# Grade-Average Method: A Statistical Approach for Estimating Missing Value for Continuous Assessment Marks 

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#### Abstract

In this paper we propose an alternative way for finding an estimate of a missing score for continuous assessment mark of an examination so as to allocate an appropriate grade. We considered four different examinations and randomly selected five students of different class of grade in each, with their actual Examinations and Continuous Assessment Scores. The proposed grade-average method (GAM) is expected to be an unbiased or best estimator among the various methods of estimating the missing Continuous Assessment score adopted by different teachers.


Keywords: Continuous assessment, grades, average score and standard deviation.

### 1.0 Introduction

In many occasions for one reason or another student misses continuous assessment (C.A), this poses a serious problem to the examiner. The problem arises on how to assess the continuous performance of the student for that period of study and allocate a mark to him. This mark must be added to the final examination result in order to make up the total final score of the student in a particular course. Whenever such problems prevail people take different options or adopt different methods to tackle them, which perhaps have statistical defects. Some of the methods they adopt are:
(i) The student whose continuous assessment score is not available is given other set of questions different from the questions given to the class. This method has problem since the questions given to the students may not be the same questions given to the class in terms of standard, conduciveness of the environment during test and the time limitation. In addition to these, there is a high tendency that the examiner could be strictly biased when marking fewer scripts.
(ii) Some lecturers use a fraction of the continuous assessment mark on the fraction of the examination score to determine the missing continuous assessment score. For instance, if C.A. is $30 \%$ of the final result, and final examination is $70 \%$ then $30 \%$ of the score in the examination is taken and considered as a representative of the student's continuous assessment mark. In fact, this is a futile effort, since there is no change effect in that regard with respect to the original result of the final examination.
(iii) Some lecturers mark student's best examination question as both an exam question as well as a continuous assessment question.
The above-mentioned two methods are definitely defective in the sense that either the student that misses the continuous assessment is cheated or the entire class (or some) is cheated. In our effort to reduce bias in the evaluation of students' performance in examination, we provide the same question(s) to them to answer, within a specified time limit, under similar environmental conditions. However, bias resulting from psychological state of individual student is virtually beyond our control. Thus the methods used by some teachers in evaluating the missing C.A. score as mentioned above could only promote bias in that respect.
In this paper we suggest an alternative approach of dealing with such problem if it exists. Perhaps, the method based on the following assumption is going to be preferable from statistical point of view.
i. Students' scores are classified according to grades A, B, C, D and E, which are non-overlapping.

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## Grade-Average Method: A Statistical...

ii. The final examination is conducted under strict measures to avoid examination mal-practice and in a favorable conducive atmosphere.
iii. Students work independently of each other during assignment, test and final examination.
iv. A student misses continuous assessment with genuine reason.

### 2.0 Related Works

Sadler and Good [1], compared teacher-as-signed grades to those awarded either by students to themselves or by their peers. By training students to grade with the help of a scoring rubric, a very high correlation ( $r=0.91$ to 0.94 ) between students and their teacher on test question was obtained. However, pattern of bias was also observed when students assigned grades to themselves; they awarded lower grades to the best performing students than their teacher did and lower performing students inflated their own low scores than their teacher did.
Their idea of making above comparison was a result of the 2001 U. S. Supreme Court's reaffirmation on the popular view that students grading each others' is valuable, saves teachers' time and augments student learning.
Weltman and Whiteside [2], showed that active learning is not universally effective and, in fact, may inhibit learning for certain types of students. Their results dictates that as increased levels of active learning are utilized, student test scores decreases for those with a high grade point average as against the increase in the test scores for students in the lower level grade point average resulted from the introduction of active learning to students. They taught three topics to every student that involve in the experiment each by a different teaching method, and administered an assessment test to students at the end of each session to assess the comprehension.
Zoller and Ben-Chaim [3], designed a special self-assessment containing higher order cognitive skills (HOCS)-type questions and administered to 71 Biology majors enrolled in a four-year college program. They accounted for the gap between students' self-assessment and that of their HOCS biased teachers (the authors). Many of the respondents were observed to have confidently evaluated themselves capable of self-assessment.
They suggested from their results that the potential for student self-assessment within college science teaching and learning exist, however, still a great purposed effort in HOCS-oriented teaching and learning is required in order for the student selfassessment practice to become a routine integral component of HOCS science examinations.

### 3.0 Grade Average Method (GAM).

Defined $X_{\mathrm{i}}$ to be score for the ith continuous assessment result of a course unit, such that $i=1,2,3, \ldots, n$, where $n$ is the number assignments. The sum of $X_{\mathrm{i}}^{\prime}$ s gives the final continuous assessment of a student in a particular course. Suppose one of the $X_{\mathrm{i}}$ 's is missing, say $X_{\mathrm{r}}$. To find $X_{\mathrm{r}}$, using GAM we follow these steps:
Step 1: Identify the final examination score of the student that missed the C.A; $X_{\mathrm{r}}$ and the grade (or class) he belongs.
Step 2: Find all the subjects that have scored the same grade with him.
Step 3: Find the average score of the rth C.A of the subjects in step 2 and
hence consider it to be the estimate of the missing value of $X_{r}$, which we denote as $\hat{X}_{r}=\bar{X}_{r}$. To obtain the average avoid repeated scores; that will make the distribution of the scores more symmetric about $\hat{X}_{r}$.

### 4.0 Illustration of GAM.

Consider four different examinations and randomly select five students from each examination. Their scores in rth C.A and final examination are recorded in Tables1a, 2A, 3A \& 4A which we denoted as $X_{\mathrm{r}}$ and $Y$; respectively. In the same table, we graded the final year results of each of the five students as A, B, C, D, or E., the last column of the table gives the rth C.A score of other students in that class.
However, we avoided repeated values in that last column, in order to make the distribution more symmetric about the mean value $\hat{X}_{r}=\bar{X}_{r}$; which we consider to be the estimate of the missing C.A. score; $\hat{X}_{r}$.
Tables $1 \mathrm{~B}, 2 \mathrm{~B}, 3 \mathrm{~B} \& 4 \mathrm{~B}$ resulted from Tables $1 \mathrm{~A}, 2 \mathrm{~A}, 3 \mathrm{~A} \& 4 \mathrm{~A}$ respectively from each of the four cases. In those tables, we obtain $\hat{X}_{r}$, which is the estimate of $X_{\mathrm{r}}$ (in table As). The difference between exact value and estimated value are given in the third column. Final grades with the true value of $X_{r}$ and the estimate $\hat{X}_{r}$ are given for the sake of comparison. Standard deviations are also given in the last column of the Tables 1B, 2B, 3B \& 4B.

Table 1A comprises of five columns as Grade, $X_{\mathrm{r}}$ and $Y$ represents the actual Continuous Assessment (C.A) and Examination scores for a randomly selected student of a corresponding grade respectively, Exam column tells us about the percentage mark of the end semester paper without the C.A score with the ranges needed for each exam score to equate its corresponding grade (i.e. $\frac{49}{70} * 100=70, \frac{42}{70} * 100=60$, etc ), and the last column gives the actual C.A scores of other students having same grade with the selected student. The same procedure applies to other grades of the table and tables 2A and 3A except in the fourth column of table 4 A where the examination is $60 \%$ not the usual $70 \%$ as in other three cases.
However, Table 1B comprises of seven columns as the serial number, $\hat{X}_{r}$ is the average C.A score from other students having the same grade with the randomly selected student, the next column informs us about the difference between the estimated C.A (average score) and the actual C.A score of a selected student, Exact grade (100) is the grade resulted from summing the actual C.A and exam scores of a selected student in Table 1A. The fifth column, Exact grade (100), gives the grade resulted from summing the estimated score in the second column of the table with the actual examination score of selected student in Table 1A, the attached * indicates where the estimated grade do not tally with the actual grade in Table 1 A . The sixth column, $n$ gives the total number of other students considered to estimate a particular grade and the last column, $\sigma_{X_{r}}$ gives the standard deviation which measures the variation between the actual observations with the estimated C.A score.

### 4.1 Definitions of Some Terms[4]

Let $x_{1}, x_{2}, \ldots \ldots \ldots, x_{n}$ be $n$ number of observations then
i. The Mean denoted by $\tilde{x}$ is defined as

$$
\tilde{x}=\frac{\sum_{i=1}^{n} x_{i}}{n}
$$

ii. The sample variance $s^{2}$ is defined as

$$
s^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\tilde{x}\right)^{2}}{n-1}
$$

and the sample standard deviation is obtained as

$$
s=\left[\frac{\sum_{i=1}^{n}\left(x_{i}-\tilde{x}\right)^{2}}{n-1}\right]^{\frac{1}{2}}
$$

### 4.2 The Analysis

Under this, we consider four different cases for the four different courses regarded case one, two, three and four marked by different lecturers on four different levels of students over a given period of time.
The analysis was done using MATLAB software.

## Case one.

Table 1A: DISTRIBUTION ON ACTUAL PERFORMANCE OF THE FIRST COURSE

| Grade | $X_{\mathrm{r}}$ | $\mathbf{Y}$ | Exam (70) | rth C.A results in the class. |
| :--- | :--- | :--- | :--- | :--- |
| A | 21 | 60 | $\mathrm{~A}(\geq 49)$ | $25 ; 15 ; 24 ; 11$ |
| B | 13 | 47 | $\mathrm{~B}(42-48)$ | $17 ; 10 ; 22 ; 15 ; 12 ; 21 ; 18 ; 14 ; 25$ |
| C | 17 | 36 | $\mathrm{C}(35-41)$ | $13 ; 14 ; 25 ; 18 ; 05 ; 11 ; 19 ; 15 ; 16 ; 23 ; 08$ |
| D | 15 | 34 | $\mathrm{D}(32-34)$ | $21 ; 17$ |
| E | 10 | 31 | $\mathrm{E}(27-31)$ | $04 ; 12$ |

Table 1B: DISTRIBUTION ON ESTIMATED PERFORMANCE OF THE FIRST COURSE

| S/N | $\hat{X}_{r}$ | $\hat{X}_{r}-\mathrm{X}_{\mathrm{r}}$ | Exact grade (100) | Grade with $\hat{X}_{r}(100)$ | $n$ | $\sigma_{X_{r}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 19 | -2 | A | A | 4 | 5.93 |
| 2 | 17 | +4 | B | B | 9 | 4.63 |
| 3 | 15 | -2 | C | C | 11 | 5.72 |
| 4 | 19 | -4 | D | C * | 2 | 2.00 |
| 5 | 08 | -2 | E | E | 2 | 4.00 |

## Case two.

Table 2A: DISTRIBUTION ON ACTUAL PERFORMANCE OF THE SECOND COURSE

| Grade | $X_{\mathrm{r}}$ | $\mathbf{Y}$ | Grade (70) | rth C.A results in the class |
| :--- | :--- | :--- | :--- | :--- |
| A | 10 | 51 | $\mathrm{~A}(\geq 49)$ | $12 ; 16 ; 18 ; 14$ |
| B | 12 | 44 | $\mathrm{~B}(42-48)$ | $14 ; 16 ; 22 ; 15$ |
| C | 14 | 37 | $\mathrm{C}(35-41)$ | $14 ; 12 ; 16 ; 13$ |
| D | 11 | 34 | $\mathrm{D}(32-34)$ | 17 |
| E | 12 | 27 | $\mathrm{E}(27-31)$ | $10 ; 12 ; 14$ |

Table 2B: DISTRIBUTION ON ESTIMATED PERFORMANCE OF THE SECOND COURSE

| S/N | $\hat{X}_{r}$ | $\hat{X}_{r}-\mathrm{X}_{\mathrm{r}}$ | Exact grade (100) | Grade with $\hat{X}_{r}(100)$ | ${ }^{\mathrm{n}}$ | $\sigma_{X_{r}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 15 | +5 | B | B | 4 | 2.23 |
| 2 | 17 | +4 | C | $\mathrm{B} *$ | 4 | 3.11 |
| 3 | 14 | 0 | C | C | 4 | 1.48 |
| 4 | 14 | +3 | D | D | 6 | 3.00 |
| 5 | 12 | 0 | E | E | 3 | 1.63 |

Case three.

Table 3A: DISTRIBUTION ON ACTUAL PERFORMANCE OF THE THIRD COURSE

| Grade | $X_{\mathrm{r}}$ | $\mathbf{Y}$ | Grade (70) | rth C.A results in the class |
| :--- | :--- | :--- | :--- | :--- |
| A | -- | -- | $\mathrm{A}(\geq 49)$ | -- |
| B | 28 | 44 | $\mathrm{~B}(42-48)$ | $18 ; 20$ |
| C | 20 | 38 | $\mathrm{C}(35-41)$ | 21 |
| D | -- | -- | $\mathrm{D}(32-34)$ | -- |
| E | 16 | 26 | $\mathrm{E}(27-31)$ | $17 ; 10 ; 21 ; 15$ |

Table 3B: DISTRIBUTION ON ESTIMATED PERFORMANCE OF THE THIRD COURSE

| $\mathrm{S} / \mathrm{N}$ | $\hat{X}_{r}$ | $\hat{X}_{r}-\mathrm{X}_{\mathrm{r}}$ | Exact <br> $(100)$ | grade | Grade with <br> $\hat{X}_{r}(100)$ | N | $\sigma_{X_{r}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | -- | -- | -- | -- | -- | -- |  |
| 2 | 19 | -9 | A | $\mathrm{~B} *$ | 2 | 1.00 |  |
| 3 | 21 | +1 | C | C | 4 | 0.5 |  |
| 4 | -- | -- | -- | -- | -- | -- |  |
| 5 | 16 | 0 | E | E | 4 | 3.96 |  |

## Case four

Table 4A: DISTRIBUTION ON ACTUAL PERFORMANCE OF THE FOURTH COURSE

| Grade | $X_{\mathrm{r}}$ | $\mathbf{Y}$ | Grade (60) | rth C.A results in the class. |
| :--- | :--- | :--- | :--- | :--- |
| A | 21 | 49 | $\mathrm{~A}(\geq 42)$ | $26 ; 36 ; 27 ; 29 ; 23 ; 25 ; 30 ; 24 ; 32 ; 28 ; 22$ |
| B | 19 | 47 | $\mathrm{~B}(36-41)$ | $25 ; 23 ; 26 ; 20 ; 30 ; 21 ; 27 ; 28 ; 18 ; 16 ; 31$ |
| C | 18 | 35 | $\mathrm{C}(30-35)$ | $15 ; 28 ; 16 ; 26 ; 20 ; 24 ; 14 ; 17 ; 13 ; 23 ; 12 ; 22 ;$ <br>  <br> D |
| E | 19 | $21 ; 9 ; 25$ | $\mathrm{D}(27-29)$ | $11 ; 18 ; 22 ; 15 ; 10 ; 29 ; 16 ; 24 ; 14 ; 13 ; 08$ |
|  | 17 | 23 | $\mathrm{E}(27-31)$ | $18 ; 14 ; 16 ; 19 ; 20 ; 12 ; 21 ; 24 ; 26 ; 23 ; 25 ; 28 ;$ |

Table 4B: DISTRIBUTION ON ESTIMATED PERFORMANCE OF THE FOURTH COURSE

| S/N | $\hat{X}_{r}$ | $\hat{X}_{r}-\mathrm{X}_{\mathrm{r}}$ | Exact grade (100) | Grade with $\hat{X}_{r}(100)$ | n | $\sigma_{X_{r}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 27 | +6 | A | A | 11 | 3.96 |
| 2 | 24 | +5 | B | $\mathrm{~A} *$ | 11 | 4.68 |
| 3 | 19 | +1 | C | C | 15 | 5.54 |
| 4 | 16 | -3 | D | E | 11 | 6.11 |
| 5 | 20 | +3 | E | E | 13 | 4.97 |

### 5.0 Discussion and Conclusion.

In this paper, we have presented GAM for estimating the missing Continuous Assessment score due acceptable reasons.
To justify the likelihood of the method we considered four different courses each with five different grades. In all the four courses, GAM estimates correctly the exact or almost-exact grade of each of the five different grades we have considered as indicated in the column five of each of table Bs. Where GAM fails to provide the exact estimate of the student's grade, it is observed that the true score is in the upper or lower margin of the grade he belongs. This leads us to the question of whether to take $\hat{X}_{r}$ or to adjust it in order to reduce bias, $\left(\hat{X}_{r}-X_{\mathrm{r}}\right)$. To adjust the estimate $\hat{X}_{r}$ we either add the standard deviation $\delta_{x}$ to it or subtract from it; since the true value is likely to be slightly above or below $\hat{X}_{r}$ by some amount $\delta_{x}$. It is difficult to make either of the above decisions, hence we suggest that the decision to take would be based on the overall performance of the student in all the other examinations and/or his status, whether he is a regular, carry over or spill over student.

### 6.0 References

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