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Application of the Verhulst Model to Population Projections and Planning Using 2006 Population Census Data of Nigeria

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Abstract

In this study, the Verhulst modeling procedure was used to project population of Imo State from 2010 to 2105 using the 2006 census value of Imo state from the Federal Republic of Nigeria official Gazette. This model which is still the best model among the population projection, as it improved upon the exponential growth of Malthus by incorporating carrying capacity that the environment can support. In this study, population projection for future years which would be used in facilitating the proper allocation of resources for the social and economic development of Imo state in particular and Nigeria in general was obtained.

Keywords: Population, Verhulst model, growth rate population projections, planning.

1.0 Introduction

Population according to the Oxford advanced learners dictionary can be defined as all the people who live in a particular area, city or country, the total number of people who live there. In sociology and biology, population is the collection of inter-breading organism of a particular species. A population shares a particular characteristics of interest most often that of living in a given geographical area. In ecology, it is said to be the entire organism that constitute a specific group or occur in a specific habitat. In statistics, it is the universe of events under investigation which a statistical sample is taken.

The components of population change includes, fertility trend which has declined more rapidly and in more countries than it was previously projected. While in some countries, fertility remains high, many countries have experienced a trend toward smaller families as a result of changes in socio-economic conditions that have reduced the demand for children and increased the use of contraceptives to avoid unwanted births, mortality trends in which mortality levels in all continents vary. In the past, mortality was mostly caused by disease; example is the black death in Europe and the arrival of old world diseases to the Americans. Also, in some continents, natural disaster causes mortality such as flood and earthquake. All these cause population declines including berth mortality, Migration trends where migrants are defined as individual who have resided for at least one year in a country other than their own. Generally, migrations have a limited effect on population growth of continents and other groupings. Relative to the number of births and deaths, the numbers of migrants are small and Age structure, the composition of a population by age has important consequences for the allocation of resources. The age structure is also a major determinant of population growth as fertility and mortality rates vary greatly with age. The low-income countries have more growth youthful population; fertility is projected to continue to decline.

The current estimated world population is 7,268,095,271 with China 1,396,232, 713, India, 1,271, 968, 563, USA. 323, 338, 633, Nigeria 179, 998, 531 etc.

According to the most recent United Nations estimates [1], the human population of the world is expected to reach 8 billion people in the spring of 2024. The last 50 years have been a rapid increase in population due to medical advance and substantial increase in Agricultural productivity.

In 1914, the protectorates of Southern and Northern Nigeria were amalgamated with the colony (Lagos) by Lord Lugard to form what is now known as Nigeria. According to Chidi Anyache [2], Nigeria is famous for her huge population of about 140million people in 2006 Nigeria census figure. The largest National population on the African continent and the largest group of people on earth. The population is made up of about 250 pure ethnic groups. Three of them, the Hausa, Igbo and Yoruba are the major groups, and constitute over 40percent of the population. The alarming news is that the population growth rate has been estimated at an average of 2.5% which would in some years cause a greater challenge to attempts at eradication of poverty.

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A re-visit of the 2006 census figure posits that Nigeria population of 140million, geographically spread thus;

i. North west – 35,789,944
ii. North Central – 20,266,257
iii. North East – 18,971,965
iv. South West – 27, 581, 992
v. South South – 21, 014, 655
vi. South East – 16, 381, 729

This gives a total population of 75, 025, 166 for Northern Nigeriaand a
total population of 64,978,378for Southern Nigeria. For this paper more emphasis will be on Imo State population which is situated in southeast.

Imo State is a state in Nigeria, located in the southern region of the country, with Owerri as its capital and largest city. Imo State was created in 1976. It was carved out from part of the East Central State with a total area of 5,530KM^2 and a population of 2,485,499 (1991 census) and 3, 934, 899 (2006 census). The state is rich in crude oil, natural gas, palm oil and fertile arable agricultural land. Imo State is predominantly Igbo speaking state with Igbo people constituting a majority of 96%.

Imo State has a population of 3,934,899 persons [3] of 2006 Nigeria census figure, the high population density has lead to intensified pressure, on land, forest and other natural resources leading to increasing rural poverty which is a characteristics of densely populated rural areas. Fallow period rarely exceeds one year and in some areas continuous cropping is the rule have combined to induce people to migrate in search of job and even farmland in other parts of the country. The population of Imo State is predominantly rural. Some of the most densely settled areas of Nigeria are found in Imo state where a direct relationship exists between population density and the degree of disposal of rural settlement. The major urban centers in Imo State are Owerri, Orlu and Okigwe. Master and development plans were prepared by the State Government to guide development within the capital territory Owerri.

From Federal Republic of Nigeria official Gazette 2007 [4], table 1 shown the names of the Imo State Local Government and their current 2006 census population figure.

Table 1: Imo State Local Government and 2006 Census Population Figures

LOCAL GOVERNMENT AREA	POPULATION	MALES	FEMALES
Ideato North	158408	81849	76557
Okigwe	132237	69232	65005
Onuimo	99247	51635	47612
Ideato South	142717	67372	75345
Orlu	120003	61950	58053
Oru East	117492	61404	56088
Oru West	159879	83072	76801
Oguta	143008	74308	68700
Mbaitoli	164468	122037	115518
Njaba	80152	75008	70102
Isu	128472	85991	78477
Nkwere	198736	41642	38510
Nwangele	130931	66990	61482
Isiala-Mbano	120744	103832	94904
Ehime-Mbano	130931	67190	63741
Ihitte Oboma	120744	62630	58114
Ezinihitte Mbaise	165593	85158	80435
Ahiaizu Mbaise	195652	101385	94267
Ikeduru	149316	76232	73084
Owerri North	175395	93093	82302
Owerri West	99265	55215	44050
Ohaji/Egbema	182538	94644	87894
Ngor-okpala	159932	81519	78413
	3934899	2,032286	1,902613

2.0 Need for Accurate Population Figure/Census

Accurate population figures are important for the following reasons:

1. Reduction in the problems that do arise when allocating resources and funds.
2. Adequate and proper investment and contributions in time of crises and future disaster by government and humanitarian services.
3. When planning of development projects for instance; a country that has a higher percentage of children in its population should make room for the contribution of schools. More so, for a country to determine the number of industries she needs to have reliable data relating to the labour force.

In other words accurate population figure is the only source of comprehensive demographic data on all persons in a territory required for planning, policy formulation and monitoring of development goals.
In order to plan effectively for the development of a country or state, planners and policy makers face a lot of great challenges in allocation of funds and resources. The problem arises because proper data and information concerning the population of the state have not been obtained. Basedon this ground, in this study, Verhulst model will be applied to estimate the population of the years ahead for proper and adequate planning.
The objectives of this article are listed below

1. Predicting the population of Nigeria in general and Imo State in particular for future years using Verhulst model.
2. Testing the accuracy and reliability of the Verhulst model.

In 2005, Igbozuruike [5] applied the Verhulst model to Nigeria population census data to project the population for various periods from 1985 to 2050. The model was applied to the Nigerian population census data of 1991 with the growth rate which was projected to be 2.89% by the National Population Commission (NPC).
In 2007, Anyanwu Uchechi [6] applied the Verhulst Model to Lagos State to predict her population at five years intervals from 1991 to 2101, but due to high inflow of immigrants in Lagos State, an immigration factor was included as a key factor in the Verhulst model to obtain the predicted population of the state.
In this paper, the 1991 census figure [7] for Lagos State was analyzed and applied in the Verhulst equation. On a historical note although Verhulst suggested the use of the simple logistic curve (Equation 2.10) as early as 1838, to describe the growth of human population, his work wasvirtually ignored until 1920 when Pearl and Reed [8] derived it as an empirical curve which meets the realistic conditions: It was not until shortly after this that Lolka [9] provided a national, as distinct from an empirical deviations. Further early references are contained in Pearl [10].

Many laboratory populations have been followed as they increase in size, and the success of the deterministic logistic curve (of Equation 2.10) in summarizing the resulting data sets depends on the circumstances surrounding each particular experiment. One particularly good example (from a selection provided by Allee etal [11]) is the data of Carlson [12] on yeast growth in laboratory cultures and the subsequent analysis, Pearl [10].

3.0 Theory and Mathematical Model

A mathematical model is an abstract model that uses mathematical language to describe a system. Mathematical models are used particularly in the natural sciences and engineering disciplines (Such as physics, Biology and Electrical engineering) but also in the social sciences (such as economics, computer science) and economists use mathematical models most extensively.
Eykhoff [13] defined a mathematical model as “a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form”.
Mathematical models are of different forms. These forms include but not limited to dynamical system, statistical models, differential equation or game theoretical models. These and other types of model can overlap, with a given model involving a variety of abstract structures. The underlying theme in all applications of mathematics to real situations is the process of mathematical modeling. By this we mean the problems of translating a real problem from its initial context into a mathematical description, as shown in figure 1

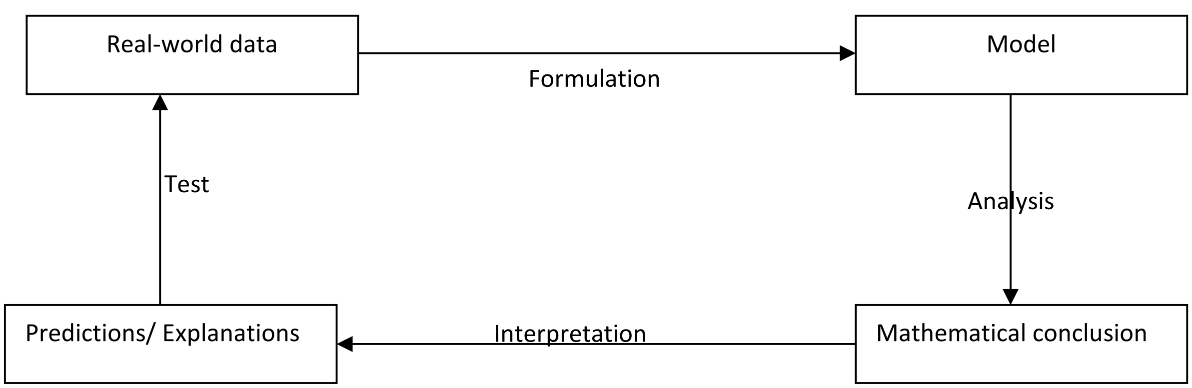

Figure 1: Process of mathematical modeling

Formulation is the main process of expressing the actual situation in mathematical terms. It became very necessary that the situation must be understood, the objectives clear and precise. Factors relevant to the situation should be identified assumptions should be realistic and each important quantity should be represented as a variable, a function or any suitable mathematical symbol. Relationship between various factors should be represented by equations, inequalities and so on.
The result from the formulation process is analyzed. It may require solving an equation or a system of equations, inequalities are solved and theorems may also be proved.
A good model should give precise and accurate result. In other words, its predictions should compare favourably with experimental results or observations. In evaluation, the model is interpreted and conclusions as regards the "real world" situation are reached.
In this section, we are going to discuss the measurement of population and models associated with it. Accurate population figures are very important for economic planning; therefore there is need for census to be carried out in every country. Population figures are also needed for many aspects of administration as well as for economic and social research, but due to the fact that census is always expensive to conduct and for predicting the population of a country for future years, production models have been proposed.
Over the years, mathematicians have built deterministic models which estimate populations of different areas. Two of such mathematicians were Thomas Robert Malthus (1766-1834) and Pierre Frances Verhulst (1804-1849). This study touches on the Malthusian population model but dwells on the Verhulst model which is an improvement on the Malthusian model.

World bodies such as the United Nations population fund, the National population council, world Health Organization and other organizations use general growth rate equation to project for the future population figures to plan for future events. The general formula that models the growth rate is

$$P_n = P_0(1+\frac{f}{100})^n = P_0(1+0.01\mathrm{f})^n \tag{2.1}$$

where

$P_0 =$ the initial population, n = number of years, f = birthrate and $P_n =$ the population in year n.

This implies that if the population of a state is P_0 , the growth rate is $f\%$ (f percent) then in the 1st year, Equation (2.1) will be

$$P_1 = P_0(1+0.01f) \tag{2.2}$$

for the 2nd year is

$$P_2 = P_0(1+0.01f)^2 \tag{2.3}$$

and for the nth year will be

$$P_n = P_0(1+0.01f)^n, n \geq 0 \quad \text{(general growth rate)}$$

4.0 Malthusian Population Model

One of the first researchers into population dynamics was Thomas Robert Malthus (1766-1834). He was concerned to point out a "principle of population" which rendered most proposed "improvement" of social institution valueless [14].
The core Principles of Malthus are

1. Food is necessary for human existence
2. Human population tends to grow faster than the power in the earth to produce subsistence, and that
3. The effect of these two unequal powers must be kept equal
4. Since humans tend not to limit their population size voluntarily, population reduction tends to be accomplished through the "positive" checks of famine disease, poverty and war.

Malthus eventually came to conclusion with the idea that the rate at which, a population grows is directly proportional to its current size. However, the model requires an equation that can find out the size of the population at any point in time. From Malthus model, we have

$$\frac{dp}{dt} = bP(t) \tag{2.4}$$

where b is a constant called the net growth rate per unit time and P represents the population size at time t that is P is a function of time (t). Solving equation (2.4) above, it integrates to the familiar exponential

$$P(t) = P_0 e^{bt} \tag{2.5}$$

If b < 0, this implies that the population decreases where , the death rate is greater than ~ ; the birth rate.

If b > 0, this implies that the population increases where ~ the birth rate is greater than, , the death rate.

If b = 0, this implies that the population remains constant where ~ , the birth rate is equal to the , death rate.

Figure 2 shows the simplest model of population growth and it given an exponentially growing population.

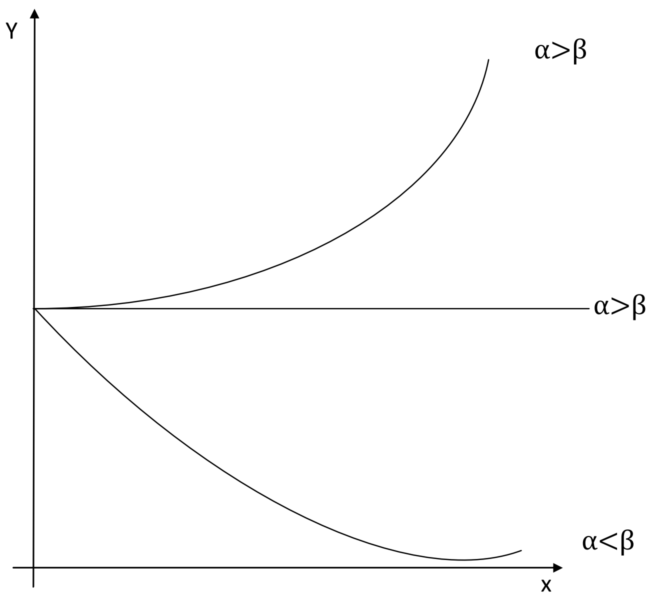

Figure 2: Population size and exponential growth

5.0 Verhulst Population Model

The Verhulst model, named Pierre-Francis Verhulst (1804-1849) of Belgium, improved upon the exponential growth model of Malthus by incorporating a limiting population value or "carrying capacity" that the environment can support. Above this value, lack of food or other resources cause the death rate to rise so that it equals the birth rate. It does not account for oscillations that may occur when food runs out suddenly but is otherwise quite accurate and has been shown to give close match to real populations. Verhulst model can also be called a logistic model.In formulating this model, the following assumptions were borne in mind.

1. The growth of a population must eventually be limited by availability of resources, which implies that the growth rate should be a function of the population thus

$$\frac{dP}{dt} = Pf(P) \tag{2.6}$$

2. When population size, P is small there will be sufficient resources, so the growth rate b will be independent of the resources and hence approximately proportional to the population

$$f(P) = b; b > 0 \tag{2.7}$$

3. As population increases, the resources available per individual will decrease and produce an effect on the population growth, therefore f(P) decreases as P increases, which implies that

$$\frac{df(P)}{dp} < 0 \tag{2.8}$$

The simplest assumption to make is that $f(P)$ is linear suice the larger the population becomes; the greater must be its inhibiting sect on further growth.
I.e.

F(p)= b –sp, s > 0 : (2.9)

Where sis a constant called the overcrowding rate, substituting Equation (2.9) into Equation (2.6) i.e.

$$\frac{dp}{dt} = p(b - sP) \tag{2.10}$$

$$P(0)=P_0;P_0>1$$

This equation is the well known Verhulst Pearl logistic equation. Solving the obtained differential equation in equation (2.10) to have

$$\frac{dp}{bdt}=P\left(1-\frac{sP}{b}\right) \tag{2.11}$$

$$dp=P\left(1-\frac{sP}{b}\right)bdt$$

$$\frac{dP}{P\left(1-\frac{sP}{b}\right)}=bdt \tag{2.12}$$

Provided P ≠ 0 ≠ b/d

Resolving $\dfrac{dP}{P\left(1-\frac{SP}{b}\right)}$ into partial fractions we have

$$\frac{dp}{P\left(1-\frac{sP}{b}\right)}=\frac{dp}{P}-\frac{\frac{sdp}{b}}{1-\frac{sP}{b}} \tag{2.13}$$

Substituting into Equation 2.12 and integrating we have

$$\frac{dp}{P}-\frac{\frac{s}{b}dP}{1-\frac{sp}{b}}=bdt\Rightarrow \ln P-\ln\left(1-\frac{sP}{b}\right)=bt+k \tag{2.14}$$

where k is a constant.

$$\ln\left(\frac{P}{1-\frac{sP}{b}}\right)=bt+k \Rightarrow \frac{P}{1-\frac{sP}{b}}=e^{bt+k}$$

$$\Rightarrow\frac{P}{1-\frac{sP}{b}}=ce^{bt} \quad \text{where } c=e^{k}$$

$$P=ce^{bt}\left(1-\frac{sP}{b}\right)=ce^{bt}-ce^{bt}\frac{sP}{b}$$

$$P=\frac{ce^{bt}}{1+\frac{s}{b}ce^{bt}}=\frac{bce^{bt}}{b+cse^{bt}}$$

Therefore $P(t)=\dfrac{bc}{be^{-bt}+cs}$ (2.16)

Using the initial condition in equation (2.10)

$$P_0=\frac{bc}{b+cs}\Rightarrow c=\frac{bP_0}{b-sP_0} \tag{2.17}$$

Substituting Equation (2.17) in Equation (2.16) to have

$$P(\mathrm{t})=\frac{b\left(\dfrac{bP_0}{b-sP_0}\right)}{be^{-bt}+s\left(\dfrac{bP_0}{b-sP_0}\right)}$$

$$P(t)=\frac{\dfrac{b^2P_0}{b-sP_0}}{be^{-bt}+s\left(\dfrac{bP_0}{b-sP_0}\right)}=\left(\frac{b^2P_0}{b-sP_0}\right)\left(\frac{b-sP_0}{be^{-bt}(b-sP_0)+bsP_0}\right)$$

$$\Rightarrow P(t)=\frac{bP_0}{sP_0+(b-sP_0)e^{-bt}} \tag{2.18}$$

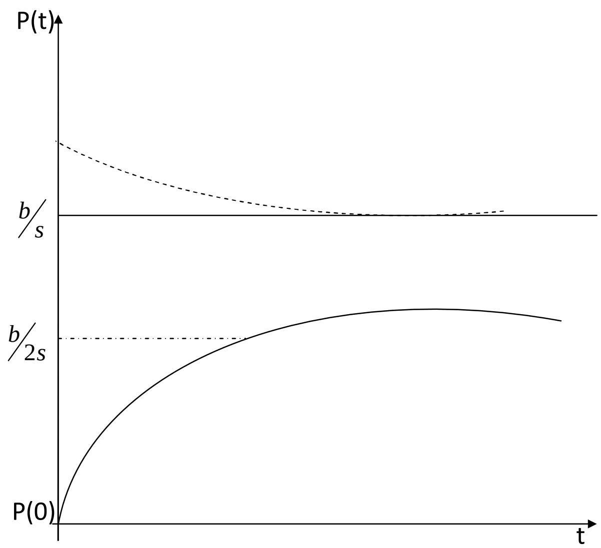

Figure 3: The logistic Curve

From equation (2.10), $\frac{dP}{dt}$ varnishes when P=0 or $P=b/s$. These points are referred to as a critical points or equilibrium points. The curve is S shaped for $P_0 > b/s$, the curve has the shape of the negative exponential curve. In both cases, however, as t$\rightarrow\infty$, P$\rightarrow b/s$, s $\neq$ 0

The ratio b/s is called the saturation level or total carrying capacity. A population growing in a limited environment can approach the ultimate carrying capacity of the environment in several possible ways. It can adjust smoothly to equilibrium below the environmental limit by means of a gradual decrease in growth rate as shown in Figure 4.

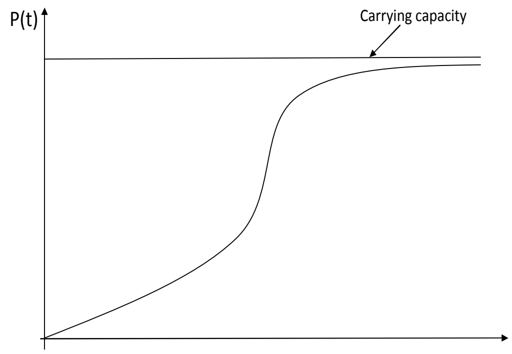

Figure 4: Population Time adjusting to equilibrium below the environmental limit

6.0 Results

This model can be used to predict the population of a State or Country provided the Parameters b, s and P_0 are known. Verhulst used this model to project the population of the United States of America with growth rate b of 0.03134 and an overcrowding factor s of 1.5887×10^{-10} with these results $b = 0.03134$ and $s = 1.5887\times10^{-10}$

Table 2: Observed and Projected Population of USA (1790 – 1870)

YEAR	OBSERVED POPULATION	PROJECTED POPULATION
1790	3,929,214	3,929,214
1800	5,308,483	5,300,510
1810	7,232,881	7,150,388
1820	9,638,453	9,645,873
1830	12,866,020	13,012,282
1840	17,069,453	17,553,569
1850	23,191,876	23,679,765
1860	31,433,321	31,944,002
1870	39,818,449	43,092,459

As Table 2 shows the projected results show a remarkable agreement with actual (experimental) results and attest to the near accuracy of the model. The model therefore satisfies a good number of qualities that make a good model viz:

1) It is based on correct assumptions
2) It is accurate; this is the projected results compared with experimental results.
3) It is precise because it gives definite figures for population at any given time (t).
4) It has useful results because a forecast can be made based on the model.

Population for future years can be estimated by substituting corresponding values for time (t). EXAMPLES OF PROJECTED POPULATION FIGURES USING VERHULST AND GROWTH RATE METHODS. Igbozurike [5] used this model to project the population of Nigeria using $b = 0.142451488$ and $S = 1.558446187\times10^{-15}$

Table 3: Projected and Estimated Population of Nigeria In Five (5) Years Interval

Time (t)	Year	Projected Population	Results for 2.8%
1	1990	86,028,284	85,993,452
2	1995	99,198,954	96,209,946
3	2000	114,386,013	115,224,312
4	2005	131,898,158	133,766,926
5	2010	152,091,357	154,801,325
6	2015	175,376,065	178,575,652
7	2020	202,225,578	205,437,131
8	2025	233,185,659	235,556,076
9	2030	265,555,611	268,813,190
10	2035	310,051,842	304,610,543
11	2040	357,518,842	341,986,344
12	2045	412,253,702	380,307,511
13	2050	475,368,238	418,006,505

With these values b and S the carrying capacity of Nigeria can be calculated as

Carrying capacity(h) $= \lim_{t\to\infty} \frac{bP_0}{sP_0 + (b - sP_0)e^{-bt}} = \frac{b}{s}$,S ≠ 0 (3.1)

I.e. using Equation (2.18)

$$\frac{0.142451488}{1.558446187\times10^{-15}} = 91,406,099,099,000$$

Another example on the accuracy of the Verhulst Model was observed by Anyanwu Uchechi [6], she used this model to project the population of Lagos State. According to 1991 population census figure, the population of Lagos State was given as 5,725,116.

Using $b = 0.028490304$ and $s = 5.495078735\times10^{-15}$ the carrying capacity of Lagos State can be calculated as

Carrying capacity(h) $= \lim_{t\to\infty} \frac{bP_0}{sP_0 + (b - sP_0)e^{-bt}} = \frac{b}{s}$; S ≠ 0 (3.2)

$$h = \frac{0.02849034}{5.495078735\times10^{-15}} = 5,184,694,410,000$$

Table 4: Projected and Estimated Population size for Lagos State in five years intervals

S/N	INTERVAL	YEAR	PROJECTED POPULATION	ESTIMATED POPULATION
1	5	1991	5725116	5725116
2	10	1996	6601613	6621613
3	15	2001	7612301	7612300
4	20	2006	8777721	8777720
5	25	2011	10121564	10121562
6	30	2016	11671145	11671141
7	35	2021	13457962	13457955
8	40	2026	15518335	15518324
9	45	2031	17894146	17894128
10	50	2036	20633655	20633657
11	55	2041	23792641	23792598
12	60	2046	27435222	27432522
13	65	2051	31635472	31635378
14	70	2056	36478767	36478631
15	75	2061	42063557	42063361
16	80	2066	48503360	48503082
17	85	2071	5592079	55928687
18	90	2076	64491652	55928687
19	95	2081	74365129	74364370
20	100	2086	85750206	85749157
21	105	2091	98878304	98876860
22	110	2096	114016275	114014296
23	115	2101	131471823	131469120

The available census data in Nigeria are the results conducted from the National census in 1952/53, 1963, 1991 and 2006. The 2006 census figure for Imo State will be analyzed and applied in the Verhulst equation to estimate the population of the State for future years in this study. The results obtained will be arranged in five years interval.

Given that the population of Imo State in 2006 was 3,934,899 by the National population Commission (NPC) and as our initial population P_0, let $P(t)$ be the population of the State in 2005, $P(t_1)$ be the population of the State in 2010 and $P(t_2)$ be the population of the State in 2015. According to Nigeria World feature article [3] on the 2006 Nigeria Census the National Population commission projected the growth rate to be 2.5%.

Applying equation (2.1) that is the growth rate equation given as $P(t) = P_0(1 + 0.01f)^n$

We obtain as follows, projecting backward using the figure of 2006 as our base year population figure, that is

$P_0 = 3,934,899$, n=-1 for 2005 projection, we will obtain as follows $P(t) = 3934899(1.025)^{-1} = 3838926$

Having calculated the projected population of Imo State in 2005 as 3,838, 926 which can now be used as our new initial population P_0. With this value, changing the values of n from 5, 10, 15… and f as 0.025, we obtained the projected population of Imo State in 2010 to be 4343392, 2015 as 4,914,150.e.t.c.

(i.e. $P(t_1) = 4,343,392$; $P(t_2) = 4,914,150$) etc

Applying Equation (2.18) i.e. the Verhulst equation method given as

$$P(t) = \frac{bP_0}{sP_0 + (b - sP_0)e^{-bt}}$$ with an interval of 5 years,

Let

$$P(t_1) = \frac{bP_0}{sP_0 + (b - sP_0)e^{-bt_1}}$$ which implies

$$bP_0 = P(t_1)\left[sP_0 + (b - sP_0)e^{-bt_1} \right] \qquad (3.3)$$

$$P(t_2) = \frac{bP_0}{sP_0 + (b - sP_0)e^{-bt_2}}$$

$$\Rightarrow bP_0 = P(t_2)\left[sP_0 + (b - sP_0)e^{-bt_2} \right] \qquad (3.4)$$

Equating Equation (3.3) and (3.4) respectively

$$P(t_1)\left[sP_0 + (b - sP_0)e^{-bt_1} \right] = P(t_2)\left[sP_0 + (b - sP_0)e^{-bt_2} \right]$$ and simplifying to obtain

$$s = \frac{b\left[P(t_2)e^{-bt_2} - P(t_1)e^{-bt_1} \right]}{P(t_1)P_0(1 - e^{-bt_1}) - P(t_2)P_0(1 - e^{-bt_2})} \qquad (3.5)$$

From equation (3.3)

$$sP_0 + (b - sP_0)e^{-bt_1} = \frac{sP_0}{P(t_1)}$$ which simplifies to

$$s = \frac{\frac{bP_0}{P(t_1)} - be^{-bt_1}}{P_0 - P_0 e^{-bt_1}} \qquad (3.6)$$

Equating equations (3.5) and (3.6)

$$\frac{\frac{bP_0}{P(t_1)} - be^{-bt_1}}{P_0(1 - e^{bt_1})} = \frac{b\left[P(t_2)e^{-bt_2} - P(t_1)e^{-bt_1} \right]}{P_0\left[P(t_1)(1 - e^{-bt_1}) - P(t_2)(1 - e^{-bt_2}) \right]}$$ which simplifies to

$$\frac{b\left[\frac{P_0}{P(t_1)} - e^{-bt_1} \right]}{P_0(1 - e^{-bt_1})} = \frac{b\left[P(t_2)e^{-bt_2} - P(t_1)e^{-bt_1} \right]}{P_0\left[P(t_1)(1 - e^{-bt_1}) - P(t_2)(1 - e^{-bt_1}) \right]}$$

Substituting the value of $P_0, P(t_1), P(t_2), t_1$ and t_2 to obtain

$$\frac{\frac{3838926}{4343392} - e^{-5b}}{1 - e^{-5b}} = \frac{4914150e^{-10b} - 4343392e^{-5b}}{4343392(1 - e^{-5b}) - 4914150(1 - e^{-10b})}$$

$$\frac{0.8838544 - e^{-5b}}{1 - e^{-5b}} = \frac{4914150(e^{-10b} - 0.8838542e^{-5b})}{4914150\left[0.8838542(1 - e^{-5b}) - (1 - e^{-10b}) \right]}$$

$$(0.8838544 - e^{-5b})\left[0.8838544(1-e^{-5b}) - (1-e^{-10b})\right] = (1-e^{-5b})(e^{-10b} - 0.8838544e^{-5b})$$

Let $(1-e^{-5b}) = A$ and $(1-e^{-10b}) = B$

$$\Rightarrow (0.8838544 - \mathrm{e}^{-5b})(0.8838544A - B) = A(e^{-10b} - 0.8838544e^{-5b})$$

$$\Rightarrow 0.8838544^2 A - 0.8838544B + Be^{-5b} = Ae^{-10b}$$

$$0.7811986(1-e^{-5b}) - 0.8838544(1-e^{-10b}) + (1-e^{-10b})e^{-5b} = e^{-10b}(1-e^{-5b})$$

$$\Rightarrow 0.1161456e^{-10b} - 0.2188014e^{-5b} + 0.102655799 = 0$$

Let $e^{-5b} = x; \Rightarrow e^{-10b} = x^2$

$$\Rightarrow 0.1161456x^2 - 0.2188014x + 0.102655799 = 0$$

Solving this resulting quadratic equation give

$e^{-5b} = 0.88358326$ with $b = 0.024692845$

Substituting b in equation (3.3) gives $S = 4.480387663 \times 10^{-14}$

Therefore the

Carrying capacity h $= \lim\limits_{t\to\infty} \dfrac{bP_0}{sP_0 + (b - sP_0)e^{-bt}} = \dfrac{b}{s}$

$$\frac{0.024692845}{4.480387663 \times 10^{-14}} = 551{,}131{,}885{,}400$$

Substituting the values of $P_0 = 3{,}838{,}926$; $b = 0.024692845$ and $s = 4.480387663 \times 10^{-14}$ into the Verhulst equation, we obtain the estimated population size for Imo State in five (5) years intervals. The results of the two methods are presented in Table 5.

Table 5: Projected Population of Imo State Using Verhulst and Growth Rate Methods in Five (5) Years Internal

S/N	INTERNAL	YEAR	2.5% GROWTH RATE	VERHULST EQUATION
1	5	2010	4343392	4343393
2	10	2015	4914150	4914152
3	15	2020	5559909	5559912
4	20	2025	6290527	6290529
5	25	2030	7117154	7117153
6	30	2035	8052407	8052401
7	35	2040	9110559	9110546
8	40	2045	10307761	10307736
9	45	2050	11662286	11662242
10	50	2055	13194806	13194735
11	55	2060	14928712	14928602
12	60	2065	16890467	16890304
13	65	2070	19110013	19109773
14	70	2075	21621226	21620880
15	75	2080	24462433	24461944
16	80	2085	27676997	2767315
17	85	2090	31313982	13130310
18	90	2095	35428897	35427607
19	95	2100	40054545	40082793
20	100	2105	45351983	45349621

7.0 Conclusion

This study has looked at the various methods of population projection such as the growth rate generally used by the world bodies such as the United Nations population funds, National Population Commission, World Health Organization etc and the Verhulst Equation model. In this Verhulst Equation, he listed certain factors that will affect the use of growth rate method in population projections. He formulated a logistic model which was supported by three assumptions listed under Verhulst Population Model of this work. Some other factors such as the overcrowding rate and carrying capacity were introduced. He postulated that a population in a limited environment can approach the ultimate carrying capacity of the environment in several possible ways. This study looked at these two different methods, derived Equation using the growth rate (h) and overcrowding factor(s) and finally established the equation of the carrying capacity of any given population.

Applied the two methods, growth rate method and Verhulst equation method to project the future population of Nigeria, Lagos State and in particular computed the carrying capacity Imo State population.

There is need for adjustment in the rate of growth of the population to the available resources so that people needs will be met. These adjustment can be made effectively only if fairly accurate censuses are taken or if reliable population projection are made for every planning period. Using theVerhulst model, such predictions can be made and errors often encountered by policy makers and planners will be limited.

The Verhulst Model is therefore being recommended to be used to project the population of a state or country for the future years. It is also recommended for good planning activities and services that require specific population data and general population information.

The history of population has shown that unforeseen events can rapidly modify the demographic environment and effect the course of a state's population dynamics in future.

Hence, the actual future population is not likely to be the same as the projected population.

However, the use of the population projections to plan for future has become a normal part of good management in Government and business.

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