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Qualitative Behaviour of a Mathematical Model of Interacting Populations with Environmental Perturbation

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Abstract

The phenomenon of the interaction between two (2) populations indexed by the unit of time is as old as the famous Lotka-Volterra formalism. However, the qualitative analysis of interacting populations under the simplifying assumption of environmental perturbation is formidable mathematical problem which requires the application of a numerical simulation. The details of this present novel contribution are presented and discussed in this study.

1.0 Introduction

The interaction between two cowpea and groundnut [1, 2] can be affected by an environmental perturbation in the form of an induced random noise characterization [3]. However, the impact of this perturbation on the type of interaction between cowpea and groundnut is rarely studied in detail. Without the inclusion of random noise driven by an environmental perturbation such as the unexpected sea level rise and other climatic factors and human activities, the interaction between cowpea and groundnut is dominantly mutualistic. In this study, we are interested to quantify the impact of changes in the intrinsic growth rates on the type of interaction between cowpea and groundnut on the simplifying assumption that the changes in the growth rates can be driven by an environment perturbation. This phenomenon of random noise is also characteristic of other scientific phenomena [4- 10].

2.0 Mathematical Formulation

The interaction between cowpea and groundnut [1] is defined by the following system of continuous non-linear first order ordinary differential equations

$$\frac{dc(t)}{dt} = \sim_1 c(t) - \ _1 c^2(t) - ¥_1 c(t) g(t) \quad (1)$$

$$\frac{dg(t)}{dt} = \sim_2 g(t) - \ _2 g^2(t) - ¥_2 c(t) g(t) \quad (2)$$

Here, the notations c(t) and g(t) define the biomass of cowpea and groundnut respectively at time t in the unit of days; the $\sim$ parameter values define the intrinsic growth rates; the parameter values define the intra-competition coefficients; the ¥ parameter values define the inter-competition coefficients. This system of model equations evolve on the basis of an initial condition c(0) > 0 and g(0) > 0 which are biologically meaningful parameter values. For the purpose of this study, we have used the following precise parameter values: $\sim_1 = 0.0225, \sim_2 = 0.0446, \ _1 = 0.0069, \ _2 = 0.0133, ¥_1 = 0.0018, ¥_2 = 0.0018,$ c(0)=0.02, g(0) =0.08.

3.0 Method of Solution

The first step involves the calculation of the limiting biomass for each type of legume without the environmental

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perturbation. In this scenario, the interaction is dominantly competitive because the biomass of the isolated legume is smaller than the legume growing together with another legume for a limited resource. The second step involves the quantification of the impact of the random noise on the competition between cowpea and groundnut on the simplifying assumption that any environmental perturbation on an ecological system will affect the intrinsic growth rates more than any other parameter value [11]. This second step was applied to select the type of interaction which responds to the changes in the intrinsic growth rate parameter values. Our results are fully presented and discussed next. The notations c_b (t) and c_{bi}(t) define the biomass of cowpea and its isolated component at time t whereas g_b (t) and g_{bi} (t) define the biomass of groundnut and its isolated component at time t.

4.0 Results and Discussion

Table 1: Environmental perturbation assessment of the intrinsic growth rate parameter values 0.0045 and 0.0089 on the type of interaction between cowpea and groundnut

Example	c_b (t) /g	c_bi (t) /g	g_b (t) /g	g_{bi} (t) /g	Type of interaction
1	0.02	0.02	0.08	0.08	Not applicable
2	0.0234	0.0206	0.1083	0.0845	Mutualism
3	0.0273	0.0213	0.1463	0.0892	Mutualism
4	0.0318	0.0219	0.1966	0.0942	Mutualism
5	0.0371	0.0226	0.2629	0.0993	Mutualism
6	0.0431	0.0233	0.3491	0.1047	Mutualism
7	0.0501	0.0240	0.4592	0.1104	Mutualism
8	0.0581	0.0247	0.5971	0.1163	Mutualism
9	0.0673	0.0255	0.7653	0.1224	Mutualism
10	0.0776	0.0263	0.9642	0.1287	Mutualism

Without the isolated components of cowpea and groundnut at time t, the biomass of cowpea generally outweighs the biomass of groundnut which is an indication of a competition interaction between cowpea and groundnut in the absence of random noise characterization caused by an environmental perturbation. On the premise that the environmental perturbation affects the intrinsic growth rate parameter values, when these parameter values are 0.0045 and 0.0089, c_b (t) >c_{bi} (t) and g_b (t) > g_{bi} (t) indicating a shift from the normal competition between cowpea and groundnut in the absence of random noise to a dominant mutualistic interaction between cowpea and groundnut. Therefore, although the environmental perturbation may be seen as a negative upset on the biomass of these interacting legumes, the reality produced by this simulation analysis is that changes in the intrinsic growth rates caused by the environmental perturbation or random noise has created instances of biodiversity gain. The same observation has been made for the scenario when the intrinsic growth rate parameter values for these combinations(0.0056, 0.0112) and (0.00067, 0.0134) [see Table 2, Table 3].

Table 2: Environmental perturbation assessment of the intrinsic growth rate parameter values 0.0056 and 0.0112 on the type of interaction between cowpea and groundnut

Example	c_b (t) /g	c_bi (t) /g	g_b (t) /g	g_{bi} (t) /g	Type of interaction
1	0.02	0.02	0.08	0.08	Not applicable
2	0.0234	0.0208	0.1083	0.0858	Mutualism
3	0.0273	0.0216	0.1463	0.0920	Mutualism
4	0.0318	0.0224	0.1966	0.0986	Mutualism
5	0.0371	0.0233	0.2629	0.1056	Mutualism
6	0.0431	0.0242	0.3491	0.1130	Mutualism
7	0.0501	0.0252	0.4592	0.1209	Mutualism
8	0.0581	0.0261	0.5971	0.1292	Mutualism
9	0.0673	0.0272	0.7653	0.1380	Mutualism
10	0.0776	0.0282	0.9642	0.1472	Mutualism

Table 3: Environmental perturbation assessment of the intrinsic growth rate parameter values 0.0067 and 0.0134 on the type of interaction between cowpea and groundnut

Example	c_b (t) /g	c_bi (t) /g	g_b (t) /g	g_{bi} (t) /g	Type of interaction
1	0.02	0.02	0.08	0.08	Not applicable
2	0.0234	0.0209	0.1083	0.0872	Mutualism
3	0.0273	0.0219	0.1463	0.0949	Mutualism
4	0.0318	0.0230	0.1966	0.1033	Mutualism
5	0.0371	0.0241	0.2629	0.1123	Mutualism
6	0.0431	0.0252	0.3491	0.1220	Mutualism
7	0.0501	0.0264	0.4592	0.1324	Mutualism
8	0.0581	0.0276	0.5971	0.1435	Mutualism
9	0.0673	0.0289	0.7653	0.1554	Mutualism
10	0.0776	0.0303	0.9642	0.1682	Mutualism

In contrast, the combination of bigger values of the intrinsic growth rates dominantly predicts a competition interaction leading to the loss of a mutualistic interaction [see Table 4, Table 5, Table 6].

Table 4: Environmental perturbation assessment of the intrinsic growth rate parameter values 0.027 and 0.0535 on the type of interaction between cowpea and groundnut

Example	c_b (t) /g	c_bi (t) /g	g_b (t) /g	g_{bi} (t) /g	Type of interaction
1	0.02	0.02	0.08	0.08	Not applicable
2	0.0234	0.0241	0.1083	0.1153	Competition
3	0.0273	0.0291	0.1463	0.1656	Competition
4	0.0318	0.0351	0.1966	0.2364	Competition
5	0.0371	0.0424	0.2629	0.3349	Competition
6	0.0431	0.0510	0.3491	0.4693	Competition
7	0.0501	0.0615	0.4592	0.6483	Competition
8	0.0581	0.0741	0.5971	0.8786	Competition
9	0.0673	0.0891	0.7653	1.1625	Competition
10	0.0776	0.1071	0.9642	1.4946	Competition

Table 5: Environmental perturbation assessment of the intrinsic growth rate parameter values 0.0281 and 0.0558 on the type of interaction between cowpea and groundnut

Example	c_b (t) /g	c_bi (t) /g	g_b (t) /g	g_{bi} (t) /g	Type of interaction
1	0.02	0.02	0.08	0.08	Not applicable
2	0.0234	0.0243	0.1083	0.1171	Competition
3	0.0273	0.0296	0.1463	0.1708	Competition
4	0.0318	0.0360	0.1966	0.2475	Competition
5	0.0371	0.0437	0.2629	0.3555	Competition
6	0.0431	0.0531	0.3491	0.5048	Competition
7	0.0501	0.0645	0.4592	0.7053	Competition
8	0.0581	0.0782	0.5971	0.9645	Competition
9	0.0673	0.0948	0.7653	1.2839	Competition
10	0.0776	0.1149	0.9642	1.6548	Competition

Table 6: Environmental perturbation assessment of the intrinsic growth rate parameter values 0.0292 and 0.058 on the type of interaction between cowpea and groundnut

Example	c_b (t) /g	c_bi (t) /g	g_b (t) /g	g_{bi} (t) /g	Type of interaction
1	0.02	0.02	0.08	0.08	Not applicable
2	0.0234	0.0245	0.1083	0.1190	Competition
3	0.0273	0.030	0.1463	0.1761	Competition
4	0.0318	0.0368	0.1966	0.2590	Competition
5	0.0371	0.0451	0.2629	0.3774	Competition
6	0.0431	0.0551	0.3491	0.5429	Competition
7	0.0501	0.0676	0.4592	0.7668	Competition
8	0.0581	0.0826	0.5971	1.0576	Competition
9	0.0673	0.1009	0.7653	1.4151	Competition
10	0.0776	0.1232	0.9642	1.8267	Competition

What meaning are these contributions providing which our previous study did not make? Our previous study has developed the mathematical equations that describe the interaction dynamics between two legumes such as cowpea and groundnut and succinctly looked at some vital mathematical analyses and numerical simulations with respect to stability conditions and the impact of a combination of the deterministic parameter values on the type of stability. But the key contribution of this present study is clearly stated as follows: empirically quantifying how the decreased and increased changes in the intrinsic growth rates affect the type of interaction between cowpea and groundnut. While a decreased intrinsic growth rate predicts more instances of mutualism, an increased intrinsic growth rate predicts more instances of competition leading to the loss of mutualism which has the potential to predict more volumes of biodiversity gain than biodiversity loss.

5.0 Conclusion

Due to the variability of the intrinsic growth rates, this study has predicted nine (9) cases of mutualism when the intrinsic growth rates were decreased while nine (9) cases of competition are predicted when the intrinsic growth rates were increased. The carrying capacities for the growth of these two interacting legumes, their doubling times and tripling times can be considered as potential factors which may affect the bifurcation from a dominant mutualism to a pure competition. This study has raised two issues namely: can a combination of the intrinsic growth rate with any other model parameter value predict more incidence of mutualism than competition or otherwise? Can the combination of the two intrinsic growth rates predict more incidence of biodiversity gain and less incidence of biodiversity loss or otherwise? The details of these pending issues which are fundamental properties of the interaction between cowpea and groundnut will be the subject of a future publication.

6.0 References

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