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Modelling the Time Series Data of the Impact of the Infection Rate on the Viral Load of the Virions

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Abstract

The critical HIV infection of decreasing viral load of the virions due to the variability of the infection rate constant is a challenging health problem that can now be tackled computationally on the implementation of a numerical simulation indexed by a shorter experimental time in the unit of days. The results of this study are novel which have not seen elsewhere with the expectation of providing an insight on how this endemic health issue can be managed.

1.0 Introduction

While the numerical simulation of the viral load of the virions of HIV infection of CD^+ T-cells and other extensions of this topic have successfully been considered as a basis to build a sound capacity building initiative as a part of HIV/AIDS intervention strategy [1, 2, 3], the critical issue of quantifying the impact of the variability of the infection rate constant on the viral load of the virions remains to be a vital open problem. This proposed problem has a strong link to the medical science HIV/AIDS intervention that requires a mathematical reasoning in order to successfully find a sustainable solution to this endemic health issue. It is against this background that one is proposing to study in great detail how the increasing value of the infection rate constant can affect the viral load of the virions for a short duration of the experimental time in the unit of days.

2.0 Mathematical Formulation

Following [1], we consider the system of time dependent non-linear first order ordinary differential equations

$$\frac{dT}{dt} = s - dT + aT\left(1 - \frac{T}{T_{max}}\right) - \quad TV \tag{1}$$

$$\frac{dI}{dt} = {}_1TV - ¢I \tag{2}$$

$$\frac{dV}{dt} = °I - cV \tag{3}$$

with the initial conditions $T(0) \geq 0, I(0) \geq 0, V(0) \geq 0$ .The notation T is called the number of target cells while the notations I and V are called the number of infected cells and the viral load of the virions at time t in the unit of days. The notation s stands for the rate at which new T cells are created from sources within the body such as the thymus whereas the notation a is called the maximum proliferation rate of target cells. The notation T_{max} stands for the T population density at which proliferation shuts off whereas the notation d stands for the death rate of the T cells.

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The notation $_1$ is represented by the exponential equation $_1 = e^{-m²}$ where is the infection rate constant whereas the term $e^{-m²}$ accounts for cells that are infected ² time units later. The notation ¢ stands for the death rate of infective cells whereas the notation ° is the reproductivity rate of the infected cell. The notation c represents the clearance rate constant of virions. The precise parameter values are

$d = 0.01, ¢ = 0.5, c = 10, a = 6.8, T_{\max} = 1300, s = 5, \quad = 0.0002, ° = 1000$

This model formulation did not look at the impact of the infection rate constant on the viral load of the virionswith its health policy [1]. This omitted idea is a vital issue for the purpose of mitigating against this endemic infection. It against this background that we have proposed the present method of tackling this problem.

3.0 Method of Analysis

In this study, the infection rate constant was first fixed and the solution trajectory of the formulated mathematical model was simulated and indexed using a short duration of twenty (20) days and the initial data of (100, 100, 100). Next, the infection rate constant was varied for 150 percent, 160 percent, 170 percent, 250 percent, 300 percent, 400 percent and other multiples of the infection rate constant such as 8 and 15. The impact of varying the infection rate constant on the viral load of the virions was calculated using the idea that the ratio of the viral load of the virions indexed by time due to the variation of the infection rate constant and the viral load of the virions indexed by time without a variation of the infection rate constant must be bounded above by a whole number one (1) under a simplifying assumption, the percentage of the viral load of the virions that was destroyed was calculated. The results of this procedure are presented and discussed in the next section of this study.

4.0 Results and Discussion

For the purpose of a clearer presentation of these results, one has considered to discuss the effect of each variation of the infection rate constant on the viral load of the virions when the experimental time ranges from one (1) day to twenty (20) days.

Scenario 1 Results

When the infection rate constant is fixed, our numerical simulation predicts that the viral load of the virions will range from the initial data of 100 to 7211 whereas when the infection rate constant is 0.0003, the predicted viral load of the virions will range from the initial data of 100 to 5962. Apart from the initial viral load of the virions which is 100, every other viral load of the virions that remains when the new infection rate constant value of 0.0003 was implemented is smaller than the value of the viral load of the virions when the infection rate constant is fixed. On the basis of these calculations, it is clear that the percentage of the viral load of the virions that has been affected when the infection rate constant is 0.0003 ranges from the value of zero to 17.3 approximately. These results are displayed in Table 1. What if the new value of the infection rate constant is 0.00032? The results of this analysis are presented in Table 2.

Table 1: Calculating the percentage of the viral load of the virions destroyed when the infection rate constant is 0.0003.

Example	Time in days	Viral load of the virions with fixed =0.0002	Viral load of the virions with changing =0.0003	Proportion destroyed
1	1	100	100	0
2	2	54365	53662	1.3
3	3	43613	41694	4.4
4	4	30698	29319	4.5
5	5	21778	20824	4.4
6	6	15826	15148	4.3
7	7	11897	11384	4.3
8	8	9354	8921	4.6
9	9	7784	7355	5.5
10	10	6905	6411	7.2
11	11	6549	5914	9.7
12	12	6578	5726	12.95
13	13	6846	5729	16.31
14	14	7181	5821	18.94
15	15	7429	5920	20.31
16	16	7524	5982	20.49
17	17	7480	6000	19.78
18	18	7375	5992	18.75
19	19	7275	5974	17.88
20	20	7211	5962	17.33

Scenario 2 Results

In this situation, the viral load of the virions when the infection rate constant is 0.00032 ranges from 100 to 5800 such that the percentage of the viral load of the virions that has been affected ranges from the value of zero to 19.6 approximately. At the end of twenty (20) days, the percentage of the viral load of the virions that has been destroyed is 19.6 in contrast to the 17.3 percentage of the viral load of the virionswhen the infection rate constant is 0.0003.

Table 2: Calculating the percentage of the viral load of the virions destroyed when the infection rate constant is 0.00032.

Example	Time in days	Viral load of the virions with fixed =0.0002	Viral load of the virions with changing =0.00032	Proportion destroyed
1	1	100	100	0
2	2	54365	53522	1.55
3	3	43613	41448	4.97
4	4	30698	29144	5.06
5	5	21778	20703	4.94
6	6	15826	15061	4.83
7	7	11897	11319	4.86
8	8	9354	8866	5.22
9	9	7784	7300	6.22
10	10	6905	6350	8.04
11	11	6549	5839	10.85
12	12	6578	5626	14.48
13	13	6846	5600	18.21
14	14	7181	5664	21.12
15	15	7429	5744	22.68
16	16	7524	5800	22.91
17	17	7480	5822	22.16
18	18	7375	5821	21.07
19	19	7275	5811	20.13
20	20	7211	5800	19.57

Scenario 3 Results

In this situation, the viral load of the virions when the infection rate constant is 0.00034 ranges from 100 to 5657 such that the percentage of the viral load of the virions that has been affected ranges from the value of zero to 21.6 approximately. At the end of twenty (20) days, the percentage of the viral load of the virions that has been destroyed is 21.6 in contrast to the 17.3 percentage of the viral load of the virions when the infection rate constant is 0.0003.

Table 3: Calculating the percentage of the viral load of the virions destroyed when the infection rate constant is 0.00034.

Example	Time in days	Viral load of the virions with fixed =0.0002	Viral load of the virions with changing =0.00034	Proportion destroyed
1	1	100	100	0
2	2	54365	53388	1.80
3	3	43613	41228	5.47
4	4	30698	28988	5.57
5	5	21778	20595	5.44
6	6	15826	14984	5.32
7	7	11897	11260	5.36
8	8	9354	8817	5.74
9	9	7784	7251	6.85
10	10	6905	6297	8.82
11	11	6549	5772	11.87
12	12	6578	5537	15.83
13	13	6846	5487	19.85
14	14	7181	5530	22.99
15	15	7429	5595	24.68
16	16	7524	5645	24.97
17	17	7480	5669	24.22
18	18	7375	5672	23.10
19	19	7275	5665	22.13
20	20	7211	5657	21.55

Scenario 4 Results

In this situation, the viral load of the virions when the infection rate constant is 0.0005 ranges from 100 to 4912 such that the percentage of the viral load of the virions that has been affected ranges from the value of zero to 31.9 approximately. At the end of twenty (20) days, the percentage of the viral load of the virions that has been destroyed is 31.9 in contrast to the 17.3 percentage of the viral load of the virions when the infection rate constant is 0.0003.

Table 4: Calculating the percentage of the viral load of the virions destroyed when the infection rate constant is 0.0005

Example	Time in days	Viral load of the virions with fixed =0.0002	Viral load of the virions with changing =0.0005	Proportion destroyed
1	1	100	100	0
2	2	54365	52555	3.33
3	3	43613	40056	8.16
4	4	30698	28161	8.27
5	5	21778	20023	8.06
6	6	15826	14577	7.90
7	7	11897	10951	7.95
8	8	9354	8557	8.53
9	9	7784	6999	10.08
10	10	6905	6017	12.87
11	11	6549	5425	17.16
12	12	6578	5102	22.44
13	13	6846	4949	27.70
14	14	7181	4894	31.85
15	15	7429	4885	34.25
16	16	7524	4892	34.98
17	17	7480	4902	34.47
18	18	7375	4908	33.47
19	19	7275	4912	32.48
20	20	7211	4912	31.88

Scenario 5 Results

In this situation, the viral load of the virions when the infection rate constant is 0.0006 ranges from 100 to 4650 such that the percentage of the viral load of the virions that has been affected ranges from the value of zero to 35.5 approximately. At the end of twenty (20) days, the percentage of the viral load of the virions that has been destroyed is 31.9 in contrast to the 17.3 percentage of the viral load of the virions when the infection rate constant is 0.0003.

Table 5: Calculating the percentage of the viral load of the virions destroyed when the infection rate constant is 0.0006

Example	Time in days	Viral load of the virions with fixed =0.0002	Viral load of the virions with changing =0.0006	Proportion destroyed
1	1	100	100	0
2	2	54365	52193	3.99
3	3	43613	39613	9.17
4	4	30698	27850	9.28
5	5	21778	19808	9.05
6	6	15826	14424	8.86
7	7	11897	10836	8.92
8	8	9354	8460	9.56
9	9	7784	6908	11.26
10	10	6905	5915	14.34
11	11	6549	5306	18.98
12	12	6578	4954	24.69
13	13	6846	4770	30.33
14	14	7181	4684	34.77
15	15	7429	4651	37.40
16	16	7524	4642	38.30
17	17	7480	4643	37.93
18	18	7375	4646	37.01
19	19	7275	4648	36.11
20	20	7211	4650	35.53

Scenario 6 Results

In this situation, the viral load of the virions when the infection rate constant is 0.0007 ranges from 100 to 4461 such that the percentage of the viral load of the virions that has been affected ranges from the value of zero to 38.2 approximately. At the end of twenty (20) days, the percentage of the viral load of the virions that has been destroyed is 38.2 in contrast to the 17.3 percentage of the viral load of the virions when the infection rate constant is 0.0003.

Table 6: Calculating the percentage of the viral load of the virions destroyed when the infection rate constant is 0.0007

Example	Time in days	Viral load of the virions with fixed =0.0002	Viral load of the virions with changing =0.0007	Proportion destroyed
1	1	100	100	0
2	2	54365	51909	4.52
3	3	43613	39282	9.93
4	4	30698	27618	10.03
5	5	21778	19648	9.78
6	6	15826	14310	9.58
7	7	11897	10750	9.64
8	8	9354	8389	10.31
9	9	7784	6840	12.13
10	10	6905	5843	15.38
11	11	6549	5221	20.27
12	12	6578	4850	26.27
13	13	6846	4645	32.15
14	14	7181	4539	36.79
15	15	7429	4489	39.57
16	16	7524	4469	40.61
17	17	7480	4462	40.35
18	18	7375	4461	39.51
19	19	7275	4460	38.69
20	20	7211	4461	38.15

Scenario 7 Results

In this situation, the viral load of the virions when the infection rate constant is 0.0008 ranges from 100 to 4321 such that the percentage of the viral load of the virions that has been affected ranges from the value of zero to 40.1 approximately. At the end of twenty (20) days, the percentage of the viral load of the virions that has been destroyed is 40.1 in contrast to the 17.3 percentage of the viral load of the virions when the infection rate constant is 0.0003.

Table 7: Calculating the percentage of the viral load of the virions destroyed when the infection rate constant is 0.0008

Example	Time in days	Viral load of the virions with fixed =0.0002	Viral load of the virions with changing =0.0008	Proportion destroyed
1	1	100	100	0
2	2	54365	51678	4.94
3	3	43613	39024	10.52
4	4	30698	27437	10.62
5	5	21778	19523	10.36
6	6	15826	14222	10.14
7	7	11897	10684	10.20
8	8	9354	8335	10.90
9	9	7784	6789	12.79
10	10	6905	5788	16.18
11	11	6549	5157	21.25
12	12	6578	4774	27.42
13	13	6846	4553	33.50
14	14	7181	4433	38.27
15	15	7429	4372	41.15
16	16	7524	4342	42.29
17	17	7480	4328	42.14
18	18	7375	4323	41.38
19	19	7275	4322	40.60
20	20	7211	4321	40.09

Scenario 8 Results

In this situation, the viral load of the virions when the infection rate constant is 0.0016 ranges from 100 to 3838 such that the percentage of the viral load of the virions that has been affected ranges from the value of zero to 46.8 approximately. At the end of twenty (20) days, the percentage of the viral load of the virions that has been destroyed is 46.8 in contrast to the 17.3 percentage of the viral load of the virions when the infection rate constant is 0.0003.

Table 8: Calculating the percentage of the viral load of the virions destroyed when the infection rate constant is 0.0016

Example	Time in days	Viral load of the virions with fixed =0.0002	Viral load of the virions with changing =0.0016	Proportion destroyed
1	1	100	100	0
2	2	54365	50704	6.74
3	3	43613	38004	12.86
4	4	30698	26727	12.94
5	5	21778	19035	12.60
6	6	15826	13877	12.32
7	7	11897	10427	12.36
8	8	9354	8126	13.13
9	9	7784	6598	15.24
10	10	6905	5590	19.05
11	11	6549	4933	24.68
12	12	6578	4511	31.43
13	13	6846	4244	38.01
14	14	7181	4078	43.21
15	15	7429	3977	46.47
16	16	7524	3916	47.95
17	17	7480	3880	48.14
18	18	7375	3858	47.69
19	19	7275	3845	47.15
20	20	7211	3838	46.79

Scenario 9 Results

In this situation, the viral load of the virions when the infection rate constant is 0.003 ranges from 100 to 3617 such that the percentage of the viral load of the virions that has been affected ranges from the value of zero to 49.8 approximately. At the end of twenty (20) days, the percentage of the viral load of the virions that has been destroyed is 49.8 in contrast to the 17.3 percentage of the viral load of the virions when the infection rate constant is 0.0003.

Table 9: Calculating the percentage of the viral load of the virions destroyed when the infection rate constant is 0.003

Example	Time in days	Viral load of the virions with fixed =0.0002	Viral load of the virions with changing = 0.003	Proportion destroyed
1	1	100	100	0
2	2	54365	50083	7.88
3	3	43613	37399	14.25
4	4	30698	26309	14.30
5	5	21778	18748	13.92
6	6	15826	13676	13.59
7	7	11897	10281	13.59
8	8	9354	8010	14.36
9	9	7784	6496	16.55
10	10	6905	5490	20.50
11	11	6549	4825	26.33
12	12	6578	4388	33.29
13	13	6846	4104	40.06
14	14	7181	3920	45.41
15	15	7429	3802	48.82
16	16	7524	3727	50.47
17	17	7480	3679	50.82
18	18	7375	3649	50.53
19	19	7275	3629	50.11
20	20	7211	3617	49.84

The emphasis of our previous work [1] was on the modelling of the stresses suffered by people living with HIV/AIDS in terms of the viral load of the virions of HIV infection of $CD4^+$ T-cells. The predicted viral loads of the virions of HIV infection of $CD4^+$ T-cells were clearly differentiated for two values of the maximum proliferation rate of target cells when the random-noise intensities were selected to be 0.8 and 8. Using the same system of model equations, this study has extended this contribution to investigate the impact of the changes of the infection rate constant on the proportion of the viral loads of the virions that can be destroyed. This is an additional cutting-edge contribution over our recent work [1] that was considered before this level of sophisticated numerical simulation. The proportions of the viral loads that can be destroyed were numerically determined when the infection rate constant takes the values of 0.0003, 0.00032, 0.00034, 0.0005, 0.0006, 0.0007, 0.0008, 0.0016 and 0.003. These nine (9) instances of the variations of the infection rate constant on the viral loads of the virions in the context of the HIV/AIDS intervention strategy are capable to provide further insight on some aspect of the effective HIV/AIDS control mechanism based on the application of a database system design upon which an early sound monitoring HIV/AIDS prevention measures can be undertaken. This is the key contribution of this study over our previous study [1].

5.0 Conclusion

On the basis of this numerical simulation, we have found that irrespective of the variation of the infection rate constant, the viral load of the virions remains at the value of 7211. However, when the infection rate constant changes as this study has clearly demonstrated, the volume of the viral load of the virions tends to respond from a mild to a relatively severe level of impact. We would expect this contribution to provide some sort of insight on how to effectively confront and manage this health problem. Increased in the viral load of virions leads to the depletion of $CD4^+$ T-Cells that can guide treatment of HIV patients.

6.0 References

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