Journal of the Nigerian Association of Mathematical Physics
Volume 29, (March, 2015), pp 267 – 274

Application of Queuing Theory and Modeling In Analyzing the Waiting Lines At Bus Rapid Transit (BRT) Transportation Services In Lagos, Nigeria

[1]Adewole A.I and [2]Egunjobi K.A

[1]Department of Mathematics, Tai Solarin University of Education Ijagun, Nigeria
[2]Physics Department, Tai Solarin University of Education Ijagun, Nigeria.

Abstract

Operation research (O.R) focuses on the application of analytical methods to facilitate better decision-making. Queuing analysis is a useful tool for estimating capacity requirements and managing demand for any system in which the timing of service needs is random. Many organizations, such as banks, airlines, telecommunications companies, and police departments, routinely use queuing models to help manage and allocate resources in order to respond to demands in a timely and cost efficient fashion. Queuing systems are important in transportation because of their effect on customers and because of the cost of providing the services. This paper uses queuing theory to study the waiting lines in Bus Rapid Transit (BRT) Services at Lagos city, Nigeria .The service time, the arrival time and the time the passengers leave the system were recorded using the stop watch.

This paper illustrated the usefulness of applying queuing theory in a real-case situation. Little's theorem and M/M/I queuing model were used to derive the arrival rate, service rate, utilization rate and waiting time in the queue. It was discovered that with a high traffic intensity or utilization factor rate of 0.6007, the probability of zero passenger on the queue is very small. Also the utilization rate is directly proportional with the number of customers' i.e. the number of customers increases as the utilization increase. We conclude by enumerating the benefits involved in performing queuing analysis to reduce the waiting lines on the BRT Services. Hence, concentration should be geared toward the time and Bus allocations for optimal utilization.

Keywords: Queuing theory, Transportation, M/M/I Queuing model, Queues, Little's theorem and waiting lines.

1.0 Introduction

Queue is a common word that means a waiting line or the act of joining a line. Queuing theory was initially proposed by Erlang in 1903. It optimizes the number of service facilities and adjusts the times of services. Queuing theory is the study of queue or waiting lines. Queuing Theory, also called random service theory, is a branch of Operation Research in the field of Applied Mathematics. It is a subject which analyzes the random regulation of queuing phenomenon. Some of the analysis that can be derived using queuing theory include the expected waiting time in the queue, the average time in the system, the expected queue length as well as the probability of the system to be in certain states, such as empty or full. Shanbhag [1] confirmed that the arrival process may depend or not on the number of customers present at the Service Centre. Sometimes refusal situations are considered: the customer arrives and refuses to enter in the Service Centre because there are too many customers waiting to be served. And also renounce situations: the customer is already in the Service Centre and leaves it because it thinks that has waited a too long time. The service process is specified indicating the length of the time probability distribution that a customer spends being attended by a server: the service time. There may be deterministic or stochastic service times.

A Service Centre which has associated a service process, a waiting capacity and a queue discipline is a node. A node with the respective arrival process is a queue. The networks of queues with infinite servers in each node have interesting applications in Logistics, based on the failures of the transport vehicles that allow computing important measure of performance [2, 3].

Corresponding author: Adewole A.I., E-mail: hayorhade2005@yahoo.com, Tel.: +2348055124368.

In queuing theory a model is constructed so that queue lengths and waiting times can be predicted [4]. The issue of queuing has been a subject of scientific debate for there is no known society that is not confronted with the problem of queuing. Wherever there is competition for limited resource queuing is likely to occur. The role of transportation in human life cannot be overemphasized. Efficient transportation system plays an important role in catering for the daily necessities in the lives of the citizens [5]. At the individual level, transportation is a crucial factor for urban insertion since it gives access to economic activity, facilitates family life and helps in spinning social networks [6].
This paper uses queuing theory to study the waiting lines in Bus Rapid Transit (BRT) Services at Lagos city, Nigeria. From the creation of human being, from time of birth to death, human being often find themselves waiting for something, the more society becomes interdependent psychologically, economically and technologically, the more individuals encounter waiting lines or queues in their daily lives. This paper seeks to illustrate the usefulness of applying queuing theory in a real- case situation, Little's theorem and M/M/I queuing model were used to derive the arrival rate, service rate, utilization rate and waiting time in the queue.
Queuing theory is the study of queue or waiting lines. Some of the analysis that can be derived using queuing theory include the expected waiting time in the queue, the average time in the system, the expected queue length, the expected number of customers served at one time, the probability of balking customers, as well as the probability of the system to be in certain states, such as empty or full. Queuing theory is basically a mathematical approach applied to the analysis of waiting lines within the field of operations management [7]. Any system in which arrivals place demand upon a finite capacity resource may be termed as a queuing system [8]. Queuing theory uses queuing models or mathematical models and performance measures to assess and hopefully improve the flow of customers through a queuing system [9].

2.0 Overview of Queuing System

Literature on queuing indicates that waiting in line or queue causes inconvenience to economic costs to individuals and organizations. Transportation system, airline companies, banks, manufacturing firms etc., try to minimize the total waiting cost, and the cost of providing service to their customers. Therefore, speed of service is increasingly becoming a very important competitive parameter [10]. Providing a faster, timely service with the ultimate goal of having zero customer waiting time, has recently received managerial attention for several reasons. First, in the more highly developed countries, where standards of living are high, time becomes more valuable as a commodity and consequently, customers are less willing to wait for service. Second, this is a growing realization by organizations that the way they treat their customers today significantly impact on whether or not they will remain loyal customers tomorrow. Finally, advances in technology such as computers, internet etc., have provided firms with the ability to provide faster services.
A queue system is a birth and death process with a population composed by customers receiving a service or waiting for it. There is a birth when a customer arrives at the Service Centre. There is a death when a customer abandons the Service Centre. The state of the system is the number of the customers in the service system. Statistical Queuing Theory is applied, for example, to intelligent transportation systems, call centers, PABXs, telecommunications networks, advanced telecommunications systems and traffic flow [11]. The networks of queues are used to reduce the waiting times in the hospitals. Researchers have work extensively on queuing system; Veeraraghavan et al. [12] analyzed studies on customers' perceptions of queue length through analytical models and suggested that waiting cost analysis be their future research. Green [13] presents the theory of queuing as applied in healthcare. She discusses the relationship amongst delays, utilization and the number of servers; the basic M/M/S model, its assumptions and extensions; and the applications of the theory to determine the required number of servers. Others study how to improve the performance of queue management [14, 15]. Furthermore, research has demonstrated that customer satisfaction is affected not just by waiting time but also by customer expectations or attribution of the causes for the waiting [16]. Consequently, one of the issues in queue management is not only the actual amount of time the customer has to wait, but also the customer's perceptions of that wait [17,18]. Obviously, there are two approaches to increasing customer satisfaction with regard to waiting time: through decreasing actual waiting time, as well as through enhancing customer's waiting experience [19, 20].

3.0 Theory and Methods

4.0 Queuing System

A queuing system can be completely described by
the input or arrival pattern (customers); ii) the service mechanism (service pattern); iii) the queue discipline & iv) customers behavior:

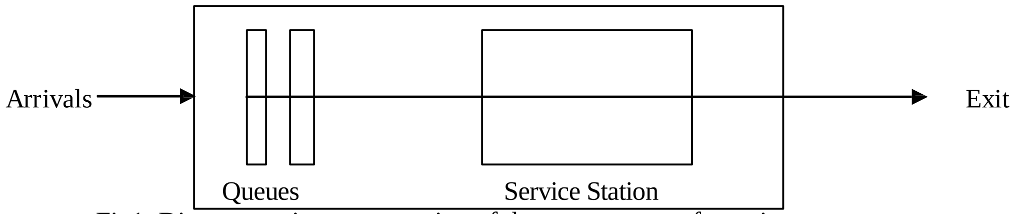

Fig1: Diagrammatic representation of the components of queuing system

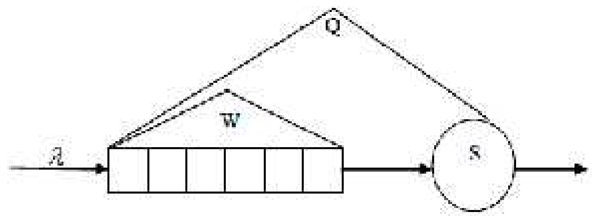

Fig2: Queuing Model

A queuing system consists of one or more servers that provide service to arriving customers. Figure 1 shows the characteristics of queuing system. The population of customers may be finite (closed systems) or infinite (open systems) [21]. The customers arrive to the service center in a random fashion.

Queue represents a certain number of customers waiting for service. The capacity of a queue is either limited or unlimited. Waiting lines in Bus Routine Transport services is an example of unlimited queue length. Figure 2 represents a queuing model.

The following nomenclature will be adopted as defined;
Arrival rate (λ) = the average rate at which customers arrive,
μ = The mean service rate,
$\frac{\lambda}{\mu}$ = Utilization factor,
Service time (s) = the average time required to service one customer,
Number waiting (W) = the average number of customers waiting,
Number in the system (Q) = the total number of customers in the system. (1)

5.0 Queuing Theory

In 1908, Copenhagen Telephone Company requested Erlang to work on the holding times in a telephone switch. He identified that the number of telephone conversations and telephone holding time fit into Poisson distribution and exponentially distributed. This was the beginning of the study of queuing theory.

6.0 Little's Theorem

Little's theorem describes the relationship between throughput rate (i.e. arrival and service rate), cycle time and work in process (i.e. number of customers/jobs in the system). This relationship has been shown to be valid for a wide class of queuing models. The theorem states that the expected number of customers (N) for a system in steady state can be determined using the following equation:

$$L = \lambda \quad (2)$$

Here, L is the expected average number of customer in a steady system, λ is the average customer arrival rate and T is the average service time for a customer.

Three fundamental relationships can be derived from Little's theorem,

I. L increases if λ or T increases
II. λ increases if L increases or T decreases
III. T increases if L increases or λ decreases

7.0 (M/M/I Queuing Model)

An M/M/I Queuing system is the simplest queuing system which has exponentially arrival distribution and exponential service time distribution. The system is always first in, first serve under the Markovian Principle on M/M/I. M/M/1 queuing model also indicated that the arrival and service time are exponentially distributed (Poisson process). For the analysis of the BRT,

M/M/1 queuing model, the following variables will be investigated:
Probability of zero customers in the system

$$P = 1-\rho \qquad (3)$$

P_n: The probability of having *n* customers in the system:

$$P_n = p_o(1-\rho)\rho^n \qquad (4)$$

L_S: The average number of customers in the system:

$$L_S = \frac{\mu}{1-\mu} = \frac{\lambda}{\mu-\lambda} \qquad (5)$$

L_q: The average number of customers in the queue:

$$L_q = \frac{\mu^2}{1-\mu} = \frac{\rho}{\mu-\lambda} \qquad (6)$$

W_q: The average waiting time in the queue:

$$W_q = \frac{\mu}{\mu-\lambda} \qquad (7)$$

W_S: The average time spent in the queue, including the waiting time:

$$W_s = \frac{1}{\mu-\lambda} \qquad (8)$$

8.0 Observation and Discussion.

The data under study was collected at Bus Rapid Transit in Ketu Lagos, Nigeria .The service time and the time passengers leave the system were recorded using stopwatch. The data was collected between the hours of 6am to 10am on tropical working days, Saturdays and Sundays.

Table1: Time Interval of Passenger Served.

Time Interval	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
6:01am-7am	342	237	286	273	225	192	99
7:01am-8:am	296	229	298	195	194	224	126
8:01am-9am	247	196	192	175	167	165	62
9:01am-10am	195	162	156	156	145	107	73
Total	1080	824	932	799	731	688	360

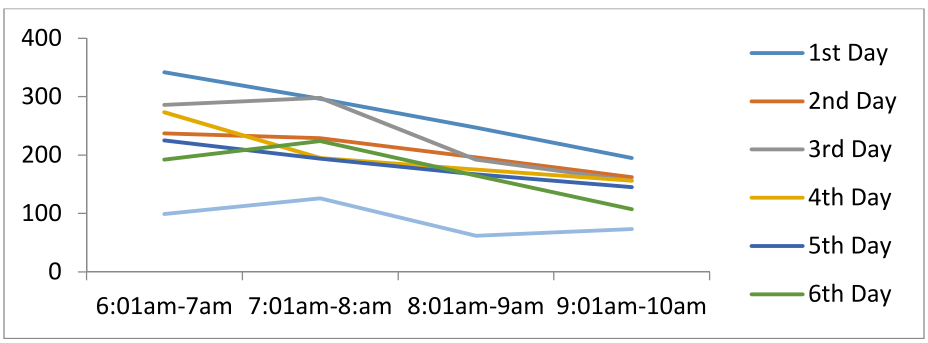

***Figure3**: Number of passengers served against time interval*

It was observed that, the busiest period for the BRT Transport Services is always on tropical [working days especially Mondays and Wednesdays, hence, the time period is very important for the research. Also, we observed from above that, after Thursday the number of customers start decreasing slowly as the week progresses. This is because the queuing length is always at reduced value at weekends.

9.0 Estimation Using Queue Measure

Computations of parameters to determine the queuing parameters were done using the M/M/1 queuing model. It is assumed that time interval between successive arrivals and serving time is independent and identically distributed. The queuing discipline observed was first-come first-served (FCFS)

From the data obtained, the total taken by 2988 passenger to be served is 720mins, and time interval for 2988 arrivals is 4hrs x 5 working days, i.e.

4 x 60 minutes x 5 working days

240 minutes x 5 = 1200 minutes

From this we can derive the arrival rate as:

$$\lambda = \frac{5}{1} = 4.517$$

Hence the mean service rate can be obtained by

$\mu = \frac{5}{7} = 7.5$

Traffic intensity or Utilization factor can be derived using (8) and (9).

$\rho = \frac{\lambda}{\mu}$ (9)

$= \frac{4.5}{7.5} = 0.6007$

With the high utilization rate of 0.6007, the probability of zero passengers on the queue is small as derived using

$P = 1 - \rho$ (10)

$P = 1 - 0.6007 = 0.3993$

i.e., the probability that the servers are idle is 0.3993.

The generic formula that can be used to calculate the probability of having *n* passengers on the queue is as follows:

$P_n = (1 - \rho)\rho^n$ (11)

$= (1 - 0.6007)\, 0.6007^n$.

Also the average number of passengers in the system was derived using

$L_S = \frac{\lambda}{\mu - \lambda}$ (12)

$\frac{4.5}{7.5 - 4.5} = 1.5046$

It can also be derived by the notation

$L_S = \frac{\rho}{1 - \rho}$ (13)

Average number of passenger in the queue or the expected (average) queue length L_q was derived by

$L_q = L_S - \frac{\lambda}{\mu}$ (14)

$= \frac{\rho^2}{1 - \rho}$

$L_q = L_S - \frac{4.517}{7.519}$

$= 1.5046 - 0.6007$

$= 0.90396$

W_q: the average waiting time in the queue or the expected waiting line in the queue is

$W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu - \lambda}$ (15)

Or

$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$ (16)

$W_q = \frac{0.6}{7.5 - 4.5}$

$= 0.2001$

W_S: the average time spent in the queue, including the waiting time is the expected waiting line in the system, it was estimated using

$W_s = W_q + \frac{1}{\mu}$ (17)

$= \frac{\lambda}{\mu(\mu - \lambda)} + \frac{1}{\mu} = \frac{1}{(\mu - \lambda)}$ (18)

$= \frac{1}{(7.5 - 4.5)}$

$= 0.3331$

The expected waiting time of a customer who has to wait (w/w>0) is

$(w/w>0) = \frac{1}{(\mu - \lambda)} = \frac{1}{\mu(1 - \rho)}$ (19)

$= \frac{1}{7.5\,(1 - 0.6)}$

$= 0.3331$

The expected length of non-empty queue is

$(L/L>0) = \frac{\mu}{(\mu - \lambda)}$ (20)

$= \frac{1}{(1 - 0.6)}$

$= 2.5043$

Interrelationship between L_S, L_q, W_s and W_q w.r.t Little's theorem is

$L_S = \lambda W_s$ (21)

Similarly,

$$L_q = \lambda W_q \quad (22)$$

$$W_q = W_s - \frac{1}{\mu} \quad (23)$$

and

$$L_q = L_s - \frac{\lambda}{\mu} \quad (24)$$

10.0 General Discussion

Queuing theory was first developed for studying queuing phenomena in commerce, telephone traffic, transportation and business-industrial servicing etc [22].

From the observation and the result in this research, the arrival rate (Lambda effective) = 4.518; the mean service rate ( ) =7.519. The system performance parameters are as follows;

Ls = 1.5046. This measures the average number of passengers in the system.

Lq = 0.9039. This implies there is average number of 0.9039 passengers in the queue waiting to be served by the BRT transport system. Wq = 0.2001, meaning that passengers spent an average time of 0.2001hour(12.006 minutes) on the queue waiting to be attended to.

Ws = 0.3331 ,this means that each passengers spent an average time of 0.331hour (19.986 minutes) in the system, i.e. the time spent before joining the queue, waiting in the queue to be served and time spent after being served before departure from BRT. Expected waiting time of a customer who has to wait (w/w>0), is19.986 minutes. Which implies that the average waiting time in the system corresponds to the expected time of the customer who has to wait?

The Traffic intensity or the utilization rate is 0.6007; the probability that the servers are idle is 0.3993. The utilization is directly proportional with the mean number of customers. It means that the mean number of customers will increase as the utilization increase. From the result above, it was observed that as ρ approaches 1 the number of customers would become very large. This can be easily justified intuitively, will approach 1 when the average arrival rate starts approaching the average service rate. In this situation, the server would always be busy hence leading to a queue build up. When the service rate is higher, the utilization will be lower which makes the probability of the customers going away decreases. For a stable system the average service rate should always be higher than the average arrival rate. (Otherwise the queues would rapidly race towards infinity). Thus ρ should always be less than one.

11.0 Conclusion and Recommendation

In respect to queuing problem, this research paper has discussed the application of queuing theory to real life problem using general characteristics of queuing system, and the service channel which is single server model with Poisson input and exponential study of Transportation system with respect to the arrival time, service time and the departure time of passengers in the Bus rapid transit (BRT) transport service in Ketu, Lagos. This research work shows that passenger have to wait for a period of time before receiving service which may not yield an effective utilization and good result for an optimum Transportation system to reduce the problem of queuing. It is highly recommended for the operator of BRT Transport services to know when to increase the number of service so that the waiting time is reduced. In other words, more buses and route terminus station should be provided to reduce the time waiting for service.

Future research will be on the survey and how to reduce waiting time in Nigeria General Hospitals using Queuing theory and models. Also research will apply M/M/S-Multiple server model with Poisson input and exponential services in reducing queuing problem.

12.0 References

[1] Shanbhag, D. N. (1966), *"On Infinite Servers Queues with Batch Arrivals"*. Journal of Applied Probability, 3, pp 274-279.

[2] Ferreira, M. A. M. and Filipe, J. A. (2010a), *"Solving Logistics Problems using M/G/M Queue Systems Busy Period"*. Aplimat- Journal of Applied Mathematics, Vol. 3 (3), pp 207-212.

[3] Ferreira, M. A. M. and Andrade, M. (2009a), *"M/*G/S *Queue System Parameters for a Particular Collection of Service Time Distributions"***.** AJMCSR-African Journal of Mathematics and Computer Science Research Vol. 2(7), pp 138-141.

[4] Sundarapandian, V. (2009). 7. Queueing Theory: Probability, Statistics and Queueing Theory. PHI Learning. ISBN 8120338448

[5] Intikhab, QA., Huapu, L., & Shi Y. (2008). Urban Transportation and Equity: A Case Study of Beijing and Karachi. In Transportation Research Part A: Policy and Practice, 42:

[6] Heidemann, D., 1991, Queue length and waiting-time distributions at priority intersections, Transportation Research-B, 25, 163-174.

[7] Wane, H.R. (2001). Urban Mobility and Emissions: Towards Accurate Standards for Sub-Saharan Africa. A Research Program and Results on a Sahelian Case: The District of Bamako. At the Session on Air Pollution, Accra, Ghana, April 18-20, 2001.

[8] Nosek, A.R., Wislon, P. J., Queuing Theory and Customer Satisfaction: A Review of Terminology, Trends and Applications to Pharmacy Practice, *Hospital Pharmacy*, 36(3):275-279, 2001.

[9] Gorney, L, *Queuing Theory: A Problem Solving Approach*, New York: Petrocelli Booko, Inc., 1981.

[10] Katz, K., Larson, K., Larson, R., Prescription for the waiting in line Blues: Entertain, Enlighten and Engage", *Sloan Management Review* (winter): 44-53, 1991

[11] Davis, M. M. and Heineke, J. (1994). "Understanding the roles of customer and the operation for better queue management". *International Journal of Operations and Production Management*, Vol. 14 No. 5, pp. 21-34

[12] Veeraraghavan, S. and Debo, L. (2009). Joining *longer queues: Information externalities in queue choice*. Manufacturing & Service Operations Management (INFORMS) 11, no. 4 (Fall 2009): 543-562

[13] Green, L. (2006a) Queueing analysis in healthcare, in Patient Flow: Reducing Delay in Healthcare Delivery, Hall, R.W., ed., Springer, New York, 281-308.

[14] Ferreira, M. A. M. (1987), "*Redes de filas de espera*". Master thesis presented at [Instituto Universitario de Lisboa (ISCTE-IUL) BRU-IUL, Lisboa, Portugal].

[15] Hassin, R. (1986), "Consumer Information in Markets with Random Product Quality": the case of queues and balking, Econometrica, vol. 54, No. 5 (September, 1986), 1185-1195.

[16] Kalai, Ehud, Morton I. Kamien, and Michael Rubinovitch (1992), "Optimal Service Speeds in a Competitive Environment," *Management Science*, 38 (8), 1154-63.

[17] Taylor, S., Waiting for Service: The Relationship between Delays and Evaluation of Service, *Journal of Marketing*, 58(2): 56-69, 1994.

[18] Davis M. M, Aquilano M. J. N, Chase B. R (2003).Fundamentals *of Operations Management*. Boston: McGraw- Hill Irwin, Fourth Edition

[19] N. K. Tiwari & Shishir K. Shandilya, *Operations Research*. Third Edition, ISBN 978-81-203-2966-9. Eastern Economy Edition, 2009.

[20] Nita H. Shah, Ravi M. Gor, Hardik Soni, *Operations Research*. 4th Edition, ISBN 978-81- 203-3128-0. Eastern Economy Edition, 2010. [21] K. Sanjay, Bose, "*An Introduction to queuing system*", Springer, 2002.

[22] M. Laguna and J. Marklund, *Business Process Modeling, Simulation and design*. ISBN 0-13-091519-X. Pearson Prentice Hall, 2005.

[23] Bunday BD. An Introduction to Queuing Theory. New York: Halsted Press; 1996