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The Impact of a Time Delay on the Depleted Proportion of the Viral Load of the Virions Due to a Decreased Reproductive Rate of the Infected Cell

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Abstract

While the impact of the variability of the reproductive rate of the infected cell on the viral load of the virions is an on-going research activity, the inclusion of a time delay which mimics the African culture of diverse health inhibiting belief system is a new numerical simulation perspective of solving the mathematical problem and the health policy dimension of HIV/AIDS intervention strategy. The full results of this study which one has not seen elsewhere are presented and discussed in this paper.

1.0 Introduction

Upon the implementation of a numerical simulation, we have clearly shown that a decreased reproductive rate of the infected cell can lead to the depletion of the viral load of the virions indexed by a time independent variable whereas an increased reproductive rate of the infected cell leads to the recovery of the viral load of the virions indexed by a time independent variable. This recent contribution and other previous studies [1-9] did not consider the inclusion of a time delay in the determination of the response of the viral load of the virions due to the variation of the reproductive rate of the infected cell. It is worth mentioning that while detailed mathematical analyses of several HIV/AIDS can be found in the literature, it is not a uniform practice to find papers in which the notion of a numerical simulation has been used to quantify the impact of a time delay on the depleted proportion of the viral load of the virions due to a decreased reproductive rate of the infected cell.

In the context of African demography, the issue of a time delay in the HIV/AIDS transmission dynamics can be attributed to several factors out of which four (4) can play significant roles in the motivation for a time delay. These factors include late reporting of the status of the illness, traditional and cultural barriers to health reporting, social stigma associated to the people living with AIDS and some sort of wrong health counselling [10].

2.0 Mathematical Formulation

Following [1, 4], we consider the system of time dependent non-linear first order ordinary differential equations

$$\frac{dT}{dt} = s - dT + aT\left(1 - \frac{T}{T_{\max}}\right) - \quad TV \tag{1}$$

$$\frac{dI}{dt} = \quad {}_1 TV - \text{¢} I \tag{2}$$

$$\frac{dV}{dt} = {}^{\circ} I - cV \tag{3}$$

with the initial conditions $T(0) \geq 0, I(0) \geq 0, V(0) \geq 0$ .The notation T is called the number of target cells while the notations I and V are called the number of infected cells and the viral load of the virions at time t in the unit of days. The notation s stands for the rate at which new T cells are created from sources within the body such as the thymus whereas the notation a is called the maximum proliferation rate of target cells.

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The notation T_{max} stands for the T population density at which proliferation shuts off whereas the notation d stands for the death rate of the T cells. The notation $_1$ is represented by the exponential equation $_1 = \; e^{-m^2}$ where is the infection rate constant whereas the term e^{-m^2} accounts for cells that are infected ² time units later. The notation ¢ stands for the death rate of infective cells whereas the notation ° is the reproductivity rate of the infected cell. The notation c represents the clearance rate constant of virions. The precise parameter values are

$d = 0.01,$ ¢ $= 0.5, c = 10, a = 6.8, T_{max} = 1300, s = 5, \; = 0.0002,$ ° $= 1000$

This model formulation did not look at the impact of the reproductive rate of the infected cell on the viral load of the virions with its time delay and health policy implication [1].

3.0 Method of Analysis

The first method of analysing this proposed problem concerns the calculation of the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 50 with a fixed duration of twenty (20) days and initial data (100, 100, 100). The second method concerns the calculation of the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell takes the values of 50 and 100 with an initial condition of (100, 100, 100) on the simplifying assumption that the time delay ranges from two (2) days to eight (8) days. These two methods were utilized to calculate the impact of the reproductive rate of the infected cell on the viral load of the virions for the instances of without and with a time delay inclusion. The results obtained will be clearly differentiated and discussed in the next section of this paper.

4.0 Results and Discussion

In the absence of a time delay (Table 1), the percentage of the percentage of the viral load of the virions that is depleted when the reproductive rate of the infected cell is 50 with a fixed duration of twenty (20) days and initial data of (100, 100, 100) ranges from a small value of 14.9 approximately at the end of twenty (20) to a big value of 97.3 approximately at the end of two (2) days.

Table 1: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 50 with a fixed duration of twenty (20) days and the initial data of (100, 100, 100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 50	Proportion depleted without a time delay
1	1	100	100	0
2	2	54365	1465	97.3
3	3	43613	1805	95.9
4	4	30698	2598	91.5
5	5	21778	4104	81.2
6	6	15826	5998	62.1
7	7	11897	7019	41.0
8	8	9354	6591	29.5
9	9	7784	5434	30.2
10	10	6905	4243	38.6
11	11	6549	3281	49.9
12	12	6578	2589	60.6
13	13	6846	2145	68.7
14	14	7181	1928	73.2
15	15	7429	1948	73.8
16	16	7524	2260	70.0
17	17	7480	2989	60.0
18	18	7375	4180	43.3
19	19	7275	5482	24.7
20	20	7211	6139	14.9

In the presence of a time delay of two (2) days as displayed in Table 2, the percentage of the percentage of the viral load of the virions that is depleted when the reproductive rate of the infected cell is 50 with a duration of eighteen (18) days and initial data of (100, 100, 100) takes the value of 43.3 approximately at the end of eighteen (18) dayscompared to a big value of 97.3 approximately at the end of two (2) days.

Table 2: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 50 with a time delay of two (2) days with theinitial data of (100,100,100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 50	Proportion depleted with a time delay of two (2) days
1	1	100	100	0
2	2	54365	1465	97.3
3	3	43613	1805	95.9
4	4	30698	2598	91.5
5	5	21778	4104	81.2
6	6	15826	5998	62.1
7	7	11897	7019	41.0
8	8	9354	6591	29.5
9	9	7784	5434	30.2
10	10	6905	4243	38.6
11	11	6549	3281	49.9
12	12	6578	2589	60.6
13	13	6846	2145	68.7
14	14	7181	1928	73.2
15	15	7429	1945	73.8
16	16	7524	2260	70.0
17	17	7480	2989	60.0
18	18	7376	4180	43.3

In the presence of a time delay of four (4) days as displayed in Table 3, the percentage of the percentage of the viral load of the virions that is depleted when the reproductive rate of the infected cell is 50 with a duration of sixteen (16) days and initial data of (100, 100, 100) takes the value of 43.3 approximately at the end of eighteen (16) days to a big value of 97.3 approximately at the end of two (2) days.

Table 3: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 50 with a time delay of four (4) days with theinitial data of (100, 100, 100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 50	Proportion depleted with a time delay of four (4) days
1	1	100	100	0
2	2	54365	1465	97.3
3	3	43613	1805	95.9
4	4	30698	2598	91.5
5	5	21778	4104	81.2
6	6	15826	5998	62.1
7	7	11897	7019	41.0
8	8	9354	6591	29.5
9	9	7784	5434	30.2
10	10	6905	4243	38.6
11	11	6549	3281	49.9
12	12	6578	2589	60.7
13	13	6846	2145	68.7
14	14	7181	1928	73.2
15	15	7429	1945	73.8
16	16	7522	2259	70.0

In the presence of a time delay of six (6) days as displayed in Table 4, the percentage of the percentage of the viral load of the virions that is depleted when the reproductive rate of the infected cell is 50 with a duration of fourteen (14) days and initial data of (100, 100, 100) takes the value of 73.2 approximately at the end of fourteen (14) days to a big value of 97.3 approximately at the end of two (2) days.

Table 4: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 50 with a time delay of six (6) days with theinitial data of (100, 100, 100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 50	Proportion depleted with a time delay of six (6) days
1	1	100	100	0
2	2	54365	1465	97.3
3	3	43613	1805	95.9
4	4	30698	2598	91.5
5	5	21778	4104	81.2
6	6	15826	5998	62.1
7	7	11897	7019	41.0
8	8	9354	6591	29.5
9	9	7784	5434	30.2
10	10	6905	4243	38.6
11	11	6549	3281	49.9
12	12	6578	2589	60.7
13	13	6846	2145	68.7
14	14	7181	1927	73.2

In the presence of a time delay of eight (8) days as displayed in Table 5, the percentage of the percentage of the viral load of the virions that is depleted when the reproductive rate of the infected cell is 50 with a duration of twelve (12) days and initial data of (100, 100, 100) takes the value of 60.6 approximately at the end of twelve (12) days to a big value of 97.3 approximately at the end of two (2) days.

Table 5: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 50 with a time delay of eight (8) days with theinitial data of (100, 100, 100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 50	Proportion depleted with a time delay of eight (8) days
1	1	100	100	0
2	2	54365	1465	97.3
3	3	43613	1805	95.9
4	4	30698	2598	91.5
5	5	21778	4104	81.2
6	6	15826	5998	62.1
7	7	11897	7019	41.0
8	8	9354	6591	29.5
9	9	7784	5434	30.2
10	10	6905	4243	38.6
11	11	6549	3281	49.9
12	12	6579	2590	60.6

From the deterministic perspective, the results that we have obtained in this study due to a time delay of 2 days, 4 days, 6 days and 8 days are consistent with the results displayed in Table 1with a zero time delay. This is a robust contribution to knowledge in the context of HIV/AIDS intervention strategy.In the scenario when the reproductive rate of the infected cell is 100 (Table 6) with a fixed duration of twenty (20) days and initial data (100, 100, 100), the percentage of the viral load of the virions depleted is 41.07 approximately compared with the proportion depleted value of 14.9 when the reproductive rate of the infected cell is 50. In essence, when the reproductive rate of the infected cell is doubled, the percentage of the viral load of the virions that is depleted will increase by 176 approximately.The impact of the variability of a time delay on the viral load of the virions has been calculated and displayed as shown in Table 7, Table 8, Table 9 and Table 10 which is consistent with the data of Table 6.

Table 6: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 100 with a fixed duration of twenty (20) days and the initial data of (100, 100, 100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 100	Proportion depleted without a time delay
1	1	100	100	0
2	2	54365	3402	93.7
3	3	43613	5037	88.5
4	4	30698	6625	78.4
5	5	21778	7079	67.5
6	6	15826	6346	59.9
7	7	11897	5175	56.5
8	8	9354	4101	56.2
9	9	7784	3275	57.9
10	10	6905	2720	60.6
11	11	6549	2426	62.96
12	12	6578	2398	63.55
13	13	6846	2698	60.59
14	14	7181	3433	52.20
15	15	7429	4633	37.64
16	16	7524	5900	21.58
17	17	7480	6455	13.70
18	18	7375	6068	17.72
19	19	7275	5179	28.80
20	20	7211	4250	41.07

Table 7: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 100 with a time delay of two (2) days with theinitial data of (100,100,100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 100	Proportion depleted with a time delay of two (2) days
1	1	100	100	0
2	2	54365	3402	93.7
3	3	43613	5037	88.5
4	4	30698	6625	78.4
5	5	21778	7079	67.5
6	6	15826	6346	59.9
7	7	11897	5175	56.5
8	8	9354	4101	56.2
9	9	7784	3275	57.9
10	10	6905	2720	60.6
11	11	6549	2426	63.0
12	12	6578	2398	63.6
13	13	6846	2698	60.6
14	14	7181	3433	52.2
15	15	7429	4633	37.6
16	16	7524	5900	21.6
17	17	7480	6455	13.7
18	18	7376	6068	17.7

Table 8: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 100 with a time delay of four (4) days with the initial data of (100,100,100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 100	Proportion depleted with a time delay of four (4) days
1	1	100	100	0
2	2	54365	3402	93.7
3	3	43613	5037	88.5
4	4	30698	6625	78.4
5	5	21778	7079	67.5
6	6	15826	6346	59.9
7	7	11897	5175	56.5
8	8	9354	4101	56.2
9	9	7784	3275	57.9
10	10	6905	2720	60.6
11	11	6549	2426	63.0
12	12	6578	2398	63.6
13	13	6846	2698	60.6
14	14	7181	3433	52.2
15	15	7429	4633	37.6
16	16	7522	5901	21.6

Table 9: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 100 with a time delay of six (6) days with the initialdata of (100,100,100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 100	Proportion depleted with a time delay of six (6) days
1	1	100	100	0
2	2	54365	3402	93.7
3	3	43613	5037	88.5
4	4	30698	6625	78.4
5	5	21778	7079	67.5
6	6	15826	6346	59.9
7	7	11897	5182	56.5
8	8	9354	4102	56.2
9	9	7784	3275	57.9
10	10	6905	2720	60.6
11	11	6549	2426	63.0
12	12	6578	2398	63.6
13	13	6846	2698	60.6
14	14	7181	3432	52.2

Table 10: Calculating the percentage of the viral load of the virions depleted when the reproductive rate of the infected cell is 100 with a time delay of eight (8) days with the initial data of (100,100,100)

Example	Time in days	Viral load of the virions with fixed ° = 1000	Viral load of the virions with changing ° = 100	Proportion depleted with a time delay of eight (8) days
1	1	100	100	0
2	2	54365	3402	93.7
3	3	43613	5037	88.5
4	4	30698	6625	78.4
5	5	21778	7079	67.5
6	6	15826	6346	59.9
7	7	11897	5181	56.5
8	8	9354	4102	56.2
9	9	7784	3274	57.9
10	10	6905	2720	60.6
11	11	6549	2426	63.0
12	12	6579	2399	63.6

In the absence of a time delay, our earlier research output has shown that a decreased reproductive rate of the infected cell can lead to the depletion of the viral load of the virions indexed by a time independent variable. The advantage of this present analysis over our previous study is that the inclusion of the time delay has shown the proportions of the viral load of the virions that can be depleted when the reproductive rate of the infected cell takes the values of 50 and 100. In the instance of a time delay of 2 days when the reproductive rate of the infected cell is 50, the approximated percentage of the depleted viral load of the virions that is 43 compared with 18 percent when the reproductive rate of the infected cell is 100. Next, in the instance of a time delay of 4 days when the reproductive rate of the infected cell is 50, the approximated percentage of the depleted viral load of the virions that is 70 compared with 22 percent when the reproductive rate of the infected cell is 100. In the instance of a time delay of 6 days when the reproductive rate of the infected cell is 50, the approximated percentage of the depleted viral load of the virions that is 73 compared with 52 percent when the reproductive rate of the infected cell is 100. Finally,in the instance of a time delay of 8 days when the reproductive rate of the infected cell is 50, the approximated percentage of the depleted viral load of the virions that is 61 compared with 64 percent when the reproductive rate of the infected cell is 100. Therefore, the differing changes in the depleted viral load of the virions that can occur as a result of the changes in the reproductive rate of the infected cell in terms of the four distinct time delay constructions which were not captured in our earlier study makes this present contribution much more beneficial than our earlier study in terms of HIV/AIDS intervention strategy and other old dimensions of modelling and analysing the complex dynamics of HIV/AIDS as found in the works of [1-9]

5.0 Conclusion

In most African health behaviour scenarios of which Nigeria is no exception, a time delay of any illness is inevitable. In this context, a lower value of the reproductive rate of the infected cell is associated with a lower proportion of the depleted viral load of the virions whereas a relatively bigger value of the reproductive rate of the infected cell is associated with a bigger proportion of the depleted viral load of the virions. A time delay ranging from 2 days to 6 days has shown higher values of the depleted proportions of the viral load of the virions when the reproductive rate of the infected cell is 50 compared to the scenario when the reproductive rate of the infected cell is 100.The implication of this contribution for the further implementation of HIV/AIDS intervention strategy is the central inference of this present study over our earlier study.

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