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Extension of Einstein's Planetary Theory Based on Generalized Gravitational Scalar Potential

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Abstract

In this article, the generalized Einstein's radial equation of motion in the equatorial plane of the Sun is transformed to obtain additional correction terms to all order of c^2 to Einstein's planetary equation of motion and hence to the planetary parameters..

Keywords: Radial Equation, Planetary Equation, Planetary parameters.

1.0 Introduction

The well - known Einstein's radial equation of motion in the equatorial plane of the Sun [1, 2, 3, 4] is given as

$$\ddot{r} = -\frac{k}{r^2} + \frac{l^2}{r^3} + \frac{3kl^2}{c^2 r^4} = 0 \qquad (1)$$

The generalized Einstein's radial equation of motion in the equatorial plane of the Sun[5, 6, 7] is given explicitly as

$$\ddot{r} = -\frac{k}{r^2} - \frac{k^2}{c^2 \mathrm{R} r^2} + \frac{2k^2}{c^2 r^3} + \frac{l^2}{r^3} + \frac{3kl^2}{c^2 r^4} = 0 \qquad (2)$$

where c is the speed of light, r is the mean distance from the Sun, R is the radius of the planets and l is the angular momentum per unit rest mass. We have in this article; formulate a generalized Einstein's planetary equation of motion and its planetary parameters by transforming the generalized Einstein's radial equation of motion.

2.0 Theoretical Analysis

By transformation that

$$\ddot{r} = \frac{du}{dr}.u = u\frac{du}{dr} \qquad (3)$$

It follows that the general solution of equation (2) may be written as in the form

$$\left(1 + \frac{u^2}{c^2}\right)^2 = \frac{4}{c^2}\left[\frac{k}{r^2} - \frac{k^2}{c^2 Rr} + \frac{k^2}{c^2 r^2} + \frac{Kl^2}{C^2 r^3} - \frac{l^2}{2r^2}\right] dr \qquad (4)$$

where A is the constant of integration.
Integrating both sides and taking the initial condition as
$\dot{r} = 0; r = r_i; i = 1 \; or \; 2$, we obtain

$$\left(1 + \frac{u^2}{c^2}\right)^2 = \frac{4k}{c^2 r} - \frac{4k^2}{c^4 Rr} + \frac{4k^2}{c^4 r^2} + \frac{4kl^2}{c^4 r^3} - \frac{2l^2}{c^2 r^2} + 1 - \frac{4k}{c^2 r_1} + \frac{4k^2}{c^4 Rr_1} - \frac{4k^2}{c^4 r_i^2} - \frac{4kl^2}{c^4 r_1^3} + \frac{2l^2}{c^2 r_1^2} \qquad (5)$$

Expanding the left hand side of equation (5) and solving quadratically, we obtain

$$\dot{r}^2 = 2k\left(\frac{1}{\mathrm{r}} - \frac{1}{\mathrm{r}_1}\right) - \frac{2k^2}{c^2 R}\left(\frac{1}{\mathrm{r}} - \frac{1}{\mathrm{r}_1}\right) + \frac{2k^2}{c^2}\left(\frac{1}{\mathrm{r}^2} - \frac{1}{\mathrm{r}_i^2}\right) + \frac{2kl^2}{c^2}\left(\frac{1}{\mathrm{r}^3} - \frac{1}{\mathrm{r}_1^3}\right) - l^2\left(\frac{1}{\mathrm{r}^2} - \frac{1}{\mathrm{r}_1^2}\right) \qquad (6)$$

This is the exact generalized radial speed of the planet in terms of radial coordinate according to the generalized Einstein's geometrical theory of gravitation. It follows from (5) and (6) that

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$$\ddot{r} = -\frac{k}{r^2}\left[1+\frac{1}{c^2}\left(\frac{2k}{r_1}-\frac{l^2}{r_1^2}-\frac{k}{R}\right)\right]+\frac{l^2}{r^3}\left[1+\frac{1}{c^2}\left(\frac{2k}{r_1}-\frac{l^2}{r_1^2}\right)\right]+\frac{l^4}{c^2r^5}-\frac{6kl^2}{c^2r^4} \qquad (7)$$

This is the exact generalized radial acceleration of the planet in terms of radial coordinate according to the generalized Einstein's geometrical theory of gravitation. Similarly, it follows from (6) and (7) that the radial equation of motion for the particle in terms of the reciprocal distance and angular coordinate is given by

$$\frac{d^2v}{d\phi^2} = \frac{k}{l^2}\left[1+\frac{1}{c^2}\left(\frac{2k}{r_1}-\frac{l^2}{r_1^2}-\frac{k}{R}\right)\right]-v\left[1+\frac{1}{c^2}\left(\frac{2k}{r_1}-\frac{l^2}{r_1^2}\right)\right]-\frac{l^2v^3}{c^2}+\frac{6kv^2}{c^2} \qquad (8)$$

This is the planetary equation of motion according to Einstein's geometrical law of motion correct to the order of c^{-2}. The approximation of equation (8) is given by

$$\frac{d^2v}{d\phi^2}-\frac{k}{l^2}+v = \frac{6kv^2}{c^2} \qquad (9)$$

This is the exact equation of motion for the planet according to the generalized Einstein's geometrical law of motion. This equation possesses the exact solution

$$r(\phi) = \frac{A}{1-\varepsilon\cos\omega\phi} \qquad (10)$$

where

$$A = \left[\frac{l^2{}_o}{k}\left(1-\frac{k}{c^2R}\right)^{-1}\right] \qquad (11)$$

and

$$A_o = \frac{l^2{}_o}{k} \qquad (12)$$

is the corresponding Newtonian constant of the motion and

$$\varepsilon = 1-\frac{1}{r_i}\frac{l^2}{k}\left[\left(1-\frac{k}{c^2R}\right)^{-1}\right] \qquad (13)$$

is the orbital eccentricity of the orbit according to Einstein's geometrical law of motion and

$$\varepsilon_o = 1-\frac{1}{r_1}\frac{l^2}{k} \qquad (14)$$

is the corresponding pure Newtonian eccentricity.

Consequently, the generalized Einstein's geometrical law of motion has predicted the orbital angular frequency of the planets and comets as

$$\omega = \left[1+\frac{1}{c^2}\left(1+\frac{2k}{r_i}-\frac{l^2}{r_i{}^2}\right)\right]^{\frac{1}{2}}\omega_o \qquad (15)$$

All the expressions obtained in this paper reduces c^0 corresponding to the exact pure Newtonian and hence does not violates the Equivalence Principle in Physics. And to the order of c^2it contains additional correction terms not found in Newton's and Einstein's expressions.

3.0 Remarks and Conclusion

We have shown how to formulate a generalized Einstein's planetary equation of motion and the planetary parameters. The generalized radial speed, radial acceleration, planetary equation of motion, orbital amplitude, orbital eccentricity and the orbital angular frequency were found to be equations (6), (7), (8), (11), (13) and (15) respectively. The door is henceforth open up for the theoretical development and experimental investigations and applications of the post Newton and post Einstein correction terms.

4.0 References

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