z_{2} + \dots + \hat{a}_{1010} Z_{10}) \dots (5)$$

Rearranging equation (5), we obtained equation (6)

$$Y = (\hat{X}_{1} \hat{a}_{11} + \hat{X}_{2} \hat{a}_{12} + \dots + \hat{X}_{10} \hat{a}_{110}) z_{1} + (\hat{X}_{1} \hat{a}_{21} + \hat{X}_{2} \hat{a}_{22} + \dots + \hat{X}_{10} \hat{a}_{210}) z_{2}
+ \dots + (\hat{X}_{1} \hat{a}_{101} + \hat{X}_{2} \hat{a}_{102} + \dots + \hat{X}_{10} \hat{a}_{1010}) z_{10} \dots (6)$$

Setting

$$\hat{X}_{1}\hat{a}_{11} + \hat{X}_{2}\hat{a}_{12} + \hat{X}_{3}\hat{a}_{13} + \dots + \hat{X}_{10}\hat{a}_{110} = \hat{b}_{1}$$

$$\hat{X}_{1}\hat{a}_{21} + \hat{X}_{2}\hat{a}_{22} + \hat{X}_{3}\hat{a}_{23} + \dots + \hat{X}_{10}\hat{a}_{210} = \hat{b}_{2}$$

$$\hat{X}_{1}\hat{a}_{101} + \hat{X}_{2}\hat{a}_{102} + \hat{X}_{3}\hat{a}_{103} + \dots + \hat{X}_{10}\hat{a}_{1010} = \hat{b}_{10}$$
..... (7)

we have the principal components estimates of the original variables, h^{-1}

2.3 Significance of the Study

This section is devoted to the justification of this research as a case study and the rationale for using principal component analysis.

- (1) To identify the percentage of the total variation in the set of X's
- (2) To develop a model from the mental illness data that can be used to make prediction.
- (3) To contribute to the growing literature in the field of forecasting using principal component regression method.

2.4 Assumptions of the Study

The following assumptions have been used for the study [8-11].

- (1) The principal component analysis (PCA) solve the problem of multicollinearity in sample data.
- (2) The model is predicated on the fact that present values can be predicted from the past history.
- (3) The principal component analysis reveal the internal structure of a data.
- (4) As in all forecasting the developed model may not give exact results always.
- (5) The primary and secondary sources of data are accurate.
- (6) The study is worth conducting for problem solving and decision making within the operation research.
- (7) The software and hardware used in this study and research are the appropriate technology.
- (8). This prototype as case study is accurate and appropriate fit.

2.5 Conceptual Framework

This present work used principal component method to estimate the regression coefficients and parameters and used least square method to fit the regression equation. This is in line with the standard procedure in literature [4,8,10]. The statistical package used is the SPSS-15.

2.6 Materials and Methods

The information source was secondary since the data were obtained from the hospital record .

Monthly total observation of patients diagnosed at the Federal Neuro-Psychiatric Hospital (FNPH) Kaduna from the month of January 2000 to the December 2009 is the source of data. The data was divided into two part. The 18,846 patients diagnosed at the hospital from January 2000-December 2007 (totaling 96 months) was used to build the model while 7,366 patients diagnosed at the hospital from January 2008 – December 2009 was used as assumed future data to compare the predicting power accuracy of the model.

v = Monthly Total Observation

Monthly observation on Organic Psychosis

 χ_2 = Monthly observation on Paranoid Schizophrenia

 χ_3 = Monthly observation on Head Trauma

Monthly observation on Seizure Disorder

 χ_5 = Monthly observation on Acute Psychosis Disorder

v = Monthly observation on Schizophrenia

r = Monthly observation on Anxiety Disorder

 χ_8 = Monthly observation on Bipolar Affective Disorder

 χ_9 = Monthly observation on Mania

 x_{10} = Monthly observation on Migraine Headache

3.0 Principal Component Regression Model

The principal component regression analysis (PCA) was used to estimate the regression coefficients and parameters. The least square method was used to fit the regression equation.

$$\hat{X}_{1}\hat{a}_{11} + \hat{X}_{2}\hat{a}_{12} + \hat{X}_{3}\hat{a}_{13} + \dots + \hat{X}_{10}\hat{a}_{110} = \hat{b}_{1}$$

$$\hat{X}_{1}\hat{a}_{101} + \hat{X}_{2}\hat{a}_{102} + \hat{X}_{3}\hat{a}_{103} + \dots + \hat{X}_{10}\hat{a}_{1010} = \hat{b}_{10}$$
(8)

The estimates computations were obtained through the SPSS package: thus

Hence, the required principal component regression model is as follows:

$$\hat{\mathbf{Y}}_{i} = 2.86 + 0.9 \,\chi_{1} + 0.85 \,\chi_{2} + 1.28 \,\chi_{3} + 1.34 \,\chi_{4} \,0.88 \,\chi_{5} + 1.0 \,\chi_{6} + 1.14 \,\chi_{7} + 0.97 \,\chi_{8} + 0.91 \,\chi_{9} + 0.88 \,\chi_{10}$$
(10)

3.1 Pearson Correlation Coefficients Significance of Factor Loadings Test

From SPSS output, more than 90% of the loadings is greater than or equal to \pm 0.19. this confirm that the loadings are very significant at 5 percent level and good enough for the model.

Table 1: Analysis of Variance (ANOVA) for Principal Component Regression Model

	Df	Sum of Squares	Mean Square	F-Ratio	Prob. Level
(Constant)	1	3548551	3548551		
Regression	10	296321	29632.1	1473.3975	0.000000
Error	85	1709.47	20.11141		
Total (Adjusted)	95	298030.5	3137.163		

Table 1 give the output for the test of significance of the regression coefficients and parameters. Since ... < 0.05, we reject H_0 and accept H_1 that all the model parameters are statistically significant.

3.2 Predicting Monthly Total Observation of Mental illness Using Principal Component Regression Model

Consider the principal component regression model as given by:

$$\hat{\mathbf{Y}}_{i} = 2.86 + 0.9 \,\chi_{1} + 0.85 \,\chi_{2} + 1.28 \,\chi_{3} + 1.34 \,\chi_{4} \,0.88 \,\chi_{5} + 1.0 \,\chi_{6} + 1.14 \,\chi_{7} + 0.97 \,\chi_{8} + 0.91 \,\chi_{9} + 0.88 \,\chi_{10}$$

It is pertinent to use the principal component regression model to predict Monthly Total observation of Mental illness for 24 months (2 years). The result obtained is tabulated in Table 2.

Table 2: Principal Component Regression Model Forecasts

Months	Actual Value (Y)	Predicted Value (ŷ)	$(Y - \hat{Y})$	$(Y - \hat{Y})^2$
Jan, 2008	237	240	- 3	9
Feb, 2008	300	310	- 10	100
Mar, 2008	301	308	- 7	49
Apr, 2008	272	278	- 6	36
May, 2008	376	381	- 5	25
Jun, 2008	331	324	7	
Jul, 2008	353	339	14	196
Aug, 2008	331	321	10	100
Sept, 2008	332	330	2	4
Oct, 2008	323	320	3	9
Nov, 2008	309	303	6	36
Dec, 2008	425	413	12	144
Jan, 2009	225	212	13	169
Feb, 2009	216	206	10	100
Mar, 2009	280	272	8	64
Apr, 2009	306	302	4	16
May, 2009	356	352	4	16
Jun, 2009	369	369	0	0
Jul, 2009	308	309	- 1	1
Aug, 2009	301	293	8	64
Sept, 2009	397	391	6	36
Oct, 2009	304	292	12	144
Nov, 2009	267	265	2	4
Dec, 2009	261	275	- 14	196

The mean absolute deviation (MAD) and sum square error (SSE) for the actual and predicted Monthly Total of Mental illness are obtained as follows;

MAD =
$$\sum_{i=1}^{n} |\ell_i| = \frac{1567}{24} = 65.3$$

SSE = $\sum_{i=1}^{n} (Y - \hat{Y})^2 = 1567$

Coefficient of MAD =
$$\underline{MAD} \times 100 = \underline{65.3} \times 100 = 21\%$$

 $\overline{\overline{Y}}$ 311.

3.3 Cattell's Scree Principal Component Extraction Test.

From the Figure 1, all the ten P's are above the point where the resulting curve has some curvature .therefore, all the ten P's are extracted and retained for the regression analysis.

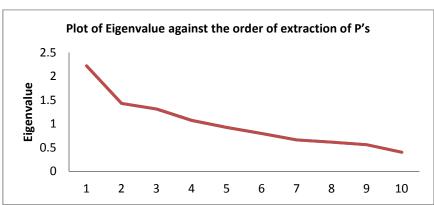


Figure 1: The Plot of Eigenvalue against order of extraction of P's

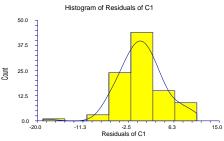


Figure 2: The Histogram of Residual Y

The plot of the Histogram of residuals and the shape of the histogram in Figure 2, showed that the residuals are normally distributed.

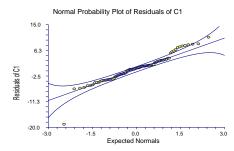


Figure 3: The Regression Normal Probability Plot

The normal probability plot of the errors indicated that by the way the errors are aligned along the straight line in Figure 3,the residuals are normally distributed are further confirmation of the assumption of linear regression.

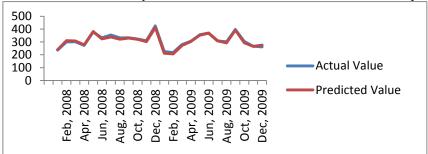


Figure 4: The plot of Forecast using Principal component regression model

The graph in Figure 4 show the variance between the actual values obtained from the hospital and the predicted value by the model for two years (24 months).

Table 3: Principal Component Regression Model Result Analysis

Model	\mathbb{R}^2	MAD	SSE	CMAD (%)
Principal Component Regression	0.9943	65.3	1567	21

R² = Coefficients of Determination MAD = Mean Absolute Deviation SSE = Sum of Square Error

CMAD = Coefficients Mean Absolute Deviation

4.0 Discussion of the Results

From Table 3,the principal component regression model has coefficient of determination of $R^2 = 0.9943$ or simply 99.43%. This implies that the ten predictor variables included in the model can explain at least 99.43% of change in the Monthly Total observation of mental illness which is good enough for the model .In the test run, the model was used to predict the Monthly Total observation of mental illness of twenty four (24) months whose actual monthly total observation were already known . The model's prediction power was good with a mean absolute deviation (MAD) of 65.3. This implies that the model can predict Monthly Total of Mental illness with an average deviation of 65.3. To express it in another form, using the coefficient of mean absolute deviation, with the value of 21%; implies that the model can predict Monthly Total of Mental illness with a percentage deviation of 21% on the average.

5.0 Research Findings

The following findings resulted from this work.

- (1) Principal component regression model has good prediction power accuracy
- (2) Principal component regression model does give better result because it solve the problem of multicollinearity in data.
- (3) Principal component regression model identified the percentage of the total variance in the set of X's
- (4) Graph of residuals gives no definite pattern, an indication of goodness of fit.

Table 4: Monthly Total Observation of Patients Diagnosed (2000 - 2009)

MONTHS	\mathcal{X}_1	\mathcal{X}_2	\mathcal{X}_3	\mathcal{X}_4	\mathcal{X}_5	$\chi_{\scriptscriptstyle 6}$	X_7	\mathcal{X}_8	χ_9	\mathcal{X}_{10}	y_{i}
1	30	20	08		10	20	10	10	10	10	128
2	18	29	10	10	06	18	08		20	20	139
3	10	40	06		08	20	20		10	10	124
4	19	32	09			22	22	16	15		135
5	22	40	02	10	09	32	23	18	16	03	175
6	23	60	10		20	40	24		18	14	209
7	29	70	18	15	02	28	28	20	20	20	230
8	10	40	20		08	29	29		22	22	180
9	10	50	21	10	19	40	30	20	18	10	228
10	20	39	08	10	14	22	42		20	10	185
11	04	60	09			24	50	02	23	15	187
12	05	29	11	10	08	40	29		41	20	193
13	40	18	09			18	40	11	24	20	180
14	10	20	10		03	20	20		20	30	133
15	06	05	08			22			18	41	100
16	11	20	20	10	04	24	03		17	20	129
17	18	10	04	03		33	04		20	33	125
18	10	18	05	04		41	06		29		113
19	20	40	20			42		41	32	40	235
20	04	28	22	10	09	29	10		24	20	154
21	09	32	20		20	40	23	20	28		192
22	40	40	09	06	18	30	21	22	38	22	246
23	18	60	12		14	25	40		24	23	216
24	20	43	14		16	40			29	10	172
25	07	30	20	02	02	20	06	16	32	20	155
26	20	29	11		09	21	07		10	20	127
27	10	16	14	06	20	22	18	18	08	22	154
28	20	18	22		18	25	18	19	19	23	182
29	15	30	09		12	32	18		09	24	150
30	16	08	09		14	40	24		10	28	145
31	20	20	22	02	18	44	29	09	11	29	204
32	20	16	28	10	20	32	10	10	02	14	162
33	30	18	20		10	40	01		04	20	143
34	50	30	13		12	32		20	03	22	182
35	52	40	18	07	14	32	03	10	05	28	209
36	40	70	19	20		40	41		20	29	279
37	20	02	22		03	21	09	20	04	10	111
38	06	19	12		04	23		16	03	22	105
39	09	20	15		11	22	01	19	04	20	121
40	29	34	24		17	28	10	20	08	23	201
41	32	32	09	10	18	33	11	14	09	24	192
42	19	35	10	11	01	48	13	05	20	28	190
43	28	10	20			44	19	06	20	29	176

	10	1.0	20	0.5	0.0	20	20	1.0	1 2 7	1.0	202
44	40	10	28	06	03	20	20	10	25	40	202
45	30	18	23		04	26		20	28	30	179
46	20	10	15	09	03	29	04	30	18		138
47	55	16	20		08	40	05		10	24	178
48	40	09	20	10		43		10	40	29	201
49	18	20	20	20	20	32	20	08		20	178
50	19	32	20		04	24	22	20	04	42	187
51	22	10	20	08	19	26	25	22	09	10	171
52	30	18	32	02	20	27		14	20	20	183
53	28	06	29		22	28	09	15		22	159
54	20	09	18		04	40	10		04	23	128
55	22	10	22		30	45	14	07	20	14	184
56	49	20	29	09		29	18	22	22	15	213
57	34	23	20	10	04	29	19		23	28	190
58	10	09	08	06	02	40	18			29	122
59	62	41	20			42	20	04	04	16	209
60	21	60	10	20	02	48	22	20	09	30	242
61	19	32	28		29	30	24			22	184
62	11	39	30		04	28	03	11	10	40	176
63	10	20	29		20	38		12	10	25	164
64	33	39	33			40		19	20	20	204
65	30	20	40			29		15	18	23	175
66	22	29	28	10	03	29	08	20	17	24	190
67	27	40	30	01	02	44	09	10	20	29	212
68	50	30	32	02	09	43	20	31	01	40	258
69	39	34	40		10	64		20	14	43	264
70	28	20	20		12	02	09	18	20	20	149
71	71	50	28	10	14	10	20	14	29		247
72	18	60	20	02	08	18			30	28	184
73	20	40	30		20		24	04	28		166
74	10	38	29	05	28	23	03		30		166
75	16	29	40		33	24			04		146
76	40	39	38		04	28		19		10	178
77	22	30	43		06	40				20	161
78	28	40	40			40	08	18	05	10	189
79	29	43	40	02	04		09		09	09	146
80	53	34	43	03		20	20	20	20	20	233
81	49	37	43	04		10				23	166
82	40	29	30	03	02	10	09		04	40	167
83	18	50	32	08	15	20	20	41	03	43	250
84	09	62	28	19	20	20		20	09	40	227
85	10	50	33		02	18	02	20	20	20	175
86	20	60	30			19	24	30	18	18	219
87	15	70	40		04	20	29	32	20	40	270
88	60	39	39	03		29	30	29	30	43	302
89	15	39	44		03	42	40	34	40	44	301
90	10	44	40	04	05	48	16	39	43	43	292
91	40	49	29		20	10	19	42	40	16	265
92	52	40	43	04	22	29	20	51	49	18	328
93	50	40	44	08	23	40	22		48	20	295
94	22	30	38	20	04	48	28	10	50	51	301
95	90	58	32	32	26	42	14	20	20	73	408
96	10	67	40	10	30	20	40	42	29	50	338
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97	05	40	40	02			40	20	30	60	237
98	06	62	48	03	02	19	42	32	32	54	300
99	12	39	50	09		28	32	43	40	48	301
100	10	39	29	20	05	39	29	20	28	53	272
101	29	39	30	32	20	48	40	42	29	67	336
102	19	45	29		18	49	40	20	40	71	331
103	29	49	30		40	19	31	32	31	92	353
104	56	43	33		04	21	28	34	33	79	331
105	55	42	45	04	03	28	28	35	34	58	332
106	58	38	38	01		70	15	38	36	29	323
107	10	59	38		09	28	28	39	39	58	309
108	20	68	40		50	80	29	40	40	58	375
109	17	50	20	08	20				30	80	225
110	12	63	10	02	18	29	20		33	29	216
111	14	38	20	06	18	40	20	20	34	70	280
112	40	40	40	09	23	44	14		33	63	306
113	40	40	30	20	23	48	18	40	32	65	356
114	23	60	28	24	27	49	40	32	30	56	345
115	29	48	48	04	29	70	20		32	28	308
116	40	50	29	06	40	16	23	34	34	29	301
117	60	50	60	20	43		29	29	36	70	397
118	49	40	28		45		40		20	82	267
119	29	60	40		20	28	18	32	40		267
120	22	72	100		30		17			20	261

6.0 Conclusion

From this study, the available values of coefficients of determination(R²), sum of square errors (SSE), mean absolute deviation (MAD) and coefficient of mean absolute deviation (CMAD) given by the model shows that principal component regression model has good prediction power accuracy. Government and Hospital management can utilize the result from this study for Mental health management and logistic planning. They can also use the result for policy formulation. The health professionals suchas clinical psychologists, clinical sociologists and health counselor can also utilize these model for patient reorientation and rehabilitation

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