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A Mathematical Model for the Prediction of Injectivity Decline

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Abstract

Injectivity impairment due to invasion of solid suspensions has been studied by several investigators and some modelling approaches have also been reported. Worthy of note is the development of analytical models for internal and external filtration coupled with transition time concept for predicting the overall decline in injectivity.

This study presents a new mathematical model which is based on mass balance of particles flowing through the porous media by coupling rate changes in injection pressure within the reservoir due to formation damage with rate changes in porosity within the invaded region. This model when fully validated will ensure the reliability of injectivity decline prediction from a well injectivity index history which will further pave way for an effective planning of water treatment whereby creating a new frontier in the management of produced water in oil and gas technology.

Keywords: Injectivity impairment; internal filtration; External filtration; water injection; filter cake; porosity reduction; formation damage; injection decline

Nomenclature

C = particle concentration in the fluid, ppm
λ = filtration coefficient, 1/cm
˜$_i$ = initial porosity
K = formation permeability, mD
K_z = Kozeny constant
K_i = initial permeability, mD
˜$_m$ = porosity of the medium
q = flowrate, c.c/day
A = cross-sectional area, cm^2
Δ_t = time step, days

ρ_s = particle density, kg/cc
C_i = particle concentration at the start of the time step, ppm
e_c = cake thickness, cm
˜$_c$ = porosity of the cake
˜*= critical porosity value
t_D= dimensionless time
r_e = drainage radius, cm
σ = particle retention coefficient, vol/vol
D = particle dispersion coefficient, cm^2/s

D_o = molecular diffusion coefficient
C_f= formation compressibility, 1/pa
μ_s = suspension viscosity, pa*s
r = radial distance, cm
β = formation damage factor
h = formation thickness, cm
r_f = radius of injection front, cm
α = ratio of injectivity at time t to initial injectivity
a_L = longitudinal dispersivity

K_D = dimensionless permeability
n = exponent
U = permeate velocity, cm/s
K_o= original formation permeability
T_{tr}= transition time, s
J = particle flux, cm/s
T= injection time, s
r_w = wellbore radius, cm
C_p = volume of deposited particles per unit bulk volume

1.0 Introduction

There are two main properties of injection water that determine the formation damage or the injectivity of water injection wells: the total dissolved solids in the injection water and the total suspended solids (solids and oil droplets) in the injection water. The Presence of suspended solids in the injected water causes formation damage, i.e., reduction of near wellbore permeability of the formation which in turn could lead to decline in well injectivity.

The problem of formation damage due to produced water re-injection (PWRI) can be decomposed into separate distinct problems. These include internal filtration, external filter cake build-up and the associated permeability reduction

The particle size distributions responsible for the formation of formation filtration is shown in **Figure 1**.

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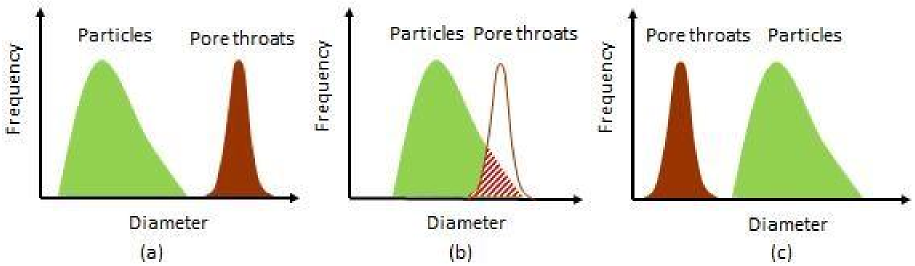

Fig.1: Particle and pore throat size distributions responsible for internal and external filtrations. (a) Pure internal filtration; Internal followed by external filtration (b) Simultaneous internal and external filtrations; (c) Pure external filtration [1].

Several papers [2-5] have been recently presented describing various mechanisms of deposition of solid particles in porous media due to invading dilute solid suspensions. Also there have been studies [6-11] of deep bed filtration process where the retention of particles is inside the porous medium resulting in the decreased porosity of the porous media. An external filter cake is deposited when the particle sizes are significantly closer [7] to the pore sizes. But there has been a lack of a single model expressed in terms of pressure to assess the formation damage relating various components of the solid invasion and deposition such as the filtration coefficient, dispersion coefficient and the porosity of the filter medium with the time of invasion and the length of the filter bed of the porous media.

This work presents a mathematical model, which is based on mass balance of particles from the injection water flowing through the porous media, by coupling rate changes in injection pressure within the reservoir due to formation damage with rate changes in porosity within the invaded region. This study develops a radial form of diffusion –convection model using the implicit finite difference method.

2.0 Problem Definition

During Produced Water Re-Injection, the inherent particles in the injected water tend to filter into the formation according to any of the following scenarios viz;

(1) Pure internal filtration of the contaminants before the transition time

(2) Pure external filter cake build-up when the pore throat is far smaller than the invading particles

(3) Initial internal filtration which terminates just at the start of external filter cake build-up (i.e the point where transition time (t *) has been reached) and

(4) Initial internal filtration followed by external cake build-up after the transition time.

The resulting effect is a net reduction in the formation porosity and permeability within the invaded region and an overall decline in well injectivity over time. To this regard, a lot of models have been developed in the literature to account for formation damage and injectivity decline due to particle invasion. But none of these models has however accounted for the changes in formation porosity as a function of both particle invasion and the prevailing injection pressure during model development. They assume a constant or known porosity value for sake of simplicity even though the fact that the formation porosity changes due to particle invasion is acknowledged. Using the third scenario, this work is aimed at relaxing that assumption by developing a model that sufficiently accounts for changes in formation porosity during deep bed filtration on the basis of the injection pressure.

3.0 Research Objective

The primary objective of this research to develop a model that can predict injectivity impairment due to internal/external filtration caused by porosity reduction during water injection and increase in cake thickness during external filtration.

3.1 Physical Model Description

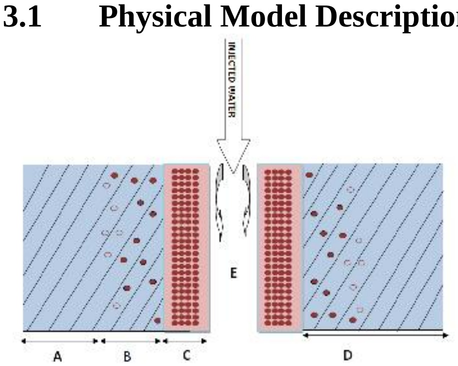

Figure 2: Diagrammatic representation of the Physical model

Figure 2 illustrates a typical injection process characterize with particle invasion. A is the Region of uninvaded zone, B is the Region where internal filtration of the water and solid particles is taking place, C is the Region of external filtration which develops into a Cake, D is the Reservoir section and E is the injection well. This study models the region of the internal filtration to the point where the external filtration begins.

3.2 Model Formulation

Based on the classical deep bed filtration theory taking into account particle dispersion, the particle invasion process is such that, initially the particles invade and deposit in the porous medium. As time progresses, more particles are retained and a point (critical porosity ˜*) will be reached where very few particles can invade the porous medium and an external filter cake begins to build on the injection face. The time at which an initial layer of external filter cake is completely formed on the injection face of the porous medium is referred to as transition time t_{tr}. Internal filtration is more predominant in the time domain $0 < t < t_{tr}$ and external filtration is more predominant in the time domain $t > t_{tr}$. The cake grows in thickness and also functions as a granular filter that retains some of the fine particles as they flow through the cake. The fine particle retention within the cake tends to decrease the cake permeability. As earlier explained, this study is aimed at examining this problem in a rigorous manner. **Fig. 3** depicts the internal filter cake formed in the porous medium in the time domain $0 < t < t_{tr}$. **Fig. 4** depicts the internal filter cake formed in the porous medium in the time domain $0 < t < t_{tr}$ and external filter cake formed on the injection face in the time domain $t > t_{tr}$.

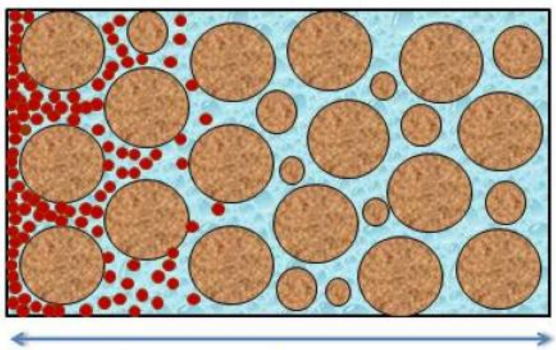

Fig. 3: Diagrammatic representation of internal filter cake formed in the porous medium in the time domain $0 < t < t_{tr}$[1].

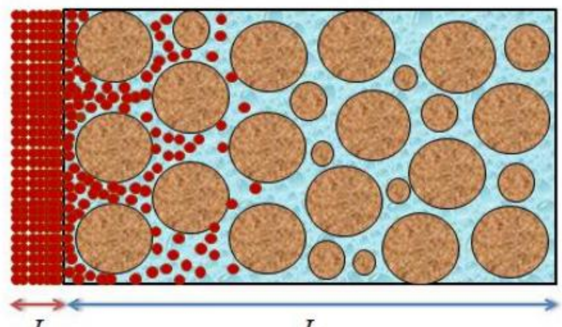

Figure 4: Diagrammatic representation of internal filter cake in the porous medium and external filter cake formed on the injection face in the time domain $t > t_{tr}$[1].

4.0 Model Assumptions

1. The porous medium is homogeneous, well consolidated sandstone with well-rounded grains and free of clay content.
2. The density of the particles is constant in deposited and suspended states.
3. Particle dispersion is taken into consideration and the filtration coefficient depends on retained particle concentration through the use of a filtration function described by Iwasaki [9]. Once the particles are deposited in the porous medium they are not released and will not change the number of available retention sites.

4.1 Model Derivation

The unsteady state radial transport equation which describes the spatial and temporal variation of injection pressure in a homogeneous porous media undergoing advection and dispersion is derived from the mass conservation equation for suspended and retained particles for an incompressible fluid. Other models were also employed in actualising our result. Some of these models include:

- **Equation for Hydrodynamic Dispersion**

Hydrodynamic dispersion denotes the spreading at macroscopic level resulting from both mechanical dispersion and molecular diffusion. The equation for hydrodynamic dispersion including molecular diffusion and mechanical dispersion is given by Van Genuchten[13], as

$$D = D_0\tau + \alpha_L V^n \tag{1}$$

- ***Equation for Retention Kinetics.***

The kinetic equation which describes the rate of transfer of particles to the porous medium [14] is given by;

$$\frac{\partial \sigma}{\partial t} = \lambda J \tag{2}$$

The particle flux J is characterised by the advective and dispersive components:

$$J = uc - D\frac{\partial c}{\partial r} \tag{3}$$

The calculations by Altoe et al [15] of the number of captured particles per unit time during flow via a sieve sequence shows that capture rate is proportional to the total particle flux . But in one study particle dispersion was taken into account in deep bed filtration modelling [14]. The dispersion term was included in the particle balance but was not accounted for in the capture rate equation. In this work, the particle capture rate is taken to be directly proportional to the total particle flux comprising of both the advection and dispersive components; i.e.

$$\frac{\partial \sigma}{\partial t} = \lambda\left(uc - D\frac{\partial c}{\partial r}\right) \tag{4}$$

- ***Filtration Coefficient Model***

In this study, the filtration function describing the ripening period as proposed by Iwasaki [16] is given as,

$$\lambda(\sigma) = \lambda_0(1 + \beta\sigma) \tag{5}$$

- ***Darcy's model***

Darcy's law for suspension flows in porous media includes the effect of permeability decline during particle retention [9].

$$u = \frac{K_0 K(\sigma)}{(1+\beta\sigma)\mu}\frac{\partial p}{\partial r} \tag{6}$$

$$K(\sigma) = \frac{1}{(1+\beta\sigma)} \tag{7}$$

Using these models in addition to the conservation of mass equation, our model was derived as:

$$\frac{\partial^2 p}{\partial r^2} + \lambda\frac{\partial p}{\partial r} = \frac{\phi_m}{D}\frac{\partial p}{\partial t} \tag{8}$$

Where, λ is the particle filtration coefficient, $\tilde{}_m$ is the original formation porosity and D is the dispersion coefficient. The detailed derivation of our model in **(8**) is given in **Appendix A**

5.0 Conclusion

1. A partial differential model representing deep bed filtration in terms of injection pressure to account for porosity during water injection was successfully developed.
2. In order for the developed model to be used for effective prediction, frantic steps are on-going in finding numerical solution to the model.

6.0 Acknowledgement

Special thanks to all lecturers of DEPARTMENT OF PETROLEUM ENGINEERING, UNIVERSITY OF BENIN for their constant guidance, from whom we had the privilege to learn advanced theoretical and practical concepts related to Petroleum Engineering. We are thankful to Prof E.S Adewole for stimulating lectures on numerical mathematics, rock fluid interaction, reservoir simulation and providing us with guidance whenever necessary. Our sincere appreciation to all the reservoir and production Engineers at **The Nigerian Petroleum Development Company Ltd**, and not forgetting our dear friend Kome Owhe, and all colleagues who supported us towards the success of this paper.

7.0 Appendix A: Model Development

The particle flux J is characterised by the advective and dispersive components:

$$J = uc - D\frac{\partial c}{\partial r} \tag{A-1}$$

Writing the mass conservation equation on the particle flux into the porous media:

$$m_{in} = 2\pi uc\rho_s rh - 2\pi D\frac{\partial c}{\partial r}\rho_s rh \tag{A-2}$$

But Area A= $2\pi rh$

$$m_{in} = ucA\rho_s - D\frac{\partial c}{\partial r}A\rho_s \tag{A-3}$$

$$m_{out} = \left[uc + \frac{\partial(uc)}{\partial r}\delta r\right] A\rho_s - \left[D\frac{\partial c}{\partial r} + \frac{\partial}{\partial r}\left(D\frac{\partial c}{\partial r}\delta r\right)\right] A\rho_s \quad \text{(A-4)}$$

$$m_{acc} = \frac{\partial}{\partial t}\,(c\phi + \sigma)A\rho_s \quad \text{(A-5)}$$

Recall: $m_{in} - m_{out} = m_{acc}$ (principle of conservation of mass)

$$-\frac{\partial(uc)}{\partial r}\delta r A\rho_s + \frac{\partial}{\partial r}\left(D\frac{\partial c}{\partial r}\right)\delta r\, A\rho_s = \frac{\partial}{\partial t}\,(c\phi + \sigma)\delta r A\rho_s \quad \text{(A-6)}$$

Dividing through by $\delta r A\rho_s$ yields;

$$-\frac{\partial(uc)}{\partial r} + \frac{\partial}{\partial r}\left(D\frac{\partial c}{\partial r}\right) = \frac{\partial}{\partial t}\,(c\phi + \sigma) \quad \text{(A-7)}$$

Accounting for Darcy's equation adjusted for permeability reduction during particle filtration i.e.

$$u = \frac{K_o K_r}{(1+\beta\sigma)\mu}\frac{\partial p}{\partial r} \quad \text{(A-8a)}$$

$$u = \frac{k}{(1+\beta\sigma)\mu}\frac{\partial p}{\partial r} \quad \text{(A-8b)}$$

Expanding (A-7);

$$c\frac{\partial u}{\partial r} + u\frac{\partial c}{\partial r} - D\frac{\partial^2 c}{\partial r^2} = -\frac{\partial}{\partial t}\,(c\phi + \sigma) \quad \text{(A-9)}$$

Substituting (A-8b) into (A-9)

$$\frac{kc}{(1+\beta\sigma)\mu}\frac{\partial}{\partial r}\left(\frac{\partial p}{\partial r}\right) + \frac{k}{(1+\beta\sigma)\mu}\frac{\partial p}{\partial r}\left(\frac{\partial c}{\partial r}\right) - D\frac{\partial^2 c}{\partial r^2} = -\frac{\partial}{\partial t}\,(c\phi + \sigma) \quad \text{(A-10)}$$

Let $R = \frac{k}{(1+\beta\sigma)\mu}$

Therefore (A-10) becomes

$$Rc\frac{\partial}{\partial r}\left(\frac{\partial p}{\partial r}\right) + R\frac{\partial p}{\partial r}\left(\frac{\partial c}{\partial r}\right) - D\frac{\partial^2 c}{\partial r^2} = -\frac{\partial}{\partial t}\,(c\phi + \sigma) \quad \text{(A-11)}$$

According to Todd and Kumar [16]

$$\frac{\partial c}{\partial \phi} = \frac{(\rho_s - c)}{\phi} \quad \text{(A-12)}$$

$$(\rho_s - c)\,\phi = (\rho_s - c_i)\,\phi_i \quad \text{(A-13)}$$

$$c\phi = \rho_s\phi - (\rho_s - c_i)\,\phi_i \quad \text{(A-14)}$$

$$\frac{\partial(c\phi)}{\partial r} = \rho_s\frac{\partial \phi}{\partial r} \quad \text{(A-15)}$$

$$\frac{\partial^2(c\phi)}{\partial r^2} = \rho_s\frac{\partial^2 \phi}{\partial r^2} \quad \text{(A-16)}$$

$$\frac{\partial c}{\partial r} = \frac{(\rho_s - c_i)\,\phi_i}{\phi^2}\frac{\partial \phi}{\partial r} \quad \text{(A-17)}$$

Substituting (A-17) into (A-11) gives

$$Rc\frac{\partial}{\partial r}\left(\frac{\partial p}{\partial r}\right) + R\frac{\partial p}{\partial r}\left(\frac{(\rho_s - c_i)\,\phi_i}{\phi^2}\right)\frac{\partial \phi}{\partial r} - D\frac{\partial^2 c}{\partial r^2} = -\frac{\partial}{\partial t}\,(c\phi + \sigma) \quad \text{(A-18)}$$

Let $m = \frac{(\rho_s - c_i)\,\phi_i}{\phi^2}$

Therefore (A-18) becomes

$$Rc\frac{\partial}{\partial r}\left(\frac{\partial p}{\partial r}\right) + Rm\frac{\partial p}{\partial r}\frac{\partial \phi}{\partial r} - D\frac{\partial^2 c}{\partial r^2} = -\frac{\partial}{\partial t}\,(c\phi + \sigma) \quad \text{(A-19)}$$

From chain rule,

$$\frac{\partial \phi}{\partial r} = \frac{\partial \phi}{\partial p}\cdot\frac{\partial p}{\partial r} \quad \text{(A-20a)}$$

But

$$c_f = \frac{1}{\phi}\frac{\partial \phi}{\partial p} \quad \text{and} \quad \text{(A-20b)}$$

$$\frac{\partial \phi}{\partial p} = \phi c_f \tag{A-20c}$$

Hence (A-20a) becomes

$$\frac{\partial \phi}{\partial r} = \phi c_f \frac{\partial p}{\partial r} \tag{A-20d}$$

Substituting (A-20d) into (A-19)

$$Rc\frac{\partial^2 p}{\partial r^2} + Rm\phi c_f\left(\frac{\partial p}{\partial r}\right)^2 - D\frac{\partial^2 c}{\partial r^2} = -\frac{\partial}{\partial t}\ (c\phi + \sigma) \tag{A-21}$$

From (A-17)

$$\frac{\partial^2 c}{\partial r^2} = \frac{(\rho_s - c_i)\,\phi_i}{\phi^2}\frac{\partial^2 \phi}{\partial r^2} \tag{A-22}$$

And

$$\frac{\partial^2 \phi}{\partial r^2} = \ \phi c_f \frac{\partial^2 p}{\partial r^2} \tag{A-23}$$

Substituting (A-23) into (A-22) gives

$$\frac{\partial^2 c}{\partial r^2} = \frac{c_f(\rho_s - c_i)\,\phi_i}{\phi}\frac{\partial^2 p}{\partial r^2} \tag{A-24}$$

Taking porosity at the initial time t=0, i.e. $\phi = \ \phi_i$

$$\frac{\partial^2 c}{\partial r^2} = c_f(\rho_s - c_i)\frac{\partial^2 p}{\partial r^2} \tag{A-25}$$

Substituting (A-25) in to (A-24) gives

$$Rc\frac{\partial^2 p}{\partial r^2} + Rm\phi c_f\left(\frac{\partial p}{\partial r}\right)^2 - Dc_f(\rho_s - c_i)\frac{\partial^2 p}{\partial r^2} = -\frac{\partial}{\partial t}\ (c\phi + \sigma) \tag{A-26}$$

Assuming $c_f\left(\frac{\partial p}{\partial r}\right)^2$ is very small and negligible, (A-26) becomes;

$$Rc\frac{\partial^2 p}{\partial r^2} - Dc_f(\rho_s - c_i)\frac{\partial^2 p}{\partial r^2} = -\frac{\partial}{\partial t}\ (c\phi + \sigma) \tag{A-27}$$

Let $Dc_f(\rho_s - c_i) = G$ (A-28)

$$(Rc - G)\frac{\partial^2 p}{\partial r^2} = -\frac{\partial}{\partial t}\ (c\phi + \sigma)$$

$$(Rc - G)\frac{\partial^2 p}{\partial r^2} = -\frac{\partial (c\phi)}{\partial t} - \frac{\partial \sigma}{\partial t} \tag{A-29}$$

Based on the work by Todd and Kumar [16]

$$\frac{\partial (c\phi)}{\partial t} = \rho_s \frac{\partial \phi}{\partial t} \tag{A-30a}$$

Applying chain rule to $\frac{\partial \phi}{\partial t}$ in (A-30a) yields;

$$\frac{\partial (c\phi)}{\partial t} = \phi c_f \rho_s \frac{\partial p}{\partial t} \tag{A-30b}$$

Considering the particle capture kinetics, the particle capture rate is related to the particle flux J as;

$$\frac{\partial \sigma}{\partial t} = \lambda J \tag{A-31}$$

where the filtration coefficient λ retains its physical meaning as a constant of proportionality.

Substituting (A-1) into A-31 yields;

$$\frac{\partial \sigma}{\partial t} = \lambda\left(uc - D\frac{\partial c}{\partial r}\right) \tag{A-32}$$

Substituting (A-8a) and (A-8b) into (A-32) respectively gives;

$$\frac{\partial \sigma}{\partial t} = \frac{\lambda kc}{(1+\beta\sigma)\mu}\frac{\partial p}{\partial r} - \lambda Dc_f(\rho_s - c_i)\frac{\partial p}{\partial r} \tag{A-33a}$$

$$\frac{\partial \sigma}{\partial t} = \left[\frac{\lambda kc}{(1+\beta\sigma)\mu} - \lambda Dc_f(\rho_s - c_i)\right]\frac{\partial p}{\partial r} \tag{A-33b}$$

But $\frac{kc}{(1+\beta\sigma)\mu} - Dc_f(\rho_s - c_i) - Rc - G$

Therefore:

$$\frac{\partial \sigma}{\partial t} = \lambda[Rc - G]\frac{\partial p}{\partial r} \qquad \text{(A-34)}$$

Substituting (A-30) & (A-34) into (A-29);

$$(Rc - G)\frac{\partial^2 p}{\partial r^2} + \lambda[Rc - G]\frac{\partial p}{\partial r} = -\phi c_f \rho_s \frac{\partial p}{\partial t} \qquad \text{(A-35)}$$

Assuming that R_c is negligible compared to G due to the conc. c_i And dispersion coefficient D respectively, we obtain;

$$-G\frac{\partial^2 p}{\partial r^2} - \lambda G\frac{\partial p}{\partial r} = -\phi c_f \rho_s \frac{\partial p}{\partial t} \qquad \text{(A-36)}$$

Recall; $Dc_f(\rho_s - c_i) = G$

Therefore, dividing through by $-c_f\rho_s$ and assuming c_i is negligible compared to ρ_s i.e particle density, gives;

$$D\frac{\partial^2 p}{\partial r^2} + \lambda D\frac{\partial p}{\partial r} = \phi\frac{\partial p}{\partial t} \qquad \text{(A-37)}$$

Taking ϕ as original formation porosity, ϕ_m, (A- 37) becomes;

$$\frac{\partial^2 p}{\partial r^2} + \lambda\frac{\partial p}{\partial r} = \frac{\phi_m}{D}\frac{\partial p}{\partial t} \qquad \text{(A-38)}$$

8.0 References

[1]. Rameshchandra Yerramilli.: "Water Injectivity Prediction: Experiments and Modeling," Thesis, Submitted To Section of Petroleum Engineering, Dept. of Apllied Earth Sciences, Delft University of Technology, The Netherlands, July 2012.

[2]. Donaldson, E.C., Baker, B.A, And Carroll, H.B: "Particle Transport In Sandstones," Peper SPE 6905 Presented At The 1977 Spe Annual Technical Conference And Exhibition, Denver, Oct. 9-12.

[3]. Huecke, T.W: "Formation Fines And Factors Controlling Their Movement In Porous Media," J. Pet. Tech. April 1979, pp. 144-50.

[4]. Davidson, D.H.: "Invasion and Impairment of Formations by Particulates," Paper SPE 8210 Presented At the 64th Annual Technical Conference and Exhibition of Society of Petroleum Engineers, Las Vegas, Nevada, Sept. 23-26, 1979.

[5]. Gruesbeck, C. And Collins, R.E.: "Entrainment and Deposition of Fine Particles in Porous Media," Soc. Of Petroleum Engineers J, Dec. 1982, pp. 247-256.

[6]. Maroudas, A.: "Particle Deposition in Granular Filter Media-1,"Filtration Separation, Vol. 2, No. 5, Sept. /Oct. 1965, Pp 369-372.

[7]. Maroudas, A.: "Particle Deposition in Granular Filter Media-2,"Filtration Separation, Vol. 3, No. 2, March/April 1966, Pp 115-21.

[8]. Eliasen, R.: "Clogging Of Rapid Sand Filters," J. Amar. Water Works Asso. Vol. 33, No. 5, 1941, Pp 926-41.

[9]. Iwasaki, T.: "Some Notes On Sand Filtration," J. Amar. Water Works Asso. Vol. 29, No. 10, 1937, Pp 1591-1602.

[10]. Ling, J.T.: "A Study of Filtration through Uniform Sand Filters," Proc. Am. Soc. Civil Eng., Vol. 81, No. 751, 1955, Pp 1-35.

[11]. Stanley, D.R.: "Sand Filtration Studied With Radiotracers," Proc. Am. Soc. Civil Eng., Vol. 81, No. 592, 1955, pp 1-23.

[12]. Genuchten, V.M., Wierenga, P.J. 1986. Solute Dispersion Coefficients and Retardation Factors. Methods of Soil Analysis, Part-1. Physical and Mineralogical Methods, 2nd Ed, Agronomy Monograph.9.

[13]. Herzig, J.P., Leclerc, D.M. And Goff, P.L.: "Flow of Suspensions through Porous Media-Application to Deep Bed Filtration," Ind. And Eng. Chem. Engrs. Chem., Vol. 62, No. 5, Way 1970, pp 8-34.

[14]. J.E Altoe F. et al.: "Role of Dispersion in Injectivity Impairment: Mathematical and Laboratory Study," SPE 88501 Presented At the Spe Formation Damage Control Symposium Held in Bakersfield, Ca. Sept. 2004.

[15]. T. Kumar and A.C Todd.: "A New Approach for Mathematical Modelling of Formation Damage Due To Invasion of Solid Suspension," SPE 18203 Presented At The Spe Formation Damage Control Symposium Held In Bakersfield, Ca. Oct. 1988