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The Effect of Shape Changing in Critical Mass of Uranium (235) In the Nuclear Fission Chain Reaction

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Abstract

In this paper the study of neutron interactions in a rectangular geometry of uranium ($U^{235)}$using Monte Carlo simulation was presented. The simulation takes as inputs the ratio of dimensions, the number of random points as well as the mass. The output taken from the simulated results shows the survival fraction as output. The ratios of length to thickness used are (0.25, 0.5, 0.75, 1.00, 1.25, and 1.50).These parameters were used to determine the critical mass of u^{235}. The effect of changing shape observed in different value of critical mass obtained.Our result shows that the ratio of 1.00 has least critical mass and that of 0.25 have the highest critical mass.

Keywords: Chain reaction, Critical mass, random number, Survival fraction.

1.0 Introduction

Uranium is among the most technologically important and study element as a nuclear energy material. It is used in the nuclear reactors for the production of power, energy and research activities. It is a major source of neutrons used in neutron activation analysis (NAA), where samples are activated by neutrons which defend on nuclear chain reaction[1-4].

In a nuclear fission chain reaction, neutrons which are emitted during first spontaneous nuclear fission collide with other U^{235} nuclei. The other U^{235} nuclei absorb the neutrons, which causes them to become highly unstable and very rapidly undergo fission, thereby emitting more which trigger more fission, and so on. We refer to each phase of this process as a generation [5].

Assume that two neutrons are emitted during each fission, and that every emitted neutron induces another fission, the starting with N spontaneous fissions, there will be 2N induced fissions after one generation, 4N after two generations, and 2^n N after n generations. Thus, the number of induced fissions grows exponentially, reaching 2^{30} (about one billion) times the original number of spontaneous fissions in only 30 generation.

Due to the small size of the uranium nucleus, neutrons emitted during nuclear fission have to travel, on the average, appreciable distances before interacting with other nuclei and inducing them to fission. In the process of spontaneous fission one or possibly both neutrons may leave the piece before encountering another uranium nucleus. In that case the average number of induced fissions caused by each spontaneous fission would be a number less than or equals to two. Let us define the quantity

$$f = \frac{N_{\text{in}}}{N} \qquad (1)$$

where N_{in} is the number of fissions induced by neutrons emitted in N fissions during the preceding generation.

We shall refer to f as the survival fraction." Starting with N fissions in the first generation, there will be (f N) fissions in the next generation, (f^2N) fission in the one after that and f^n N fission in the n^{th} generation. Thus, the number of induced fission is growth exponential if and only if f is greater than one.

The value of *f* for a particular piece of uranium is determined by its mass, shape, and purity. A piece of uranium for which f = 1.0 is said to have a critical mass M_c. If a piece of uranium has a mass greater than the critical mass M_c, it will spontaneously undergo a chain reaction.

So critical mass is the smallest amount of fissile material needed for a sustained nuclear chain reaction. Several uncertainties contribute to the determination of a precise value for critical masses of uranium (235). Among thus uncertainties including detailed knowledge of cross sections and calculation of geometric effects.

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The geometric effects provided significant motivation for the development of the Monte Carlo method [6]for the determination of critical mass.

2.0 Simulation of the Fission Process

Simulation is the representation of elements of a system by logical and arithmetic processes that can be executed on a computer in order to predict the behavior of a system.

The basic steps in the simulation are: Generate the neutrons at random positions with energies then assign each neutron an interaction based on its energy, assign each neutron a distance to travel based on the assigned interaction. Select a direction to travel in for each neutron and update its position. Remove the neutrons that have escaped the system. Interact each remaining neutron at their new positions in accordance with their assigned interactions.

To generate a random fission is to choose the location of the nucleus undergoing fission to be a random point (x_0, y_0, z_0), lying within the boundaries of the piece of uranium.

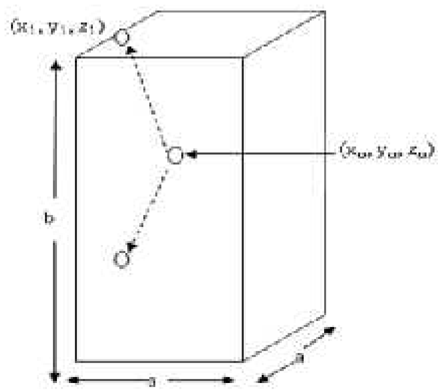

Fig. 1: Fission of two neutrons travelling along random directions as indicated by dotted lines.

If we assume that the piece of uranium is a rectangular slab of dimensions

$a \times a \times b$.

then we must choose random values for the coordinates x_0, y_0, and z_0, subject to the conditions

$$\left.\begin{array}{l} -\frac{a}{2} < x_0 < +\frac{a}{2} \\ -\frac{a}{2} < y_0 < +\frac{a}{2} \\ -\frac{b}{2} < z_0 < +\frac{b}{2} \end{array}\right\} \qquad (2)$$

This is contained in volume $a \times a \times b$.

The only fission fragments that we are concerned with are the neutrons, the two neutrons emitted during the fission process may travel in many directions. We shall ignore the fact that the number of emitted neutrons is not always two, and we shall also ignore possible correlations between the two neutron directions. A direction in three dimensions can be specified by two angles: the polar angle θ, and the azimuthal angle Φ.

If the emitted neutrons have an "isotropic" distribution, i.e., all directions are equally likely, then the probability of a neutron emitted from the point (x_0, y_0,z_0) hitting any area on a surrounding unit sphere depends only on the size of the area. This implies that the azimuthal angle Φ is uniformly distributed between 0 and 2π, and that cosθ, is uniformly distributed between -1 and 1.Note that it is cosθ, not θ itself, which is uniformly distributed due to the fact that equal intervals in cosθ contain the same surface area on a sphere of unit radius. Whether an emitted neutron hits another nucleus before leaving the block depends only on the distance along its line of flight to the boundary of the block. We shall assume that a neutron emitted during fission can hit another nucleus after it travels any distance between 0 and 1 centimeter, with equal probability.

For example, if a neutron travels along a direction such that it would leave the block after traveling only 0.6 cm, then there is a 60% chance of it hitting a nucleus in the block. Our procedure, therefore, is to choose a random number between 0 and 1 for d, the distance traveled by each neutron.

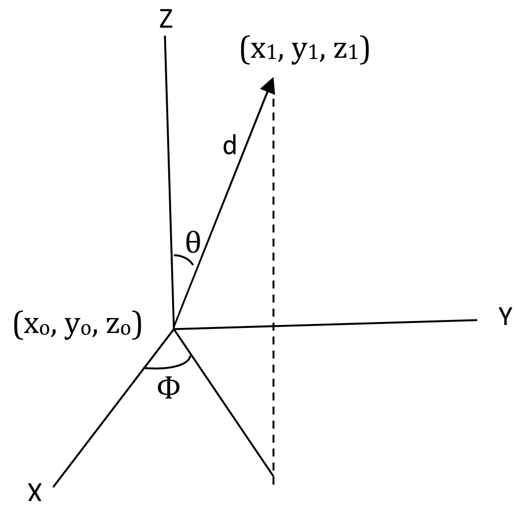

Fig. 2: Directions of travel(θ, Φ) by a neutron emitted at the point (x_o, y_o, z_o).

Since the neutron starts at the point (x_0, y_0, z_0) and travels along a direction (θ, Φ), we can find the coordinates of the point (x_1, y_1, z_1) where the neutron would hit another nucleus using the geometrical relations:

$$\left.\begin{aligned} x_1 &= x_0 + d\sin\theta\cos\Phi \\ y_1 &= y_0 + d\sin\theta\sin\Phi \\ z_1 &= z_0 + d\cos\theta. \end{aligned}\right\} \qquad (3)$$

Whether the neutron actually hits a nucleus at the point (x_1, y_1, z_1) and causes it to fission depends on whether the point lies within the block. if we count the total number of times Nin that the "n" end points (x_1, y_1, z_1) lie within the block for all N random fission, we can compute survival fraction using equation (1)

3.0 Methodology

4.0 The Monte Carlo code

The Monte Carlo code is a computer programming that is generating random number which proceeds to generate N random fissions in form of Monte Carlo simulation, and use it to calculate the survival fraction. A large number of random fission (N) was generated and keep a count of the number of neutron endpoints (N_{in}) which lie inside the block of uranium (235). To generate each random fission, it will be necessary to define nine random numbers $r_1, r_2, r_3 \ldots, r_9$, which lie between 0 and 1.

The nine quantities needed for each random fission are obtained from the random numbers $r_1, r_2, r_3 \ldots, r_9$ according to the following equations:

$$\left.\begin{aligned} X_0 &= a\left(r_1 - {}^1\!/_2\right) \\ y_0 &= a\left(r_2 - {}^1\!/_2\right) \\ z_0 &= b\left(r_3 - {}^1\!/_2\right) \end{aligned}\right\} \text{Coordinates of the nucleus undergoing fission} \qquad (4)$$

$$\left.\begin{aligned} \Phi &= 2\pi r_4 \\ \cos\theta &= 2\left(r_5 - {}^1\!/_2\right) \end{aligned}\right\} \text{are two angles for one emitted neutron} \qquad (5)$$

$$\left.\begin{aligned} \Phi^I &= 2\pi r_6 \\ \cos\theta^I &= 2\left(r_7 - {}^1\!/_2\right) \end{aligned}\right\} \text{are two angles for the other emitted neutron} \qquad (6)$$

$$\left.\begin{aligned} d &= r_8 \\ d^I &= r_9 \end{aligned}\right\} \text{Distance traveled by each neutron} \qquad (7)$$

Where Φ, θ, and d are used to described first neutron and Φ^I, θ^I, and d^I are used to described the second neutron.

These formulas insure that each of the nine parameters will lie within the proper range. For each neutron from a random fission we need to calculate the neutron endpoint from Eq. (3), and then test whether the point is inside or outside the block. The survival fraction f is then given byeq. (1)

In using the Monte Carlo method to find the survival fraction f, we are actually integrating a function F of many variables:

$$\mathbf{F} = F(x_0; y_0; z_0; \Phi; \theta; d; \Phi^I; \theta^I; d^I); \qquad (8)$$

This represents the number of fissions induced by two emitted neutrons for particular values of the variables $x_0, y_0, \ldots, d_0$. The value of **F** is zero, one, or two, depending on these variables.

In order to obtain the survival fraction f, we, must integrate the function F over all nine variables. The advantages of the Monte Carlo technique over conventional integration techniques are quite apparent in a case such as this.
In order to evaluate a nine-dimensional integral using a finite sum, each of the nine variables must be allowed to take on some number of values.
If only two values are used for each variable, it becomes necessary to evaluate the function 2^9 times or 512 calculations.

5.0 Calculation of Critical Mass

The critical mass for rectangular shape of uraniumU^{235} was determined by running computer program using ratio of length to thickness (S) =0.25,Number of random fission(N)= 100 and varying the mass from 1kg to 100kg.we then tabulate the reading obtained. Critical masses were found by making graphs of the calculated value of “f” against the mass “M” from the reading obtained. The critical mass is that value of M_c which correspond to f =1.0.
In order to see the effect of changing the shape clearly, the above procedure was repeated using different values of this ratio of length to thickness (S) such that S=0.50, 0.75, 1.00, 1.25 and 1.50.the critical masses corresponding to each of these shapes were found and then the minimum critical mass of the uranium U^{235} was determined from the graph of the critical mass M_c against the ratio of length to thickness S.

6.0 Results and discussion

7.0 Introduction

The output files of the computation were use to deduce the tables of masses against the survival fraction and the graphs were plotted to obtain the critical mass. The critical masses were then used to obtain the minimum critical mass of uranium (235).

8.0 Graphical representation of data

The following graphs summarize the output data obtained during the Monte Carlo simulation, and are used in obtaining the critical mass of U^{235}.

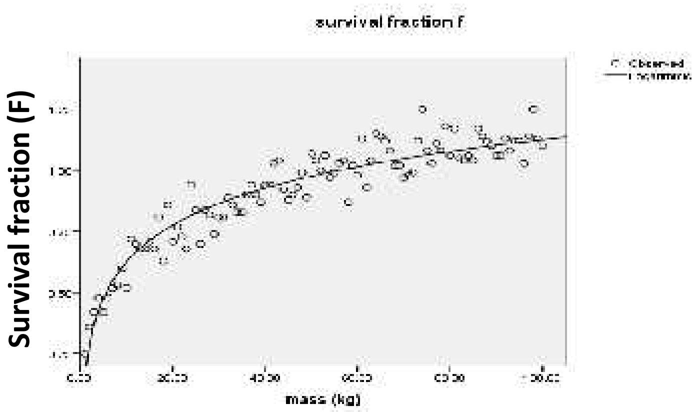

Figure 3: Graph of survival fraction against mass of uranium (235), for the ratio of length to thickness (S) 0.25.

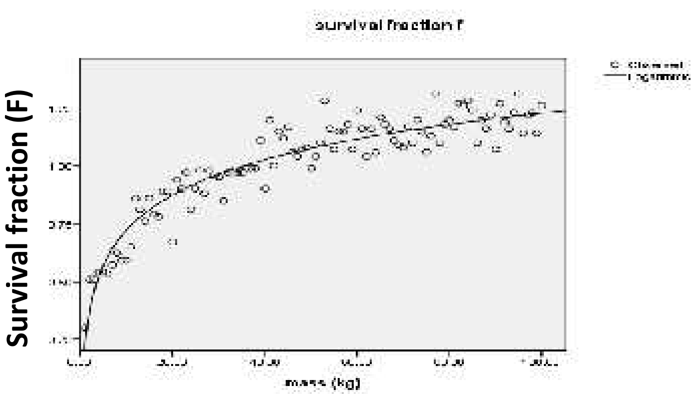

Figure 4: Graph of survival fraction against mass of uranium (235), for the ratio of length to thickness (S) 0.50.

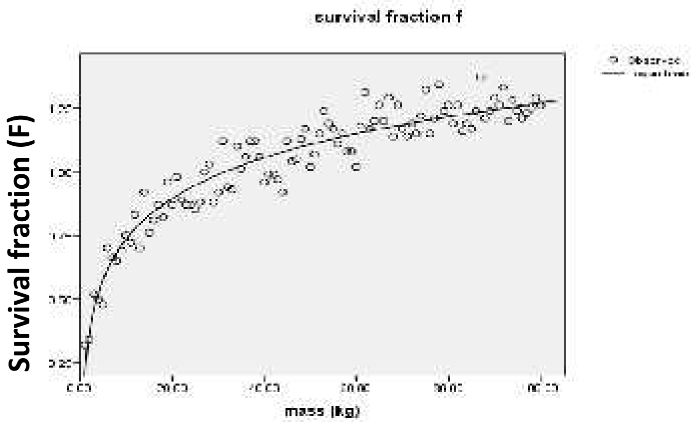

Figure 5: Graph of survival fraction against mass of uranium (235), for the ratio of length to thickness (S) 0.750.

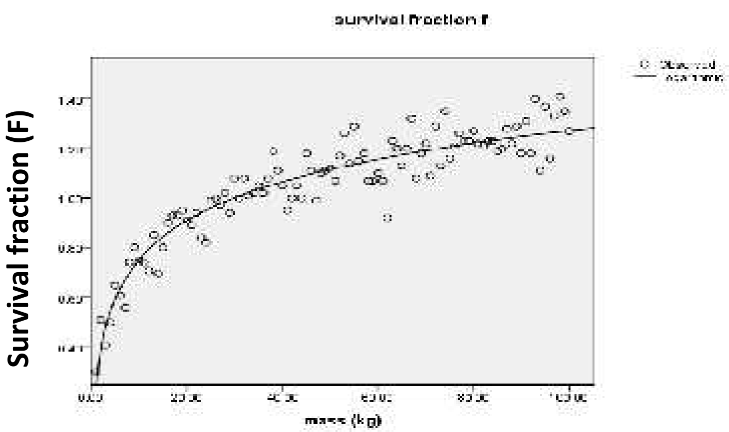

Figure 6: Graph of survival fraction against mass of uranium (235), for the ratio of length to thickness (S) 1.00.

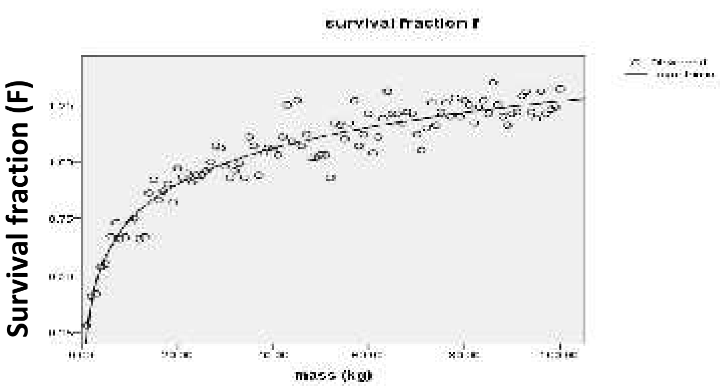

Figure 7: Graph of survival fraction against mass of uranium (235), for the ratio of length to thickness (S) 1.25.

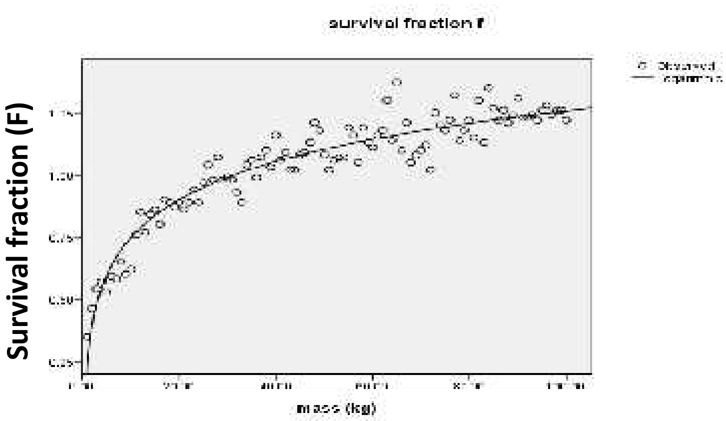

Figure 8: Graph of survival fraction against mass of uranium (235), for the ratio of length to thickness (S) 1.50

Table 1: shows the summary of the results for model equations at the respective ratio of length to thickness using regression analysis with SPSS 16.0

	Logarithmic Summary			**Critical mass**
S	R^2	a	b	M_c
0.25	0.902	0.129	0.216	57
0.50	0.898	0.213	0.221	35
0.75	0.913	0.200	0.232	32
1.00	0.889	0.233	0.226	30
1.25	0.913	0.220	0.227	32
1.50	0.911	0.235	0.222	32

Table 2: Show critical mass of uranium (235) obtained using other method by different authors.

S/N	**Method**	**Critical mass (kg)**
1	Monte Carlo Method (cylindrical shape)[6]	34.33
2	Apollo2 Normes[7]	46.56
3	Apollo2 Keff[7]	48.24
4	Standard Route CRISTAL [7]	46.56
5	Apollo2 TRIPOLI 14[7]	47.31
6	Apollo MORET(2003)[7]	44.32
7	Diffusion Theory 2[8]	45.90
8	Diffusion Theory 1[9]	60.00
9	Neutrons Transport Equation[10]	50.00
10	Monte Carlo simulation [11]	52.18

9.0 Discussion

Looking at the above graphs from figure 3 to 8 it’s difficult to notice the difference between the models graph in describing observed data, because all our graphs are in good agreement with observed data, however looking at Table 1.It is observed that the regression parameter R^2 in the logarithmic equation gives most of the values of the ratios of length to thickness closer to unity. This proves that the logarithmic equation gives a relationship between survival fraction and critical mass of uranium (235).

Comparing Table 1 and Table 2, we observed that our results are in good agreement with other results obtained using other methods by different authors.This shows that our method is a good tool for calculating critical mass of fissile materials.
Table 1 show us that the result with ratio of length to thickness of unity has least critical mass and that of 0.25 have the highest critical mass.
This results also proved that the neutron escape probability from the slab in which the ratio of length to thickness equals to 1.00.(i.e. cube) is lower, that is why it has lower critical mass and it can be the best choice for fuel processing. Varying the ratio of length to thickness from unity will increase the neutron escape probability consequently will increase the critical mass.

10.0 Conclusion

We study neutron interactions in a rectangular geometry of uranium (**$U^{235)}$**) using Monte Carlo simulation. The effect of changing shape observed in different value of critical mass obtained shows that the ratio of 1.00 has least critical mass and that of 0.25 have the highest critical mass.
This shows that the escape probability of neutron from the slab of ratio 1.00.(cube) is lower when we compared to others geometries and also neutron escape probability is higher in a slab of ratio of length to thickness 0.25.
It is observed that mass of uranium may be critical with a particular shape of geometry. Refining the shape of the same geometry makes the mass to be supercritical. Conversely changing the shape to another also makes it to subcritical. This shows that critical mass of u^{235} depends on its shape and geometry of the fuel.

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