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Variation of Rainfall in Three Nigerian Stations, Using harmonic Analysis.

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Abstract

This work is on the variation of rainfall using harmonic analysis for Portharcourt, Kano and Makurdi data, for the three stations the period of study covered 1977 to 2010, for which the time series plot, the amplitude, the first, second and third harmonics were generated. Portharcourt has a gently increasing trend with periodicities of 5 years for the first harmonic, 3.5 years for the second harmonic and seventeen years for the third harmonic respectively. Kano showed a declining trend, while the periodicities were 3 years for the first harmonic, 3 years for the second harmonic and 18 years for the third harmonic respectively. Makurdi's semi-cyclical pattern has periodicities of 3 years for the first harmonic, 4 years for the second harmonic and 19 years for the third harmonic respectively.

Keywords: Rainfall, Variation, Harmonics, Periodicities.

1.0 Introduction

Rainfall takes on a cyclical pattern of variation spanning across seasons. Concern over climate change and its impact has given rise to the increasing drive to understand the spatiotemporal changes in global rainfall regimes. Variation in climate conditions in the Northern Nigeria is characterized by droughts, storms and even floods. This variation takes on cyclical oscillation mode [1]. Studying rainfall is of great significance to Man owing to its environmental impacts and the effect on the hydrological cycle [2]. The Inter Tropical Discontinuity (ITD) as a crucial factor in rainfall moves from the Southern Nigeria to the Northern Nigeria from January to August, it is the boundary between the continental tropical air mass originating from the North and the moistly maritime air mass originating from the South of Nigeria. By August the position of the ITD is at the North with the rainfall at its peak there. Agricultural activities generally depend on rainfall in Nigeria [3,11]. Nigeria is distinctly characterized by the following rainfall types, namely convectional rainfall which occurs as a result of the heating process that give rise to currents of air that are set up as vertical columns, while highland features serve as barriers to moist winds rising upwards and dropping as rain currents, frontal rainfall is the result of difference between wet warm air overlying cold air [4]. The distribution of rainfall in Nigeria was studied in [5] using different statistical tools for the analysis and ascertaining the frequencies and peaks of rainfall occurrence.

Theoretical Analysis

Harmonic analysis gives waves as superposed signals generated from Fourier transform process to study time series data [6]. Harmonic analysis is used to study meteorological parameters to see how they vary seasonally and the possible effects of such variations [7].It is a veritable tool for studying the variability of climate parameters; the Fourier transform method is employed for which a time-dependent atmospheric phenomenon like rainfall, temperature etc are decomposed into functions of sinusoidal dimensions generating amplitudes for the parameter being studied [8]. Fourier analysis grants a complex domain representation of an addition of sine and cosine waves characterized by amplitudes and phase angles and the cyclical sinusoids points to the frequency domain characterization of the time data [9].Trigonometric functions are the fundamental basis for Harmonic analysis; it has been used extensively as in the analysis of different atmospheric parameters [7].

The Harmonic Analysis of Time Series (HANTS) model used for analyzing time series data has the following mathematical structure as in [6].

$$\tilde{y}(t_j) = a_0 + \sum_{i=1}^{nf}[a_i \cos(2\pi f_i t_j) + b_i \sin(2\pi f_i t_j)] \quad (1)$$

$$y(t_j) = \tilde{y}(t_j) + \varepsilon(t_j) \quad (2)$$

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Where j = 1… N, y is the original series and $\tilde{y}$ the reconstructed series while a_i and b_i are the coefficients of the Fourier series. Varying features on the Earth surface influence its climate, hence the need to study these parameters [8].

2.0 Materials and Method

The data for this analysis was obtained from the Nigeria Meteorological Agency (NIMET), it is made up of data from three Nigerian stations of Porthacourt, Kano and Makurdi, for each station the data covered the period 1977 -2009

2.1 Harmonic Analysis

The subject of Fourier analysis covers a broad spectrum of Mathematics and its applications, the Fourier series stems from the need to represent a periodic function say $f(x)$ as Sine and Cosine functions, the ensuing trigonometric series has coefficients determined from the Euler formula. As earlier reviewed data size is critical to the integral multiples of frequencies from the fundamental; fluctuations in the variable of interest are smoothened through Harmonic analysis [10].
For an integrable periodic function $f(x)$ having period 2π, the following holds;

$$f(x) = a_o + \sum_{n=1}^{\infty}(a_n Cosnx + b_n Sinnx) \tag{3}$$

Where a_o is a constant term evaluated by

$$a_o = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx \; n = 1,2 \dots \tag{4}$$

While the real and imaginary coefficients a_n and b_n are given by

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)Cosnxdx \; n = 1,2 \dots \tag{5}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)Sinnxdx \; n = 1,2 \dots \tag{6}$$

The above set of equations is for the continuous Fourier series, for our study we must shift to the discreet Fourier series which is suitable for the generation of the coefficients, phase angles and amplitudes respectively.

The discreet Fourier transform

$$X_t = \sum_{t=1}^{n}[A_k \, Cos\left(\frac{2\pi kt}{n}\right) + B_k Sin(\frac{2\pi kt}{n})] \;\; t = 1,2, \dots n \tag{7}$$

$$X_t = \bar{X} + C_k Cos\left(\frac{2\pi t}{n} - \phi_k\right) \tag{8}$$

Where C_k gives the amplitude and ϕ is the phase angle or the phase shift, with the coefficients given as follows;

$$A_k = \frac{2}{n}\sum_{t=1}^{n} X_t \, Cos\left(\frac{2\pi kt}{n}\right) \tag{9}$$

$$B_k = \frac{2}{n}\sum_{t=1}^{n} X_t \, Sin\left(\frac{2\pi kt}{n}\right) \tag{10}$$

For which the amplitude and phase angles are thus realized

$$C_k Cos\left(\frac{2\pi t}{n} - \phi_k\right) = A_k Cos\left(\frac{2\pi t}{n}\right) + B_k Sin\left(\frac{2\pi t}{n}\right) \tag{11}$$

And the coefficients are so resolved

$$A_k = C_k Cos(\phi_k)$$
$$B_k = C_k Sin(\phi_k)$$

Therefore the amplitude is $C_k = ({A_k}^2 + {B_k}^2)^{\frac{1}{2}}$ (12)

And the phase angle is

$$\phi_k = \tan^{-1}(\frac{B_k}{A_k}) \tag{13}$$

$$H_i = A_i Sin(\theta_i + \phi_i) \qquad i = 1,2, \dots \tag{14}$$

Equation 14 is used to generate the harmonics from the results obtained in equations (9), (10) and (13).

3.0 Results and Discussion

The following charts gives the results of the analysis of the rainfall (mm) data from three stations namely; Portharcourt, Kano and Makurdi respectively, the categorization is based on time series, amplitude and harmonic representation respectively. Figure1shows the time series chart for Portharcourt rainfall. The oscillating pattern has a slight decreasing trend covering the study period. Figure 2represents the amplitudes of Portharcourt rainfall from 1977 to 2010, the significant amplitudes fall in the years 1992, 1995, 1998 and 2008 respectively.

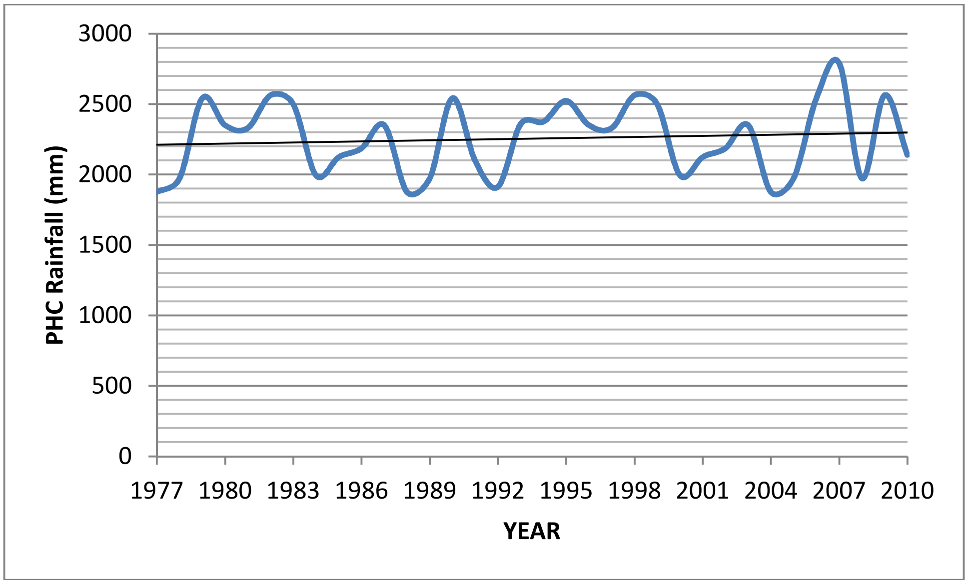

Fig. 1: Portharcourt Rainfall (mm)

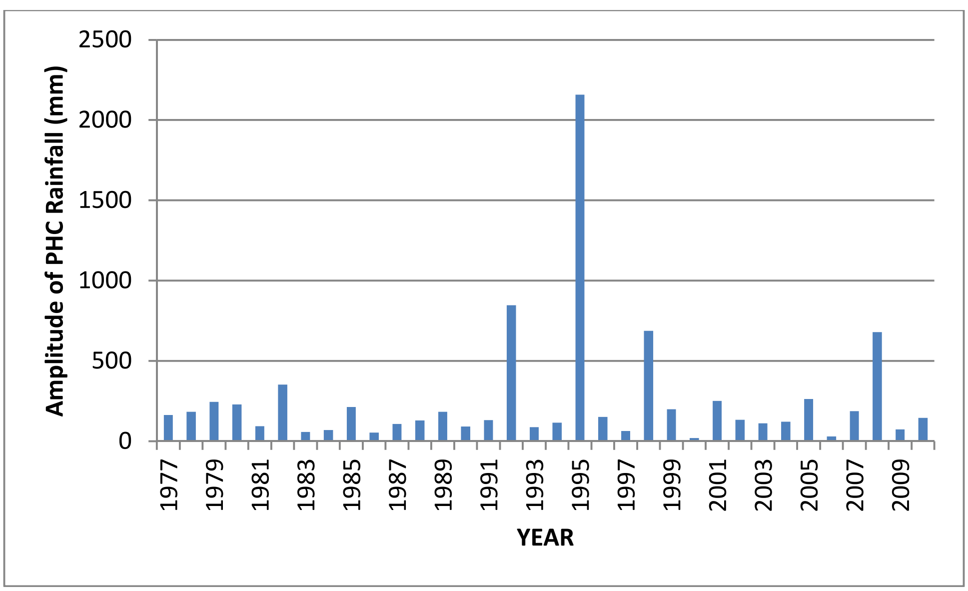

Fig. 2: Amplitude of Portharcourt Rainfall (mm)

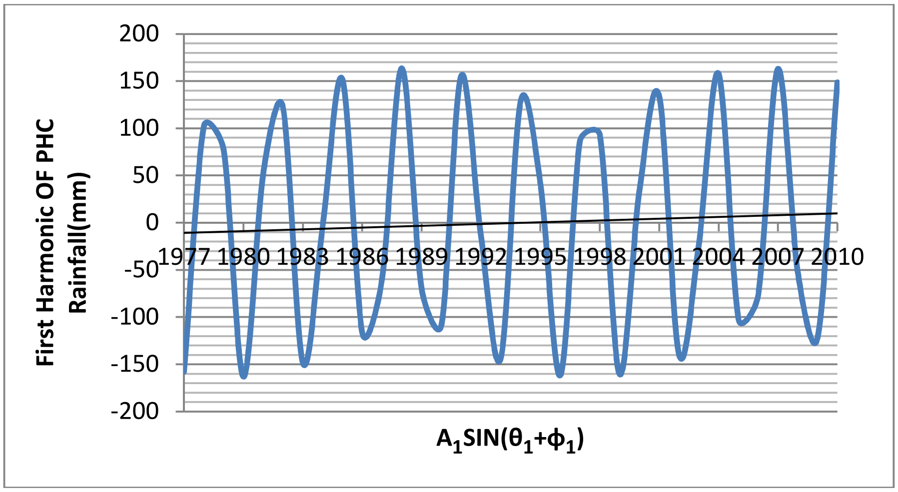

Fig. 3: First Harmonics of Portharcourt Rainfall (mm)

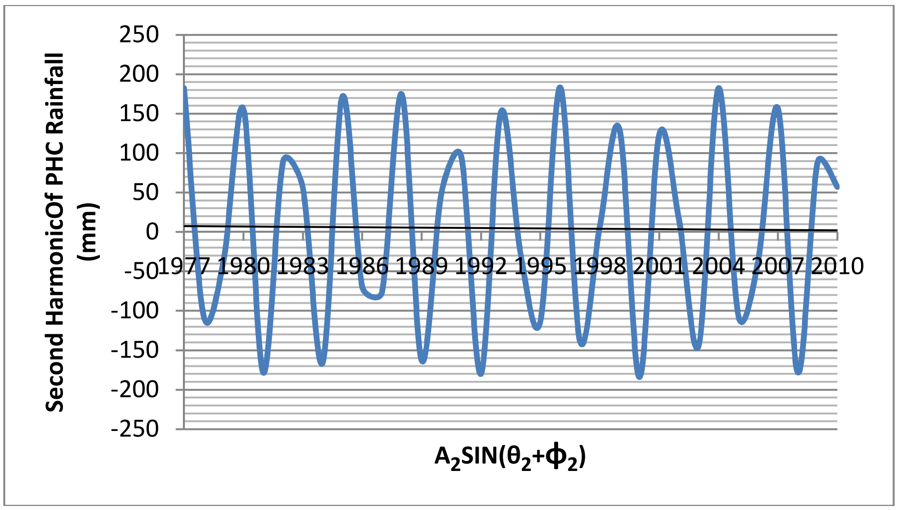

Fig.4: Second Harmonics of Portharcourt Rainfall (mm)

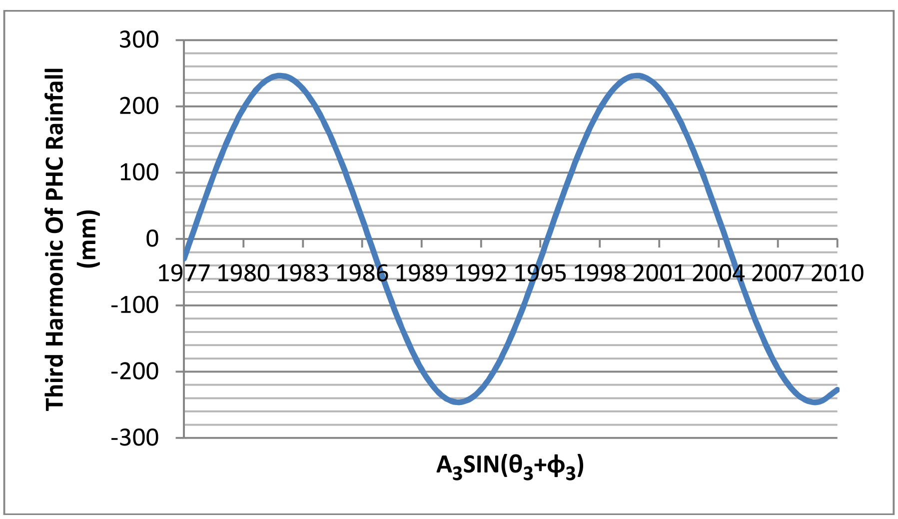

Fig. 5: Third Harmonics of Portharcourt Rainfall (mm)

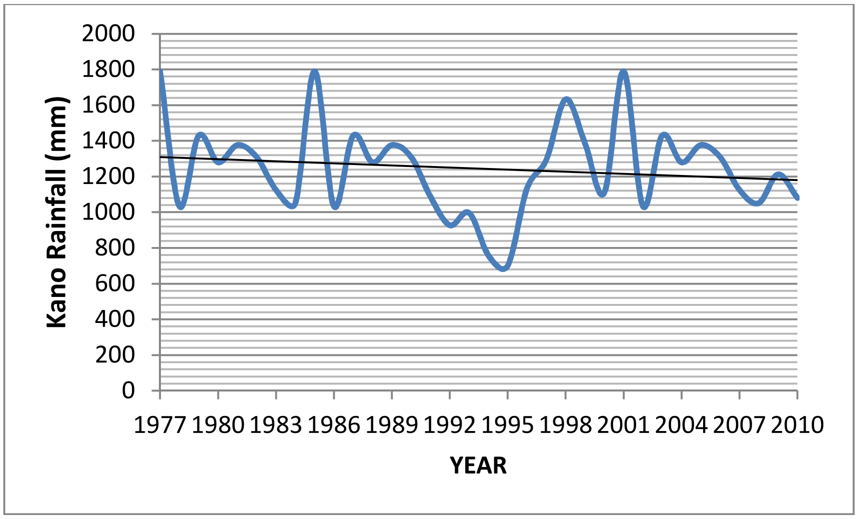

Fig. 6: Kano Rainfall(mm)

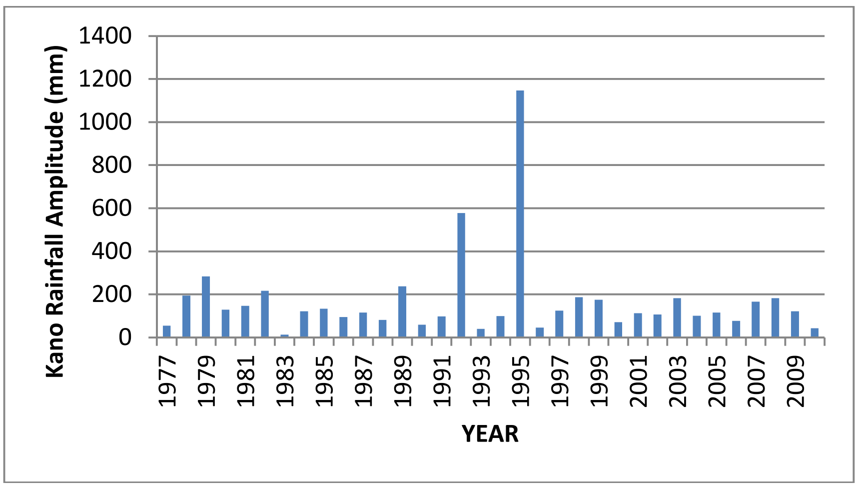

Fig. 7: Amplitude of Kano Rainfall(mm)

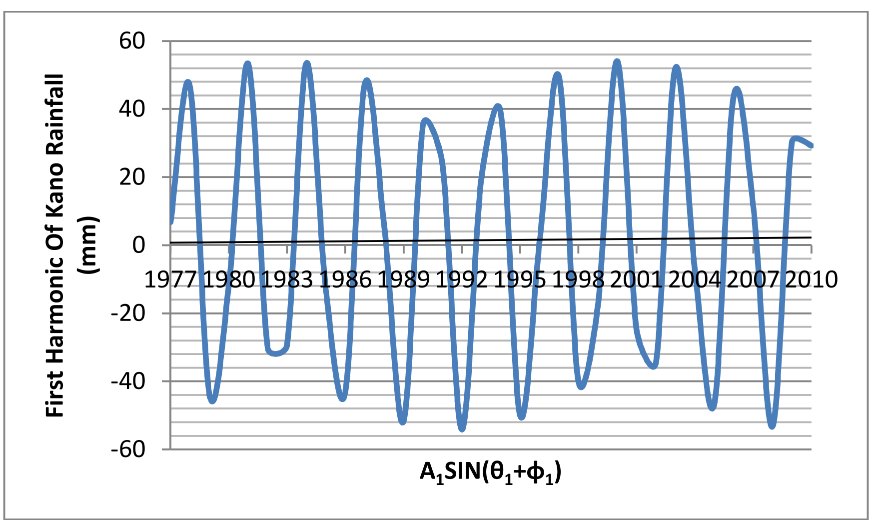

Fig. 8: First Harmonics of Kano Rainfall (mm)

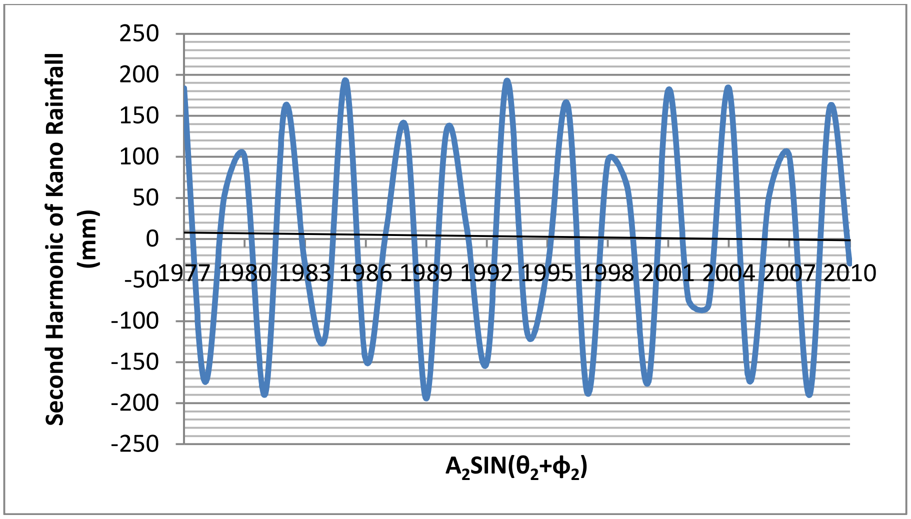

Fig. 9: Second Harmonics of Kano Rainfall(mm)

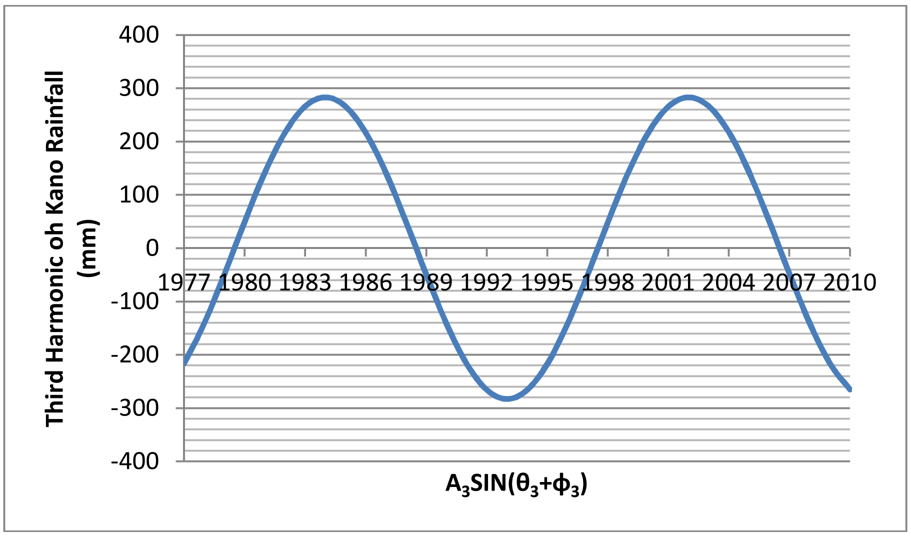

Fig. 10: Third Harmonics of Kano Rainfall(mm)

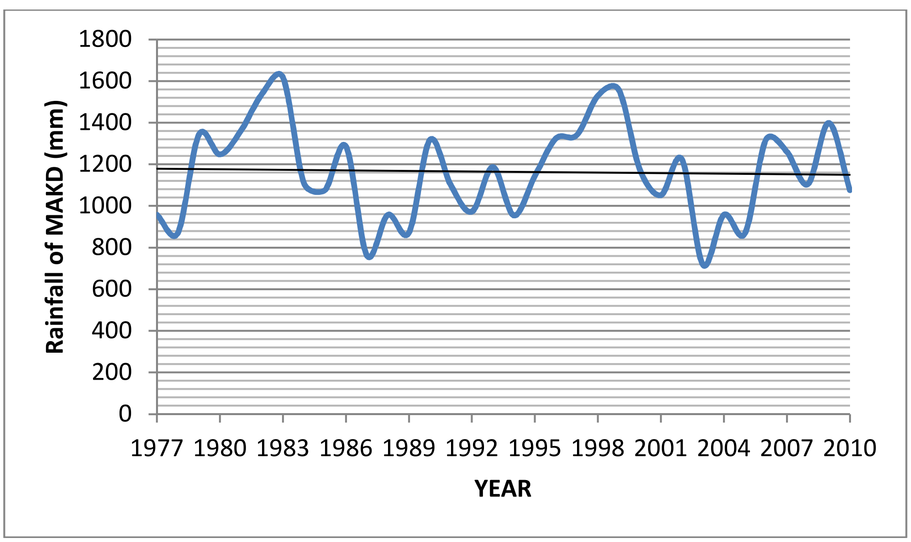

Fig. 11: Makurdi Rainfall(mm)

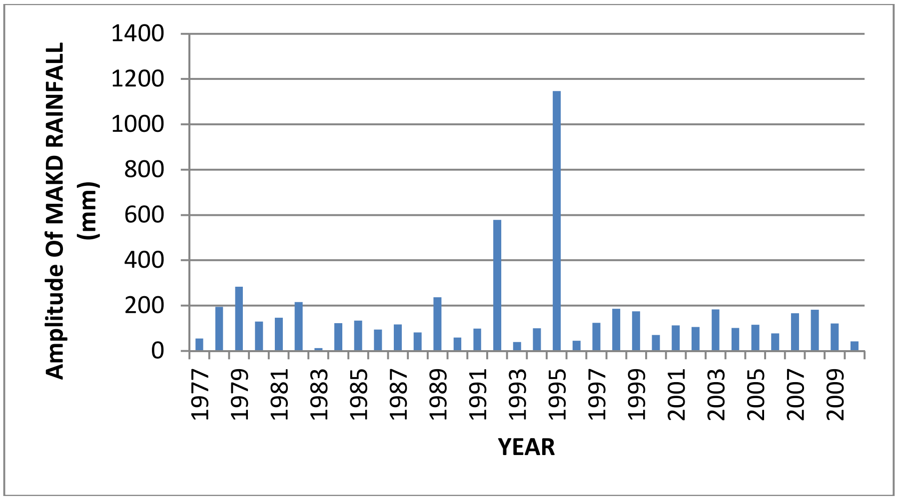

Fig. 12: Amplitude of Makurdi Rainfall(mm)

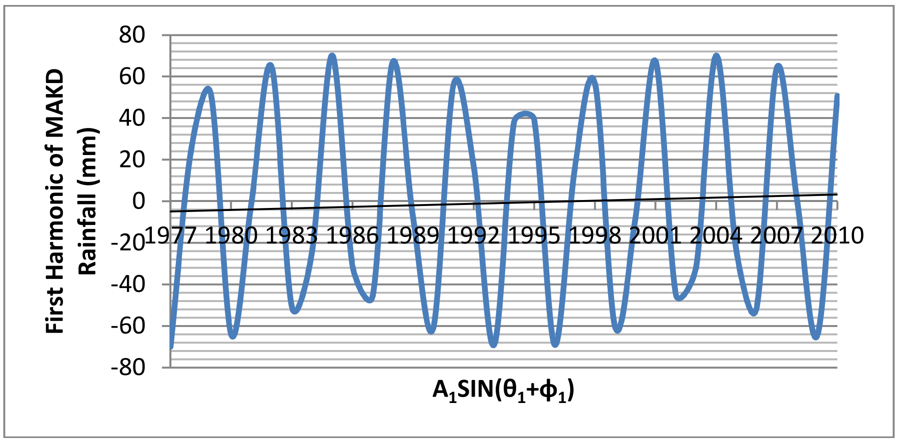

Fig. 13: First Harmonics of Makurdi Rainfall(mm)

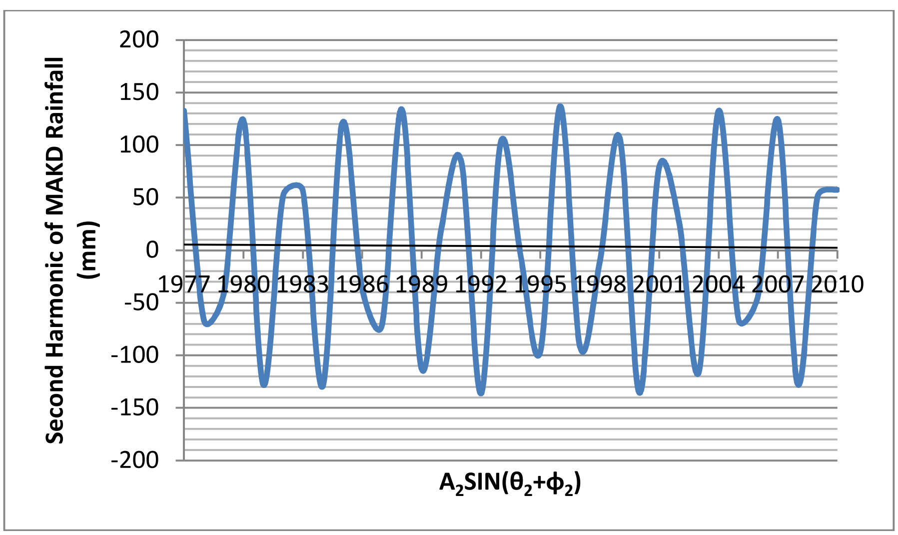

Fig. 14: Second Harmonics of Makurdi Rainfall(mm)

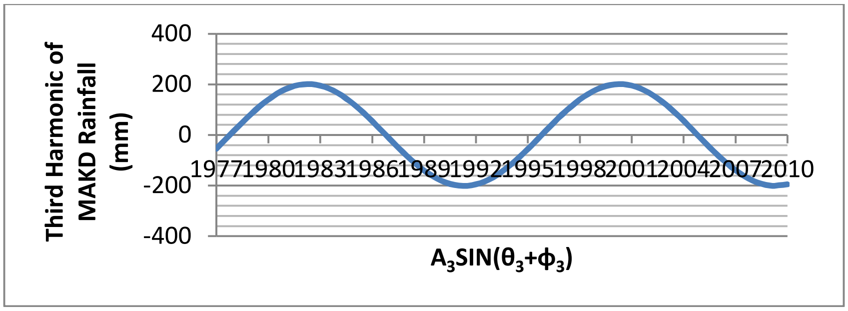

Fig. 15: Third Harmonics of Makurdi Rainfall(mm)

Fig, 3 is the First harmonic chart of Portharcourt rainfall showing the sinusoids having periodicity of 5 years; this signifies that the cycle is repeated in every five years through the 34 years period. Fig, 4 has the second harmonics for Portharcourt rainfall with a periodicity of 3.5 years while the third harmonic for Portharcourt is shown in Fig 5 having a seventeen years periodicity.

Fig, 6 gives the time series chart for Kano rainfall showing much variation and generally a declining trend in the rainfall regime. Fig, 7 is the amplitude of Kano rainfall; here the amplitude of significance fall in the years 1992 and 1995, and Fig, 8 is the first harmonic of Kano rainfall with periodicity of 3 years, the second harmonics for Kano rainfall is shown in Fig, 9 having a periodicity of 3 years while Fig, 10 is the third harmonic of Kano rainfall showing periodicity of 18 years.

Fig, 11 is the time series chart of Makurdi rainfall from 1999 to 2010; it has a semicyclical pattern of variation and generally shows a decreasing trend. Fig, 12 is the amplitude chart for Makurdi rainfall showing two amplitudes of significance in 1992 and 1995 respectively. Fig, 13 is the first harmonic chart for Makurdi rainfall showing a periodicity of 3 years cycle. Fig, 14 is the second harmonic for Makurdi rainfall with a periodicity of 4 years while, Fig, 15 is the third harmonics for Makurdi rainfall with a periodicity of 19 years.

4.0 Conclusion

The rainfall pattern for the three stations considered have clear variability with Kano rainfall as the most variable. The amplitude of significance was also determined and Portharcourt has more amplitude of significance, while the harmonics worked on for the stations fall within the range of 3, 5 and 18 years periodicity.Considering the wide margin between the first and third harmonics, the second and third harmonics and the close margin between the first and the second harmonics for all the stations, we draw the conclusion that the periodicities worth considering are as given by the first and second harmonics respectively. For the three stations studied, the rainfall variability takes on a seasonal change pattern occurring in the range of 3 to 4 years. This implies that the magnitude of rainfall experienced by these areas undergoes a significant change after a period of 3 to 4 years.

5.0 References

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