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Computation and Simulation of Circuit Topology Describing Secular Equilibrium Decay

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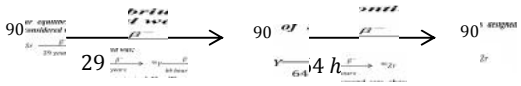
Abstract

A circuit topology for simulation of artificial coupled differential equation of Secular equilibrium decay of Strontium was designed. The equilibrium decay considered was;



An integrating time constant of 33 millisecond was chosen so as to minimize integrating error and a maximum input voltage level of 10V was chosen for the design in order to avoid saturation of the Operational Amplifiers used for integrators, Amplitude and time scaling where employed on the differential equations in order to convert the equations to voltage sources, the amplitude scale factors of $K_{A1} = 3.3070x10^{-3}Vsec$, $K_{A0} = 1.002x10^{-1}V$, $K_{B1} = 13.1475Vsec$ ***and*** $K_{B0} = 398.41V$ ***were computed from the problem parameters, The Percentage error on the average of*** $\pm 0.9\%$ ***obtained in this research means that the circuit designed simulated the differential equation describing the Secular equilibrium decay for Strontium with a fairly reasonable accuracy.***

1.0 Introduction

Equilibrium is used to describe the condition where the derivative of a function is equal to zero. In radioactive decay it relates to the situation where the number of atoms of a particular kind does not change with time. Clearly, this is a contradiction, since radioactive decay implies change, whereas equilibrium does not. Whereas absolute equilibrium is not possible, equilibrium is adequately approximated in many important situations whenever the parent species is substantially longer-lived than the daughter nuclides[1].In 2002, Nichol [2]revealed that energy release resulting from the decay of the radioactive nuclides formed in the reactor core is extremely important in formulating safe procedures for the operation of nuclear facilities and the handling of irradiated fuel. Accurate estimates of this resulting decay heat are needed for a wide range of applications, including safety assessments of all types of nuclear plant and the management of the resulting radioactive waste. More specifically, the nuclear power community must ensure accurate and reliable calculations of the decay heat of irradiated fuel in order to maintain credibility and confidence in the safe and reliable performance of the various nuclear fuel cycles. To achieved safety simulation process was involved, a process that employs a computerized model of certain significant features of some physical system. The object of the process of simulation is to provide an experimental model for the accumulation of data on target system. This paper presents a design of simple circuit topology to simulate and solve a set of coupled differential equations describing an artificial radioactive decay.

2.0 Theoretical Analysis

Radioactive decay series: In a number of cases a radioactive nuclide A decays into a nuclide B which is also a radioactive; the nuclide B decays into C which also a radioactive, and so on. For example Strontium 90 ^{90}Sr decays into a series of ten successive radionuclides. Substantially all the primary products of nuclear fission are negatron beta-ray emitters [3].

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Which decays through a chain of series two to six successive beta- ray emitters before a stable nuclide is reached as an end product, as in equation (1).

(1)

For the initial part of such series atoms of type A, B, C, D ... have radioactive decay constants $\lambda_A, \lambda_B, \lambda_C, \lambda_D$ ... then if there are initially present, at time $t = 0$ atoms of N_0, the number N_A, N_B, ... of types A, B, C ... when there are only three species, which is primary importance to nuclear medicine, and according to John [1], the differential equation describing the decay and the buildup of various nuclides are

$$\frac{dN_A}{dt} = -\lambda_A N_A \qquad (2)$$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \qquad (3)$$

$$\frac{dN_C}{dt} = -\lambda_C N_C \qquad (4)$$

Whose solutions are

$$N_A(t) = N_0 e^{-\lambda_A t} \qquad (5)$$

$$N_B(t) = N_0 \frac{\lambda_A}{\lambda_B - \lambda_A}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right) \qquad (6)$$

$$N_C(t) = N_0\left(\frac{\lambda_A}{\lambda_C - \lambda_A}\frac{\lambda_B}{\lambda_B - \lambda_A}e^{-\lambda_A t} + \frac{\lambda_A}{\lambda_A - \lambda_B}\frac{\lambda_B}{\lambda_C - \lambda_B}e^{-\lambda_B t} + \frac{\lambda_A}{\lambda_A - \lambda_C}\frac{\lambda_B}{\lambda_B - \lambda_C}e^{-\lambda_C t}\right) \qquad (7)$$

Radioactive Equilibrium: The ratio of the activities of the parent A and the daughter product B change with time. The activity is zero initially and also after a very long time, when all the atoms have decayed. Thus it passes through a maximum value, and it can be shown that this occurs at a time t_{max} given by

$$t_{max} = \frac{\ln(\lambda_B/\lambda_A)}{(\lambda_B - \lambda_A)} \qquad (8)$$

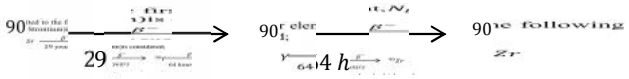
The situation in which the activities are exactly equal to each other is called **Ideal Equilibrium**, and exist only at the moment t_{max}. If the parent A is longer-lived than the daughter B, as occur in many cases, then at a time which is long compared with the mean time of B, the activity ratio approaches a constant value given by

$$\frac{B\lambda_B}{A\lambda_A} = \frac{\lambda_B}{\lambda_B - \lambda_A} = \frac{T_A}{T_A - T_B} \qquad (9)$$

Where T_A and T_B are the half-life of A and B. When the activity ratio $B\lambda_B/A\lambda_A$ is constant, a particular equilibrium exists; this is spoken as **Secular Equilibrium** if the activity ratio is experimentally indistinguishable from unity, as occur when T_A is very much greater than T_B.

However, when T_B is comparable with T_A to some extent the equilibrium ratio will clearly exceed unity, this situation is spoken as **Transient Equilibrium**[4-6].

Thus, this analysis will be limited to the first daughter element, N_B, and the following radioactive decay illustrating artificial Secular equilibrium decay (for Strontium)is considered;



(10)

The design seeks to obtain $N_A(t)$ and $N_B(t)$ In form of voltage level which could be simulated and measured on the proposed analogue circuit. In practice it is easier to implement integral operation than the differential operation since computer signal are considered to be real voltages and, therefore, are corrupted by noise to some extent. Since integration has a tendency to average out the effects of noise (while differentiation will accentuate it), a more precise solution can be obtained using integration techniques by Paz[7]. In order to limit the integrating error which increases with time, an integrating time constant of $33msec$ was carefully chosen for each integrator, such that the feed-back capacitors and input resistors take values of 1.0 μF and 100Ω respectively. To avoid saturation of the 741 operational amplifiers used a maximum voltage level of 10 $volts$ was chosen for this design [8-11].

Amplitude (magnitude) scaling: amplitude scaling according to Oroge in [12] is given as

$$\textbf{Scale factor, } K = \frac{\textbf{1 machine Unit } (M.U)}{\textbf{maximum value of variable}} \qquad (11)$$

Which are defined as

$$K_{A1} = \frac{|V|_{max}}{\left|\frac{dN_A}{dt}\right|_{max}} \qquad (12)$$

$$K_{B1} = \frac{|V|_{max}}{\left|\frac{dN_B}{dt}\right|_{max}} \quad (13)$$

$$K_{A0} = \frac{|V|_{max}}{|N_A|_{max}} \quad (14)$$

$$K_{B0} = \frac{|V|_{max}}{|N_B|_{max}} \quad (15)$$

Generally, time scaling for the derivative of x is given by

$$\frac{d^n x}{dt^n} = \alpha^n \frac{d^n x}{dT^n} \quad (16)$$

Where t is the time at which a phenomenon actually occurs and T is the time required for the phenomenon to occur on the computer, $\alpha$is the scaling factor given as

$$\alpha = \frac{T}{t_0} \quad (17)$$

Combining amplitude and time scaling, the computer patched program from equation (2) and (3) is obtained as follow, replacing dt withdT

$$\left(K_{A1}\frac{dN_A}{dT}\right) = \left[-\frac{\lambda_A K_{A1}}{\alpha K_{A0}}\right](K_{A0}N_A) \quad (18)$$

$$\left(K_{B1}\frac{dN_B}{dT}\right) = \left[\frac{\lambda_A K_{B1}}{\alpha K_{A0}}\right](K_{A0}N_A) - \left[\frac{\lambda_B K_{B1}}{\alpha K_{B0}}\right](K_{B0}N_B) \quad (19)$$

Terms in brackets are computer terms while terms in square brackets are potentiometer coefficients and amplifier gains. The corresponding patch program is shown in Figure 1.

The Integrator is an essential circuit component in any analog circuit that performs mathematical operation of Integration mainly in solving differential equation [10]. Many systems in nature are describedby differential equations, differential equations havebeen a widely applied language used in modelingour world. Simulation requires solution of thesedifferential equations to provide the informationa model contains [8-13].

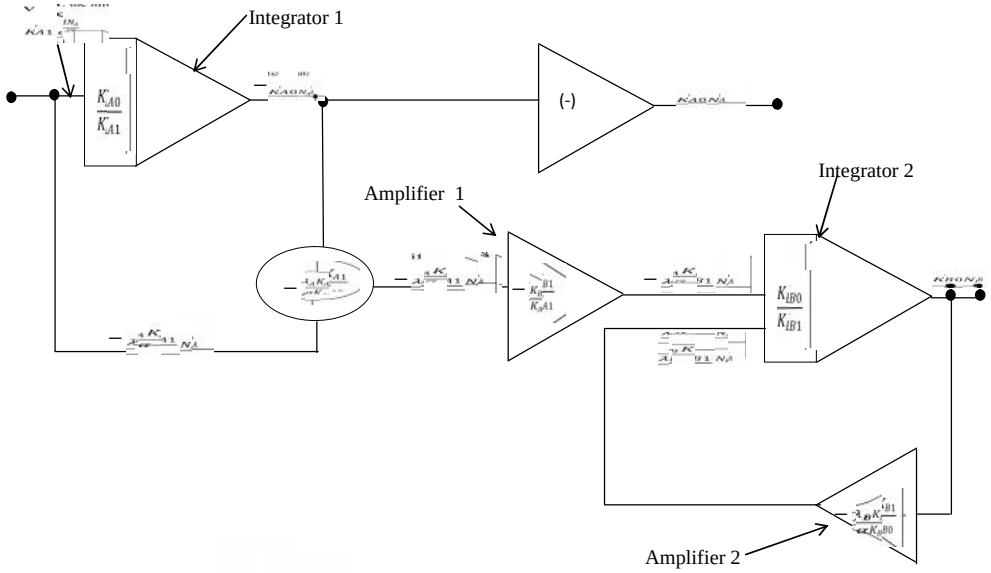


Figure 1: The complete corresponding computer patched program

When a voltage signal of $K_{A1}\frac{dN_A}{dT}$ volts is applied at the input of the integrator 1, with a gain of $\frac{K_{A0}}{K_{A1}}$, an inverted out signal of $-K_{A0}N_A$volt would be obtained which can be inverted by an amplifier of unity gain, whose input resistance equals its feed-back resistance, a positive signal of $K_{A0}N_A$ volt would be obtained. The output $K_{A0}N_A$ represents the solutionof the differential equation of the first radioactive decay of equation (5).The scaled functionsof $\lambda_A N_A$and$\lambda_B N_B$ are applied to theinput network of the summing integrator 2, the inputresistors and the integrator feed-back capacitors are carefully chosen defending on the gain of each integrator according to equation (19).

For the daughter cell, the function $K_{B1}\frac{dN_B}{dT}$ is integrated and is inverted in sign by the second integrator to an output voltage of $K_{B0}N_B$. This output signal voltage represents the solution of the differential equation of second radioactive decay of equation (6), $-\frac{\lambda_A K_{A1}}{\alpha K_{A0}}$ represents the potentiometer coefficient, $-\frac{K_{B1}}{K_{A1}}$ and $-\frac{\lambda_B K_{B1}}{\alpha K_{B0}}$ represent the gains of amplifier 1 and 2, $\frac{K_{A0}}{K_{A1}}$and $\frac{K_{B0}}{K_{B1}}$ represent the gains of the integrators 1 and 2 respectively.

3.0 Experimental Analysis

Using an initial number of nuclides of $|N_o|_{max} = 100$ the scale variables and other parameters for the computer patched program were obtained, at secular equilibrium $t = t_{max}$, as follows:

$T(computer\ time) = 33x10^{-3} sec$ $\quad t_0(real\ time) = 9.1515x10^{8}\ sec$

$t_{max} = 2.7559x10^{6}\ sec$ $\quad \alpha = 3.6060x10^{-11}$

$\lambda_A = 7.5725x10^{-10} sec^{-1}$ $\quad \lambda_B = 3.0078x10^{-6} sec^{-1}$

$N_A = 99.79$ $\quad N_B = 2.51x10^{-2}$

$\left|\frac{dN_A}{dT}\right|_{max} = 3023.90$ $\quad \left|\frac{dN_B}{dT}\right|_{max} = 0.7606$

$K_{A1} = 3.3070x10^{-3} Vsec$ $\quad K_{A0} = 1.002x10^{-1} V$

$K_{B1} = 13.1475 Vsec$ $\quad K_{B0} = 398.41 V$

Gains of Integrators and Amplifiers;

Gain of integrator1; $\beta_1 = \frac{K_{A0}}{K_{A1}} = 30.36 = \frac{1}{j\omega R_1 C_1}$

Gain of integrator2; $\beta_2 = \frac{K_{B0}}{K_{B1}} = 30.30 = \frac{1}{j\omega R_2 C_2}$

Where $j\omega = 2\pi f$

Choosing $R_1 = R_2 = 100\Omega$ and $C_1 = C_2 = 1.0\mu F$ are good choice for integrator1 and 2 to have gains of 30.36 and 30.30 respectively.

For the inverting amplifier $R_i = R_f = 1.0K\Omega$

Gain of Amplifier1; $A_{V1} = \frac{K_{B1}}{K_{A1}} = 3975.7$

Gain of Amplifier2; $A_{V2} = \frac{\lambda_B K_{B1}}{\alpha K_{B0}} = 2752.6$

For Amplifier 1 with a gain; $\boldsymbol{A_{V1} = 3975.7}$

$\boldsymbol{A_{V1} = -\frac{R_{f1}}{R_{i1}}}$**; Choosing** $\boldsymbol{R_{i1} = 68\Omega, R_{f1} = 270K\Omega}$**. (Preferred)**

For Amplifier 2 with a gain, $\boldsymbol{A_{V2} = 2752.6}$

$\boldsymbol{A_{V2} = -\frac{R_{f2}}{R_{i2}}}$**; Choosing** $\boldsymbol{R_{i2} = 68\Omega, R_{f2} = 180K\Omega}$**. (Preferred)**

4.0 Results and Discussion

The circuit topology was developed, coupled and simulated using Multisim Simulation and capture software version 8 as shown on Figure 2. The integrating time of $33msec$ where used and obtained the the value of the time scaling factor $\alpha = 3.6060x10^{-11}$. The potentiometer coefficient obtained were approximately 0.7 (70%) with a gain of approximately 30 for each integrator. This value will ensure that the scale factors would be limited within the voltage range and the solution would not be outside the voltage range of operation. Scaled quantities in the form of $\frac{K_{B0}N_B}{K_{A0}N_A}$ were compared with corresponding decay constant ratio $\frac{\lambda_A}{\lambda_B}$ and a good agreement to within percentage error of 0.9%on the average was obtained, as can be seen on Table 1.

The result obtained also agreed with the **secular Equilibrium Approximation** $\frac{N_B}{N_A} \approx \frac{\lambda_A}{\lambda_B}$.

Table 1: Simulation Result.

S/N	$\frac{K_{B0}N_B}{K_{A0}N_A}$	$\frac{K_{B0}N_B}{K_{B0}N_B}$	$\frac{K_{B0}N_B}{K_{A0}N_A}$	$\frac{\lambda_A}{\lambda_B}$
1	5.000	0.001248900	0.00024978	0.00025176
2	2.500	0.000635050	0.00025402	-
3	1.250	0.0003125250	0.00025002	-
4	0.625	0.00016001875	0.00025603	-
5	0.312	0.00007834944	0.00025112	-

5.0 Conclusion

The Percentage error on the average of $\pm 0.9\%$ obtained in this research means that the circuit designed simulated the differential equation describing the Secular equilibrium decay for Strontium with a fairly accuracy. This showed that risks involved in safety experiment can be greatly reduced and understanding the equilibrium for a given decay series through simulation, scientists can estimate the amount of radiation that will be present at various stages of the decay, therefore making some industries accident free.

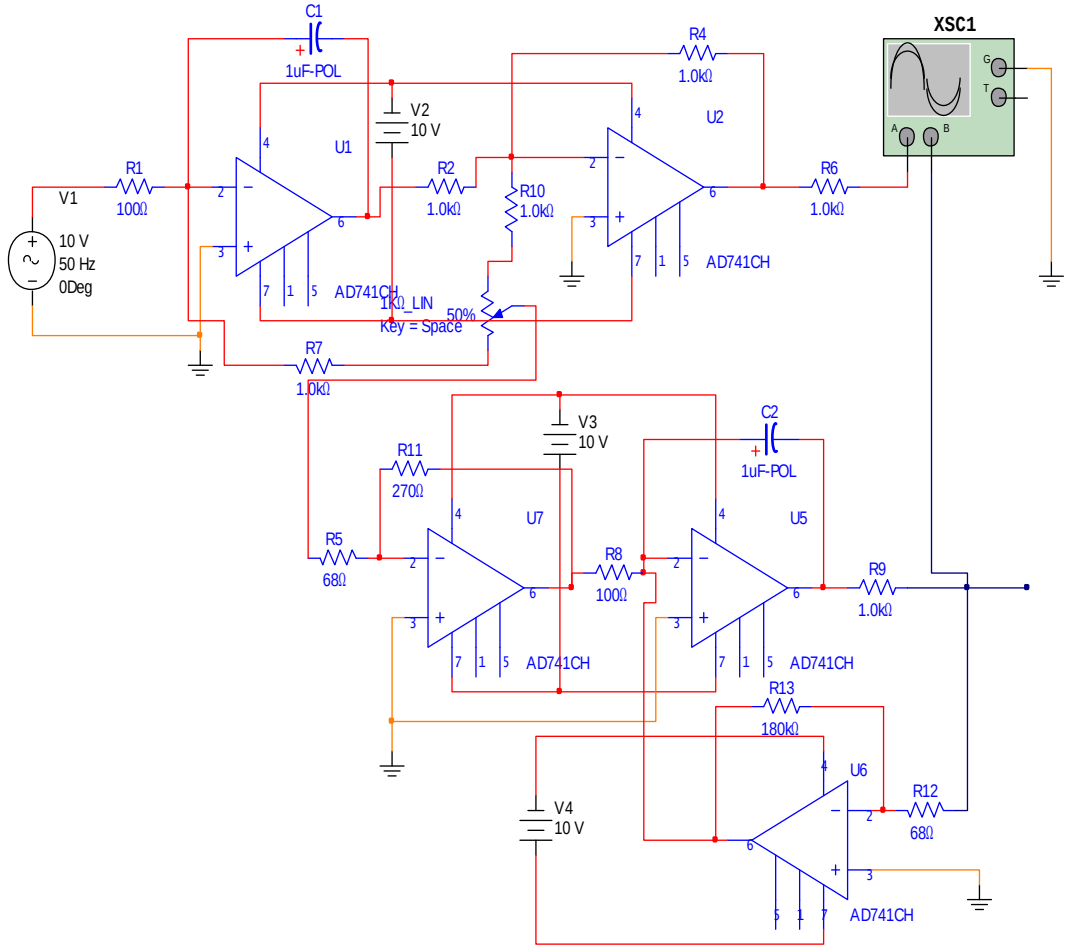


Figure 2: Circuit topology for Simulation of coupled differecial equition

6.0 References

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