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# On Ionization and Porosity in MHD Couette Flow of a Two-Component Plasma

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## *Abstract*

***The effect of ionization and porosity on the field structure of two-component plasma is studied. The flow is initiated by a relative motion of two horizontal flat plates in the presence of a uniformly applied magnetic field. A fully developed, viscous, incompressible flow with no convection current applies and the profile together with the effect of ionization and porosity on the velocity, temperature and induced magnetic field studied for astrophysical and geophysical systems.***

**Keywords:** Couette flow, two-component plasma, MHD, porosity, ionization

**Nomenclature**

$u'$ dimensional velocity component
$(x', y', z')$ dimensional cartesian coordinates
k thermal conductivity M Hartmann number
$D'$ dimensional electric displacement
$C_p$ specific heat at constant pressure
Ec Eckert number
$H_0$ constant transverse magnetic field
$R_H$ Magnetic pressure number
$H_x$ induced magnetic field
$T_0$ constant temperature at the lower wall
Pr Prandtl number
Q volumetric heat source
f collision frequency
$R_M$ Magnetic Reynolds number
$A_0$ Alfven speed
N Ratio of Alfven to wall velocity
$M_A$ dimensionless parameter
$k_1$ dimensional porosity parameter
$T_1$ constant temperature at the upper wall

**Greek symbols**

σ electrical conductivity of ionized particles
ε electric permittivity
Γ dimensionless temperature gradient
$\pm_{u,l}$ electrical conductance of plate's
ν kinematic viscosity
ρ hydrodynamic density
$\kappa_{in}^2$ ***Dacy number***
$\mu_{in}$ coefficient of viscosity
$\mu_m$ magnetic permeability
$\pm_{i,n}^2$ Wormesley frequency type parameter
$¤_{u,l}$ electrical conductance ratio
β measure of degree of ionization
$\beta_3$ heat source parameter
a magnetic diffusivity

## 1.0 Introduction

Usually in classical magneto hydrodynamics and plasma physics, a fully ionized plasma is assumed to simplify the real-life situation of partial ionization and hence neutral-ionized particle interaction. In this study which is an extension of the work of Alabraba et al. [1] a partially ionized plasma is assumed and the following procedure is adopted. First the formulation of the problem is done by describing the regime of application and derivation of governing equations with solutions of velocity and induced magnetic fields. Using these solutions; the expressions for the temperatures are obtained. Finally comments on the profile and effects of ionization and porosity are presented quantitatively.

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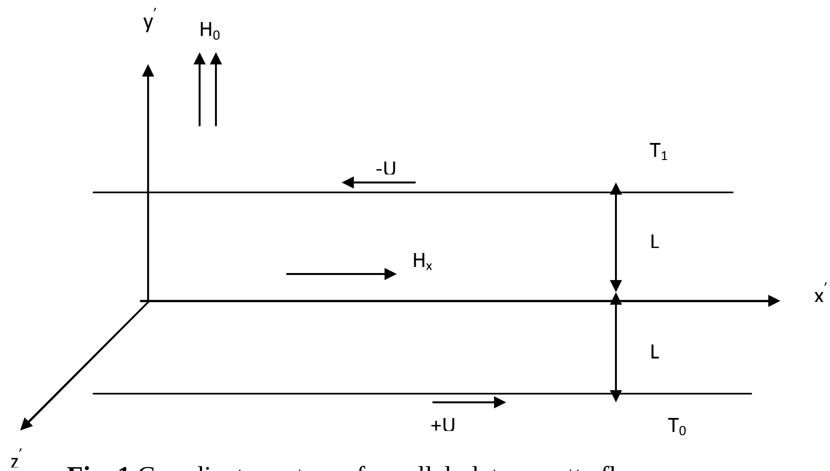



**Fig. 1** Coordinate system of parallel plate couette flow

## 2.0 Mathematical Formulation

We consider the flow of partially ionized plasma. The flow is viscous, incompressible and bounded by two horizontal parallel plates moving relative to each other along their common axis with velocities $\mp U_0$. The plates are maintained at temperatures $T_1$ and $T_o$ $(T_1 > T_0)$ and employing Cartesian coordinate system $(x', y', z')$ with one plate at $y = L$ while the other at $y = -L$. The system is infinite along the x and z direction (See Fig. 1). Such systems are obtained in geophysics where geophysical fluids like crude oil is trapped between tectonic plates moving relative to each other with constant earth's magnetic field, constant heat source and very low ionization (1% to 2%). Also in astrophysics as found in the ionosphere trapped between 90km to 500km above the earth surface; having high degree of ionization (90% to 99%) and the upper and lower layers moving relative to each other with constant heat source.

Assuming a steady and fully developed flow along the x direction, all the physical variables will depend on y except temperature, With such a flow the governing equations are:

Continuity

$$\nabla \bullet \vec{V}_{in}' = 0 \qquad (1)$$

Modified Navier- Stokes equation

$$\rho_{in}\left(\vec{V}_{in}' \bullet \nabla\right)\vec{V}_{in}' = \mu_{in}\nabla^2\vec{V}_{in}' + \mu_m\vec{J}_{in}' \times H' + F\left(\pm\vec{V}_{in}'\right) - \frac{v_{in}}{k_1}\vec{V}_{in}' \qquad (2)$$

Energy equation

$$\rho_{in}c_p\left(\vec{V}_{in}' \bullet \nabla\right)T_{in}' = k_{in}\nabla^2T_{in}' + \mu_{in}\left(\frac{\partial U_{in}'}{\partial y'}\right)^2 + \frac{J'^2}{\sigma} + Q_{in}' \qquad (3)$$

the substantial derivative (i.e. derivative following the motion) is used

The system is infinite in x and z direction puts $\frac{\partial}{\partial x} = \frac{\partial}{\partial z} = 0$ and since it is steady $\frac{\partial}{\partial t} = 0$

where $J_n' = 0$, the subscripts and later superscripts i, n are ionized and neutral components respectively. $\frac{J^2}{\sigma}$ is the Ohmic dissipation $F(\pm\vec{V}_{in}')$ is the collision frictional force term, $\mu_{in}\nabla^2\vec{V}_{in}'$ is the viscous force term, $\mu_m J_{in}' \times \vec{H}'$ is the magnetic body force term and $\frac{v_{in}}{k_1}\vec{V}_{in}'$ is the term introduced due to porosity. The electromagnetic field equations take the form :

$$\nabla \times \vec{H} = \vec{J} \qquad (4)$$

$$\nabla \bullet \vec{B} = 0, \; \nabla \bullet \vec{D} = 0 \qquad (5)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0 \qquad (6)$$

$$\vec{B} = \mu_m\vec{H}, \; \vec{D} = \varepsilon\vec{E} \qquad (7)$$

and the modified Ohms law $\vec{J} = \sigma\{\vec{E} + \mu_m(\vec{V} \times \vec{H})\}$ (8)

From Eqs. (4), (6) and (8) the interaction between flow velocity and magnetic field becomes

$$\nabla \times \left(\nabla \times \vec{H}\right) = \mu_m\sigma\nabla \times \left(\vec{V} \times \vec{H}\right) \qquad (9)$$

From Eqs. (5) and (7) we get $H_y = constant$ which we write as $H_0$ , the applied magnetic field. Consequently we set

$$\vec{V}_{in}' = U'(y')\hat{\imath}, \; \vec{H} = H_x'(y')\hat{\imath} + H_0'\hat{\jmath} \qquad (10)$$

where $H_x'$ is the induced magnetic field and $\hat{i}$ , $\hat{j}$ are respectively unit vectors along x' and y' directions.

It can be deduced that all variables are functions of $y'$ only except $T'$ which has an additional dependence on $x'$.
Splitting Eqs. (2), (3) and (9) into ionized particle and neutral specie equations and writing them in component form gives the dimensional equations:

$$0 = \mu_i \frac{d^2 U_i'}{dy'^2} + \mu_m H_0' \frac{dH_x'}{dy'} + f_i \rho_i' \beta' (U_n' - U_i') - \frac{\nu_i}{k_1} U_i' \quad (11)$$

$$0 = \mu_n \frac{d^2 U_n'}{dy'^2} - f_n \rho_n' \beta' (U_n' - U_i') - \frac{\nu_n'}{k_1} U_n' \quad (12)$$

$$0 = \frac{d^2 H_x'}{dy'^2} + \mu_m \sigma H_0 \frac{dU_i'}{dy'} \quad (13)$$

$$\circ_i C_p U_i' \frac{\partial T_i'}{\partial x'} = k_i \left( \frac{\partial^2 T_i'}{\partial x'^2} + \frac{\partial^2 T_i'}{\partial y'^2} \right) + \text{«}_i \left( \frac{dU_i'}{dy'} \right)^2 + \frac{1}{\pm} \left( \frac{dH_x'}{dy'} \right)^2 + Q \quad (14)$$

$$\circ_n C_p U_n' \frac{\partial T_n'}{\partial x'} = k_n \left( \frac{\partial^2 T_n'}{\partial x'^2} + \frac{\partial^2 T_n'}{\partial y'^2} \right) + \text{«}_n \left( \frac{dU_n'}{dy'} \right)^2 + Q \quad (15)$$

where the collision frictional force which is the coupling factor between the ionized and neutral species is defined as
$\vec{F}(\vec{V}') = f_i \circ_i \left( \vec{V}_n' - \vec{V}_i' \right)$[2]
To facilitate analysis it is expedient to introduce the non-dimensional variables:

$$x, y = \frac{x', y'}{L} \;\; ; U_{i,n} = \frac{U_{i,n}'}{U_0} \;\; ; \;\; H_x = \frac{H_x'}{H_0}$$

Substituting this into Eqs. (11), (12) and (13) we get

$$\frac{d^2 U_i}{dy^2} + R_M N \frac{dH_x}{dy} + \sigma_i^2 \beta (U_n - U_i) - \kappa_i^2 U_i = 0 \quad (16)$$

$$\frac{d^2 U_n}{dy^2} - \sigma_n^2 \beta (U_n - U_i) - \kappa_n^2 U_n = 0 \quad (17)$$

$$\frac{d^2 H_x}{dy^2} + \frac{R_M M_A}{N} \frac{dU_i}{dy} = 0 \quad (18)$$

subject to the following boundary conditions:

(i) Hydrodynamic $\quad U_{i,n} = \mp 1 \quad$ on $\quad y = \pm 1$

(ii) Magneto-hydrodynamic [3]

$$\frac{dH_x}{dy} + \frac{1}{¤_u} H_x = 0 \text{ on } y=1$$

$$\frac{dH_x}{dy} - \frac{1}{¤_l} H_x = 0 \text{ on } y=-1$$

subscripts u and l stand for upper and lower plates and $¤_{u,l} = \frac{\pm_{u,l}}{\pm}$

We now solve the equations simultaneously and after integration, substitution and reconciliation of the constants we get the expressions

$$U_i = A_1 Cosh(m_1 y) + A_2 Sinh(m_1 y) + A_3 Cosh(m_2 y) + A_4 Sinh(m_2 y) + A_5 \quad (19)$$

$$U_n = A_1 B_1 Cosh(m_1 y) + A_2 B_1 Sinh(m_1 y) + A_3 B_2 Cosh(m_2 y) + A_4 B_2 Sinh(m_2 y) + A_5 \frac{\sigma_i^2 \beta}{\sigma_n^2 \beta + \kappa_n^2} \quad (20)$$

$$H_x = -\frac{R_M M_A}{N} \left\{ \frac{A_1 Sinh(m_1 y)}{m_1} + \frac{A_2 Cosh(m_2 y)}{m_2} + \frac{A_3 Sinh(m_2 y)}{m_2} + \frac{A_4 Cosh(m_2 y)}{m_2} \right\} + A_5 y \left\{ \sigma_i^2 \beta + \kappa_i^2 + R_M^2 M_A - \frac{\sigma_i^2 \beta \sigma_i^2 \beta}{\sigma_n^2 \beta + \kappa_n^2} - \frac{R_M M_A}{N} \right\} + A_6 \quad (21)$$

Where $A_1$ ; $A_2$ ; $A_3$ ; $A_4$ ; $A_5$ ; $A_6$ are arbitrary constants which are determined from the boundary conditions as a 6X6 determinant.

## 3.0 Fluid Temperature Under Variable Wall Temperature

We assume that the temperature varies linearly along the wall with a constant heat source. We therefore set

$$T'(x', y') = \bar{T}'(y') + x'\Gamma' \quad (22)$$

and write

$$T_{i,n}'(x', -L) = \bar{T}_{i,n}'(-L) + x'\Gamma' = T_0 \quad (23)$$

$$T_{i,n}'(x',L)=\overline{T}_{i,n}'(L)+x'\Gamma'=T_1 \tag{24}$$

$T_1 > T_0$

Differentiating Eq. (22) twice and substituting in Eq. (14) and (15) we get

$$\rho_i C_p U_i'\Gamma' = k_i\frac{d^2\overline{T}_i'}{dy'^2}+\mu_i\left(\frac{dU_i'}{dy'}\right)^2+\frac{1}{\sigma}\left(\frac{dH_x'}{dy'}\right)+Q \tag{25}$$

$$\rho_n C_p U_n'\Gamma' = k_n\frac{d^2\overline{T}_n'}{dy'^2}+\mu_n\left(\frac{dU_n'}{dy'}\right)^2+Q \tag{26}$$

In addition to the non-dimensional variables we include these two

$$\theta_{i,n}(y)=\frac{\overline{T}_{i,n}'-T_0}{T_1-T_0}$$

$$\Gamma=\frac{\Gamma' L}{T_1-T_0}$$

Eqs. (25) and (26) yield the following equations after substituting the above dimensionless variables

$$\frac{d^2\theta_i}{dy^2}+\beta_1^i\left(\frac{dU_i}{dy}\right)^2+\beta_2^i\left(\frac{dH_x}{dy}\right)^2+\beta_3^i-\beta_4^iU_i=0 \tag{27}$$

$$\frac{d^2\theta_n}{dy^2}+\beta_1^n\left(\frac{dU_n}{dy}\right)^2+\beta_3^n-\beta_4^nU_n=0 \tag{28}$$

Substituting for $\left(\frac{dU_i}{dy}\right)^2$ and $\left(\frac{dH_x}{dy}\right)^2$ in Eq. (27) and integrating twice we get an equation for $\theta_i$ with two arbitrary constants subject to the boundary conditions:

$\theta_i = 0$ on $y=-1$

$\theta_i = 1$ on $y=1$

This gives the final expression for $\theta_i$ as

$$\theta_i=\frac{1}{2}(1+y)+\beta_4^i\begin{bmatrix}\frac{A_1}{m_1^2}\{Cosh(m_1y)-Cosh(m_1)\}+\frac{A_2}{m_1^2}\{Sinh(m_1y)-ySinh(m_1)\}+\\ \frac{A_3}{m_2^2}\{Cosh(m_2y)-Cosh(m_2)\}+\frac{A_4}{m_2^2}\{Sinh(m_2y)-ySinh(m_2)\}+\frac{A_5}{2}(y^2-1)\end{bmatrix}-$$

$$-\frac{\beta_3^i}{2}(y^2-1)-\left(\beta_2^iM_AM^2\right)\begin{bmatrix}\frac{A_1^2}{2}\left\{\frac{Cosh(2m_1y)}{4m_1^2}+\frac{y^2}{2}-\frac{Cosh(2m_1)}{4m_1^2}-\frac{1}{2}\right\}+\frac{A_2^2}{2}\left\{\frac{Cosh(2m_1y)}{4m_1^2}-\frac{y^2}{2}-\frac{Cosh(2m_1)}{4m_1^2}+\frac{1}{2}\right\}+\\ \frac{A_3^2}{2}\left\{\frac{Cosh(2m_2y)}{4m_2^2}+\frac{y^2}{2}-\frac{Cosh(2m_2)}{4m_2^2}-\frac{1}{2}\right\}+\frac{A_4^2}{2}\left\{\frac{Cosh(2m_2y)}{4m_2^2}-\frac{y^2}{2}-\frac{Cosh(2m_2)}{4m_2^2}+\frac{1}{2}\right\}\end{bmatrix}$$

$$-\beta_2^iM_AM^2\begin{bmatrix}\frac{A_1A_2}{4m_1^2}\{Sinh(2m_1y)-ySinh(m_1)\}+\\ A_1A_3\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}-\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\ A_1A_4\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}+\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\ A_2A_3\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}-\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\ A_2A_4\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}+\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\ \frac{A_3A_4}{4m_2^2}\{Sinh(2m_2y)-ySinh(2m_2)\}\end{bmatrix}$$

$$
-\;{}_1^{\;i}\left[\begin{array}{l}
\frac{A_1^2 m_1^2}{2}\left\{\frac{Cosh(2m_1y)}{4m_1^2}-\frac{y^2}{2}-\frac{Cosh(2m_1)}{4m_1^2}+\frac{1}{2}\right\}+\frac{A_2^2 m_1^2}{2}\left\{\frac{Cosh(2m_1y)}{4m_1^2}+\frac{y^2}{2}-\frac{Cosh(2m_1)}{4m_1^2}-\frac{1}{2}\right\}+\\
\frac{A_3^2 m_2^2}{2}\left\{\frac{Cosh(2m_2y)}{4m_2^2}-\frac{y^2}{2}-\frac{Cosh(2m_2)}{4m_2^2}+\frac{1}{2}\right\}+\frac{A_4^2 m_2^2}{2}\left\{\frac{Cosh(2m_2y)}{4m_2^2}+\frac{y^2}{2}-\frac{Cosh(2m_2)}{4m_2^2}-\frac{1}{2}\right\}+\\
\frac{A_1A_2}{4}\left\{Sinh(2m_1y)-ySinh(2m_1)\right\}+A_1A_3m_1m_2\left\{\begin{array}{l}\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\\ -\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}+\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\end{array}\right\}+\\
A_1A_4m_1m_2\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}-\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\
A_2A_3m_1m_2\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}+\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\
A_2A_4m_1m_2\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}-\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\
\frac{A_3A_4}{4}\left\{Sinh(2m_{2y})-ySinh(2m_2)\right\}
\end{array}\right]
$$

In similar vein we deduce $\theta_n$

$$
-_n=\frac{1}{2}(1+y)+\;{}_4^{\;n}\left[\begin{array}{l}
\frac{B_1A_1}{m_1^2}\left\{Cosh(m_1y)-Cosh(m_1)\right\}+\frac{B_1A_2}{m_1^2}\left\{Sinh(m_1y)-ySinh(m_1)\right\}+\\
+\frac{B_2A_3}{m_2^2}\left\{Cosh(m_2y)-Cosh(m_2)\right\}+\frac{B_2A_4}{m^2}\left\{Sinh(m_2y)-ySinh(m_2)\right\}+\frac{A_5}{2}(y^2-1)
\end{array}\right]-
$$

$$
-\frac{{}_3^{\;n}}{2}(y^2-1)-\;{}_1^{\;n}\left[\begin{array}{l}
\frac{B_1^2A_1^2m_1^2}{2}\left\{\frac{Cosh(2m_1y)}{4m_1^2}-\frac{y^2}{2}-\frac{Cosh(2m_1)}{4m_1^2}+\frac{1}{2}\right\}+\frac{B_1^2A_2^2m_1^2}{2}\left\{\begin{array}{l}\frac{Cosh(2m_1y)}{4m_1^2}+\frac{y^2}{2}-\\ -\frac{Cosh(2m_1)}{4m_1^2}-\frac{1}{2}\end{array}\right\}+\\
+\frac{B_2^2A_3^2m_2^2}{2}\left\{\frac{Cosh(2m_2y)}{4m_2^2}-\frac{y^2}{2}-\frac{Cosh(2m_2)}{4m_2^2}+\frac{1}{2}\right\}+\frac{B_2^2A_4^2m_2^2}{2}\left\{\begin{array}{l}\frac{Cosh(2m_2y)}{4m_2^2}+\frac{y^2}{2}\\ -\frac{Cosh(2m_2)}{4m_2^2}-\frac{1}{2}\end{array}\right\}\\
+\frac{B_1^2A_1A_2}{4}\left\{Sinh(2m_1y)-ySinh(2m_1)\right\}
\end{array}\right]
$$

$$
-\;{}_1^{\;n}\left[\begin{array}{l}
B_1B_2A_1A_3m_1m_2\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}+\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\
B_1B_2A_1A_4m_1m_2\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}-\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\
B_1B_2A_2A_3m_1m_2\left\{\frac{Sinh[y(m_1+m_2)]}{(m_1+m_2)^2}-\frac{Sinh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{ySinh(m_1+m_2)}{(m_1+m_2)^2}+\frac{ySinh(m_1-m_2)}{(m_1-m_2)^2}\right\}+\\
B_1B_2A_2A_4m_1m_2\left\{\frac{Cosh[y(m_1+m_2)]}{(m_1+m_2)^2}+\frac{Cosh[y(m_1-m_2)]}{(m_1-m_2)^2}-\frac{Cosh(m_1+m_2)}{(m_1+m_2)^2}-\frac{Cosh(m_1-m_2)}{(m_1-m_2)^2}\right\}
\end{array}\right]
$$

$$- {}_1^n\left[\frac{B_2A_3A_4}{4}\{Sinh(2m_2y) - ySinh(2m_2)\}\right]$$

## 4.0 Results and Discussion

The effects of ionization and porosity on the Couette flow of a two-component plasma is presented in Astrophysical and Geophysical systems. To have a feel of the parameters used, numerical results are presented with constants as follows:

N=2.0, $\Pr^i = \Pr^n = 0.71$ , $R_M$=3.0 , $\mathrm{Re}^i = \mathrm{Re}^n = 1.0$ , $R_H$=2.0 , M=3.0 , $\Gamma = M_A = \pm_i^2 = \pm_n^2 = 1.0$ , $Ec^i = Ec^n = 0.1$ , ${}_3^i = {}_3^n = 2.0$, $¤_u = ¤_l = 1.0$ (upper and lower walls are conducting). Table 1 shows different values ofκ and β combined to give the graphs.

**(A)** Astrophysical Plasma

A striking feature is that this plasma which is about 90% (β= 0.11) to 99%(β=0.01) ionized, there is separation of the ionized specie from the neutral, as can be seen in **Fig.2a**with the neutral on either side of the channel higher in velocity. This is in good agreement with the result of Alabraba et al [1] where the ionization is 50% (β=1).

For this type of plasma since the ionized specie is dominant, we concentrate on it.

**(i)** The velocity is higher the greater the degree of ionization {from graph I(90%) to II(99%)}(See **Fig. 2b.)**

**(ii)** The induced magnetic field like the velocity is higher for graph II than graph I and becomes more appreciable as it gets close to the center of the channel.(See **Fig.2c**.)

**(iii)** The temperature difference is not appreciable for the level of ionization though graph II (99%)ionized is higher than graph I (90%) ionized.(See **Fig. 2d)**

**(B)** Geophysical Plasma (Crude Oil)

By the same reasoning as in astrophysical plasmas, the ionization in geophysical fluids is very low (1% to 2%) and so we consider the neutrals to be dominant. For this type of fluids we notice that the ionized particles have the same profile as the neutrals and so virtually no separation.(See**Fig. 3.)** Graphs III and IV show the effect of porosity which is measured by the Dacy number on the flow.

**(i)** Increase in porosity causes a decrease in velocity.(See **Fig. 4a.)**

**(ii)** Increase in porosity causes the induced magnetic field to be higher only beyond half the channel on both sides. (See **Fig 4b.)**

**(iii)** There is a slight increase in temperature beyond the positive half of the channel and peaks up to 1.64 on y=0.3. It then decreases to 1.0 on y=1. (See **Fig. 4c.)**

Finally the effect of ionization on geophysical fluids are shown in graphs III and V.
(See **Fig. 5a to 5c.)**

There is no noticeable effect on the velocity of the neutral particles, the induced magnetic field and the temperature.

**Table 1:** Combination of κ and β in the graphs

| **κ** | **β** | **Degree of Ionization** | **Graphs** |
|---|---|---|---|
| 0.1 | 0.11 | 90% | **I** |
| 0.1 | 0.01 | 99% | **II** |
| 0.1 | 99 | 1% | **III** |
| 2.0 | 99 | 1% | **IV** |
| 0.1 | 49 | 2% | **V** |

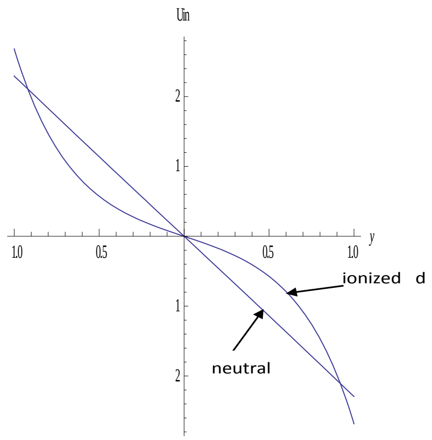



**Fig. 2a:** Velocity profile in couette flow of a two component plasma in Astrophysical fluids

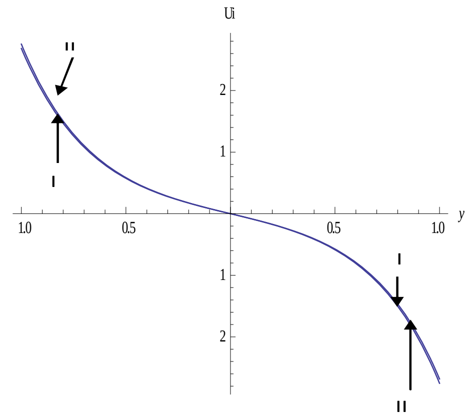



**Fig. 2b:** Effect of ionization on velocity of ionized specie in Astrophysical fluids

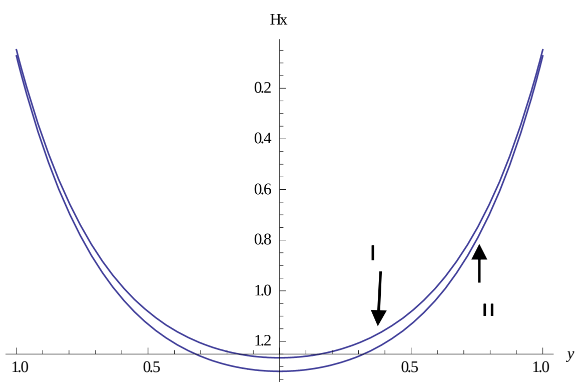



**Fig. 2c:** Effect of ionization on induced magnetic field of the ionized specie in Astrophysical fluids

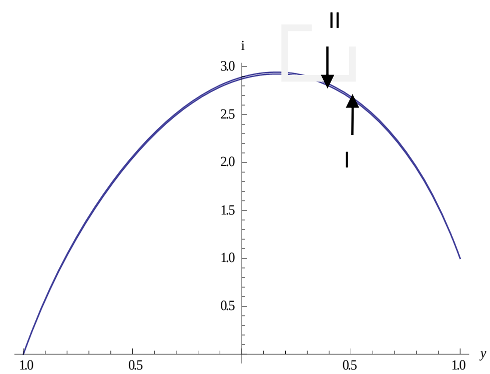



**Fig. 2d:** Effect of ionization on temperature of the ionized specie in Astrophysical fluid

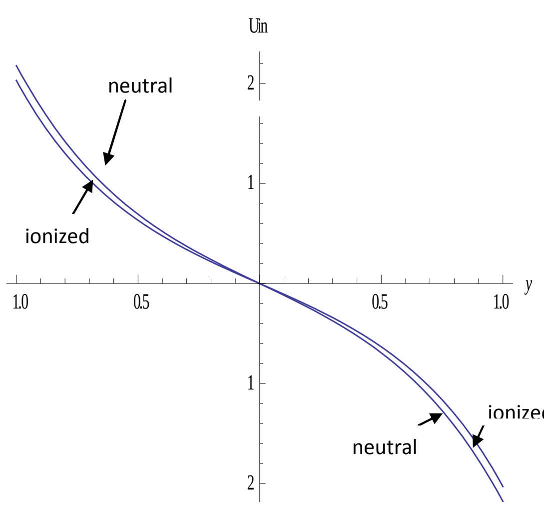
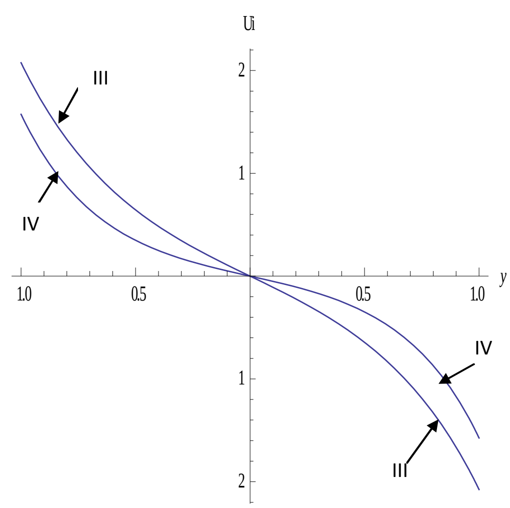
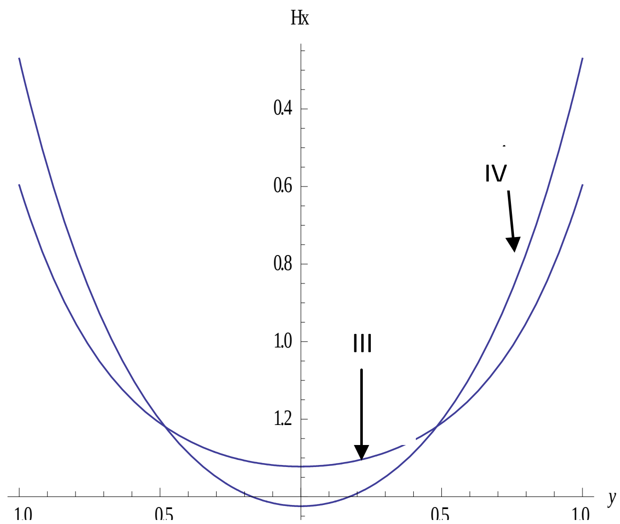



**Fig. 3:** Velocity profile in couette flow of a two component plasma in Geophysical fluid.




**Fig, 4a:** Effect of porosity on velocity in Geophysical fluid




**Fig, 4b:** Effect of porosity on induced magnetic field in Geophysical fluid.

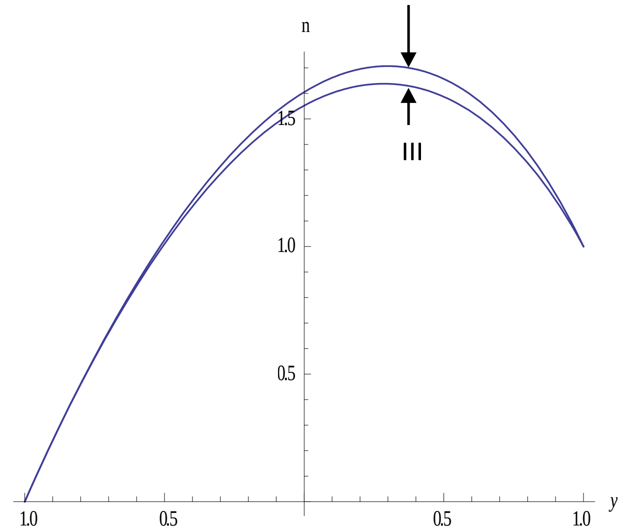



**Fig. 4c:** Effect of porosity on temperature in Geophysical fluid

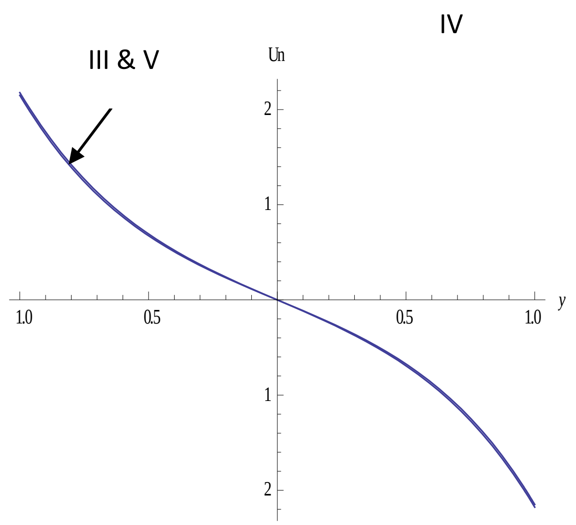



**Fig. 5a:** Effect of ionization on velocity in Geophysical fluids

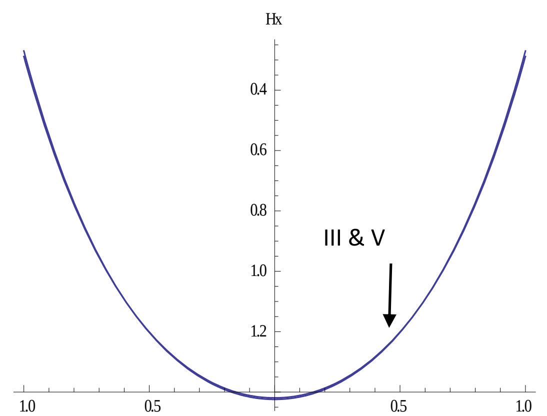



**Fig. 5b:** Effect of ionization on induced magnetic field in Geophysical fluids

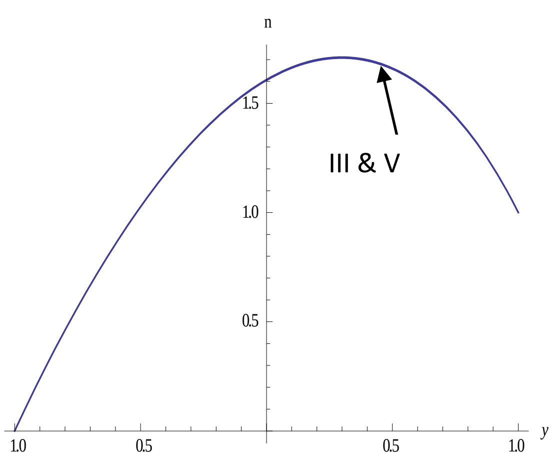



**Fig. 5c:** Effect of ionization on temperature in Geophysical fluids

## 5.0 Conclusion

We make the following conclusion by studying the effects of ionization and porosity in MHD parallel plate couette flow on astrophysical and geophysical systems:

**(i)** There is separation of the components in astrophysical systems but not in geophysical systems which has not been expressed in any literature.

**(ii)** Increased porosity in geophysical systems cause a decrease in velocity in accordance with flow in a porous medium, increase in induced magnetic field and slight increase in temperature while increase in ionization does not have effect on velocity, induced magnetic field or temperature.

**(iii)** For astrophysical systems increased ionization brings about increased velocity and temperature only slightly and increased induced magnetic field close to the center of the channel.

## 6.0 Appendix A

The following constants have been used

$$a = \sigma_i^2\beta + \kappa_i^2 + \sigma_n^2\beta + \kappa_n^2 + R_M^2 M_A$$

$$b = \kappa_n^2(\sigma_i^2\beta + \kappa_i^2 + R_M^2 M_A) + \kappa_i^2\sigma_n^2\beta + R_M^2 M_A \sigma_n^2\beta$$

$$A_0 = \sqrt{\frac{\mu_m H_0^2}{\rho_i}}$$

$$\nu_i = \frac{\mu_i}{\rho_i}$$

$$N = \frac{A_0}{U_0}$$

$$\sigma_{in}^2 = \frac{f_{in}L^2}{\nu_{in}}$$

$$M_A = \frac{\nu_i}{\lambda}$$

$$\lambda = \frac{1}{\mu_m \sigma}$$

$$\kappa_{in}^2 = \frac{\nu_{in}L^2}{k_1\mu_{in}}$$

$$\beta = \frac{\rho_n}{\rho_i}$$

$$\beta_1^{in} = \frac{\mu_{in}U_0^2}{k_{in}(T_1 - T_0)} = Pr^{in}Ec^{in}$$

$$\beta_2^{i} = \frac{H_0^2}{\sigma k_i(T_1 - T_0)} = \frac{R_H Pr^i Ec^i}{M_A}$$

$$\beta_3^{in} = \frac{Q_{in}L^2}{k_{in}(T_1 - T_0)}$$

$$\beta_4^{in} = \frac{\rho_{in}c_P U_0 L\Gamma}{k_{in}} = Pr^{in}Re^{in}\Gamma$$

$$R_M = \frac{A_0 L}{\nu_i}$$

$$R_H = \frac{\mu H_0^2}{\rho_i U_0^2}$$

$$m_1 = \sqrt{\frac{a + \sqrt{(a^2 - 4b)}}{2}}$$

$$m_2 = \sqrt{\frac{a - \sqrt{(a^2 - 4b)}}{2}}$$

$$B_1 = 1 - \frac{m_1^2}{\sigma_i^2 \beta} + \frac{\kappa_i^2 + R_M^2 M_A}{\sigma_i^2 \beta}$$
$$B_2 = 1 - \frac{m_2^2}{\sigma_i^2 \beta} + \frac{\kappa_i^2 + R_M^2 M_A}{\sigma_i^2 \beta}$$

## 7.0 References

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[2] R. K. Chhajlani , D.S. Vaghela, Gravitational Stability of finitely conducting two-component plasma through porous medium. Astrophys. Space Sci. 139 (1987) 337-352

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