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Comparative Analysis of Some Reliability Characteristics of Deteriorating Systems

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Abstract

In this paper, probabilistic models for a system with different stage deteriorations have been developed to analyze and compare some reliability characteristics. Three configurations are studied under the assumption that each state that is working in reduced capacity is minimally repaired and the system is replaced at failure. Configuration 1, configuration 2 and configuration 3 have one (major), two (minor and major) and three (mild, minor and major) stage deteriorations respectively. Explicit expressions for mean time to system failure (MTSF) and steady state availability ($AV(\infty)$ ***) are analyzed using kolmogorov's forward equation method. Comparisons are performed for specific values of system parameters. Finally, the configurations are ranked based on MTSF and*** $AV(\infty)$ ***and the results show that configuration 3 is optimal.***

Keywords: Reliability, Availability, Deterioration, Repair, Replacement.

1.0 Introduction

System failure is an inevitable phenomenon which could be costly and sometimes dangerous. System maintenance results in high reliability, availability and profit. Many researchers have studied reliability problem of different systems [1-4]. System availability represents the percentage of time the system is available for operation.. As the age of equipment increases, the equipment gradually deteriorates and subsequently leads to failure. In manufacturing industries, system condition has significant impact on the quality of produced products. Most of these systems are subject to random failures and disastrous effect on safety and cost. Maintenance models assume minimal repair, perfect repair and imperfect repair [5,6]. This paper considers a deteriorating system with three configurations. Configuration 1 has three states visa-vie; working, working with deterioration and failed states. Configuration 2 has four states visa-vie; working, working with minor deterioration, working with major deterioration and failed states while configuration 3 has five states namely; working, working with mild deterioration, working with minor deterioration, working with major deterioration and failed states respectively. The contribution of the paper is of threefold. The first is to develop the explicit expressions for the Mean time to system failure (MTSF) and Availability ($AV(\infty)$) for the three configurations. The second is to perform numerical investigation on the system parameters for mean time to system failure and availability. The third is to compare the MTSF and $AV(\infty)$ of all the three configurations under study.

2.0 Notations

a_{ij} : Deterioration/failure rate of the system from state S_i to state S_j

«$_{ji}$: Repair/replacement rate of the system from state S_j to state S_i

S_i : State of the system, $i = 1, 2....m$

$MTSF_n$: Mean time to system failure for configuration n

$AV_n(\infty)$: Steady-state availability for configuration n

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3.0 Description of the System and Assumptions

1. For configuration 1: Perfect (S_1), Major deterioration (S_2), Failure (S_3)
2. For configuration 2: Perfect (S_1), Minor deterioration (S_2), Major deterioration (S_3), Failure (S_4)
3. For configuration 3: Perfect (S_1), Mild deterioration (S_2), Minor deterioration (S_3), Major deterioration (S_4), Failure (S_5)
4. At any given time the system is either in the operating state, deteriorating state or in the failed state.
5. The state of the system changes as time progresses.
6. The transition from one state to another takes place instantaneously.
7. The failure and repair rates are constant.

Table 1: Transition rates for configuration 1

	S_1	S_2	S_3
S_1		a $_{12}$	a $_{13}$
S_2	« $_{21}$		a $_{23}$
S_3	« $_{31}$		

Table 2: Transition rates for configuration 2

	S_1	S_2	S_3	S_4
S_1		a $_{12}$	a $_{13}$	a $_{14}$
S_2	« $_{21}$		a $_{23}$	
S_3		« $_{32}$		a $_{34}$
S_4	« $_{41}$			

Table 3: Transition rates for configuration 3

	S_1	S_2	S_3	S_4	S_5
S_1		a $_{12}$	a $_{13}$	a $_{14}$	a $_{15}$
S_2	« $_{21}$		a $_{23}$		
S_3		« $_{32}$		a $_{34}$	
S_4			« $_{43}$		a $_{45}$
S_5	« $_{51}$				

4.0 Mean Time to System Failure

5.0 Mean Time to System Failure Calculations for Configuration 1

Let $P(t)$ be the probability row vector at time $t\,(t \geq 0)$, then the initial conditions for this problem are as follows:

$$P(0) = [P_1(0), P_2(0), P_3(0)] = [1,0,0] \quad (1)$$

We obtain the following differential equationsfor configuration 1 from Table 1

$$P'_1(t) = -(\text{a}_{12} + \text{a}_{13})P_1(t) + \text{«}_{21}P_2(t) + \text{«}_{31}P_3(t)$$

$$P'_2(t) = \text{a}_{12}P_1(t) - (\text{a}_{23} + \text{«}_{21})P_2(t) \quad (2)$$

$$P'_3(t) = \text{a}_{13}P_1(t) + \text{a}_{23}P_2(t) - \text{«}_{31}P_3(t)$$

The differential equation in matrix form can be expressed as

$$P' = AP \quad (3)$$

Where,

$$A = \begin{bmatrix} -(a_{12} + a_{13}) & «_{21} & «_{31} \\ a_{12} & -(a_{23} + «_{21}) & 0 \\ a_{13} & a_{23} & -«_{31} \end{bmatrix}$$

To evaluate the transient solution is too difficult. Therefore, to calculate the MTSF, we follow refs.[7,8]. We take the transpose of matrix A and delete the row and column of absorbing state i.e. state 3. The new matrix is called Q and the expected time to reach an absorbing state is given by

$$E[T_{P(0)\to P(arbsorbing)}] = P(0)(-Q^{-1})\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (4)$$

Where,

$$Q = \begin{bmatrix} (a_{12} + a_{13}) & a_{12} \\ «_{21} & -(a_{23} + «_{21}) \end{bmatrix}$$

Therefore, the explicit expression for the mean time to system failure for configuration 1 is given by

$$MTSF_1 = \frac{N_1}{D_1} \quad (5)$$

Where,

$$N_1 = a_{12} + a_{23} + «_{21}$$

$$D_1 = a_{12}a_{23} + a_{12}a_{23} + a_{13}«_{21}$$

6.0 Mean Time to System Failure Calculations for Configuration 2

Let $P_n(t)$ be the probability that the system is working at time t ($t \geq 0$). The initial conditions are

$$P(0) = [P_1(0), P_2(0), P_3(0), P_4(0)] = \{1, 0, 0, 0\} \quad (6)$$

We obtain the following differential equations for configuration 2 from Table 2

$$P'_1(t) = -(a_{12} + a_{13} + a_{14})P_1(t) + «_{21}P_2(t) + «_{41}P_4(t)$$

$$P'_2(t) = a_{12}P_1(t) - (a_{23} + «_{21})P_2(t) + «_{32}P_3(t) \quad (7)$$

$$P'_3(t) = a_{13}P_1(t) + a_{23}P_2(t) - (a_{34} + «_{32})P_3(t)$$

$$P'_4(t) = a_{14}P_1(t) + a_{34}P_3(t) - «_{41}P_4(t)$$

The differential equation in matrix form can be expressed as

$$P' = AP \quad (8)$$

Where,

$$A = \begin{bmatrix} -(a_{12} + a_{13} + a_{14}) & «_{21} & 0 & «_{41} \\ a_{12} & -(a_{23} + «_{21}) & «_{32} & 0 \\ a_{13} & a_{23} & -(a_{34} + «_{32}) & 0 \\ a_{14} & 0 & a_{34} & -«_{41} \end{bmatrix}$$

To evaluate the transient solution is too difficult. Therefore, to calculate the MTSF, we take the transpose of matrix A and delete the row and column of absorbing state i.e. state 4. The new matrix is called Q. The expected time to reach an absorbing state is given by

$$E[T_{P(0)\to P(arbsorbing)}] = MTSF_2 = P(0)(-Q^{-1})\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (9)$$

Where,

$$Q = \begin{bmatrix} -(a_{12} + a_{13} + a_{14}) & a_{12} & a_{13} \\ «_{21} & -(a_{23} + «_{21}) & a_{23} \\ 0 & «_{32} & -(a_{34} + «_{32}) \end{bmatrix}$$

Therefore, the explicit expression for the mean time to system failure for configuration 2 is given by

$$MTSF_2 = \frac{N_2}{D_2} \tag{10}$$

Where,

$$N_2 = a_{12}a_{23} + a_{13}a_{23} + a_{13}«_{21} + a_{12}a_{34} + a_{12}«_{32} + a_{13}«_{32} + a_{23}a_{34} + a_{34}«_{21} + «_{21}«_{32}$$

$$D_2 = a_{12}a_{23}a_{34} + a_{13}a_{23}a_{34} + a_{14}a_{23}a_{34} + a_{13}a_{34}«_{21} + a_{14}a_{34}«_{21} + a_{14}«_{21}«_{32}$$

7.0 Mean Time to System Failure Calculations for Configuration 3

Let $P_n(t)$ be the probability that the system is working at time t ($t \geq 0$). The initial conditions are

$$P(0) = [P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1,0,0,0,0] \tag{11}$$

We obtain the following differential equations for configuration 3 from Table 3

$$\begin{aligned}
P'_1(t) &= -(a_{12} + a_{13} + a_{14} + a_{15})P_1(t) + «_{21}P_2(t) + «_{51}P_5(t) \\
P'_2(t) &= a_{12}P_1(t) - (a_{23} + «_{21})P_2(t) + «_{32}P_3(t) \\
P'_3(t) &= a_{13}P_1(t) + a_{23}P_2(t) - (a_{34} + «_{32})P_3(t) + «_{43}P_4(t) \\
P'_4(t) &= a_{14}P_1(t) + a_{34}P_3(t) - (a_{45} + «_{43})P_4(t) \\
P'_5(t) &= a_{15}P_1(t) + a_{45}P_4(t) - «_{51}P_5(t)
\end{aligned} \tag{12}$$

The differential equation in matrix form can be written as

$$P' = AP \tag{13}$$

Where,

$$A = \begin{bmatrix} -(a_{12}+a_{13}+a_{14}+a_{15}) & «_{21} & 0 & 0 & «_{51} \\ a_{12} & -(a_{23}+«_{21}) & «_{32} & 0 & 0 \\ a_{13} & a_{23} & -(a_{34}+«_{32}) & «_{43} & 0 \\ a_{14} & 0 & a_{34} & -(a_{45}+«_{43}) & 0 \\ a_{15} & 0 & 0 & a_{45} & -«_{51} \end{bmatrix}$$

To evaluate the transient solution is too difficult. Therefore, to calculate the MTSF, we take the transpose of matrix A and delete the row and column of absorbing state i.e. state 5. The new matrix is called Q. The expected time to reach an absorbing state is given by

$$E[T_{P(0)\to P(arbsorbing)}] = MTSF_3 = P(0)(-Q^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \tag{14}$$

Where,

$$Q = \begin{bmatrix} -(a_{12}+a_{13}+a_{14}+a_{15}) & a_{12} & a_{13} & a_{14} \\ «_{21} & -(a_{23}+«_{21}) & a_{23} & 0 \\ 0 & «_{32} & -(a_{34}+«_{32}) & a_{34} \\ 0 & 0 & «_{43} & -(a_{45}+«_{43}) \end{bmatrix}$$

Therefore, the explicit expression for the mean time to system failure for configuration 2 is given by

$$MTSF_3 = \frac{N_3}{D_3} \tag{15}$$

Where,

$$N_3 = a_{12}a_{23}a_{34} + a_{13}a_{23}a_{34} + a_{14}a_{23}a_{34} + a_{13}a_{34}«_{21} + a_{14}a_{34}«_{21} + a_{14}«_{21}«_{32} + a_{23}a_{34}a_{45} + a_{34}a_{45}«_{21}$$
$$+ a_{45}«_{21}«_{32} + «_{21}«_{32}«_{43} + a_{12}a_{34}a_{45} + a_{12}a_{45}«_{32} + a_{13}a_{45}«_{32} + a_{12}«_{32}«_{43} + a_{13}«_{32}«_{43} + a_{14}«_{32}«_{43}$$
$$+ a_{12}a_{23}a_{45} + a_{13}a_{23}a_{45} + a_{12}a_{23}«_{43} + a_{13}a_{23}«_{43} + a_{13}a_{45}«_{21} + a_{14}a_{23}«_{43} + a_{13}«_{21}«_{43} + a_{14}«_{21}«_{43}$$

$$D_3 = a_{14}a_{45}«_{21}«_{32} + a_{15}a_{45}«_{21}«_{32} + a_{15}«_{21}«_{32}«_{43} + a_{12}a_{23}a_{34}a_{45} + a_{13}a_{23}a_{34}a_{45} + a_{14}a_{23}a_{34}a_{45} + a_{15}a_{23}a_{34}a_{45}$$
$$+ a_{13}a_{34}a_{45}«_{21} + a_{14}a_{34}a_{45}«_{21} + a_{15}a_{34}a_{45}«_{21}$$

8.0 Availability analysis

9.0 Availability Calculations for configuration 1

For the availability of configuration 1, we use the same initial conditions (1) and the differential equations (2). The differential equations (2) can be expressed in matrix form as

$$\begin{bmatrix} P'_1(t) \\ P'_2(t) \\ P'_3(t) \end{bmatrix} = \begin{bmatrix} -(a_{12}+a_{13}) & «_{21} & «_{31} \\ a_{12} & -(a_{23}+«_{21}) & 0 \\ a_{13} & a_{23} & -«_{31} \end{bmatrix} = \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}$$

The steady- state probability can be obtained using the following procedure. In the steady-state, the derivatives of the state probabilities become zero which allows us to calculate the steady -state probabilities .The states S_0 and S_1 are the only working states of the system. The steady-state availability is sum of the probability of operational states. Thus,

$$AV_1(\infty) = P_1(\infty) + P_2(\infty) \tag{16}$$

and

$$AP = 0,$$

or in matrix form

$$\begin{bmatrix} -\left(a_{12}+a_{13}\right) & «_{21} & «_{31} \\ a_{12} & -\left(a_{23}+«_{21}\right) & 0 \\ a_{13} & a_{23} & -«_{31} \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

Using the following normalizing condition:

$$P_1(\infty) + P_2(\infty) + P_3(\infty) = 1 \tag{18}$$

We substitute (18) in any one of the redundant rows in (17) to obtain

$$\begin{bmatrix} -\left(a_{12}+a_{13}\right) & «_{21} & «_{31} \\ a_{12} & -\left(a_{23}+«_{21}\right) & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{19}$$

The solution of (19) provides the steady-state probabilities and the explicit expression for availability is given by

$$AV_1(\infty) = \frac{N_4}{D_4} \tag{20}$$

Where,

$$N_4 = a_{23}«_{31} + «_{21}«_{31} + a_{12}«_{31}$$

$$D_4 = a_{12}a_{23} + a_{13}a_{23} + a_{13}«_{21} + a_{12}«_{31} + a_{23}«_{31} + «_{21}«_{31}$$

10.0 Availability Calculations for Configuration 2

For the availability of configuration 2, we use the same initial conditions (6) and the differential equations (7). The differential equations (7) can be expressed in matrix form as

$$\begin{bmatrix} P'_1(t) \\ P'_2(t) \\ P'_3(t) \\ P'_4(t) \end{bmatrix} = \begin{bmatrix} -(a_{12}+a_{13}+a_{14}) & «_{21} & 0 & «_{41} \\ a_{12} & -(a_{23}+«_{21}) & «_{32} & 0 \\ a_{13} & a_{23} & -(a_{34}+«_{32}) & 0 \\ a_{14} & 0 & a_{34} & -«_{41} \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix}$$

The steady- state probability can be obtained using the following procedure. In the steady-state, the derivatives of the state probabilities become zero which allows us to calculate the steady -state probabilities. The states S_0 , S_1 and S_2 are the only operational states of the system. The steady-state availability for configuration 2 is the sum of the probability of operational states. Thus,

$$AV_2(\infty) = P_1(\infty) + P_2(\infty) + P_3(\infty) \tag{21}$$

and

$$AP = 0,$$

or in matrix form

$$\begin{bmatrix} -(\alpha_{12}+\alpha_{13}+\alpha_{14}) & \beta_{21} & 0 & \beta_{41} \\ \alpha_{12} & -(\alpha_{23}+\beta_{21}) & \beta_{32} & 0 \\ \alpha_{13} & \alpha_{23} & -(\alpha_{34}+\beta_{32}) & 0 \\ \alpha_{14} & 0 & \alpha_{34} & -\beta_{41} \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{22}$$

Using the following normalizing condition:

$$P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) = 1 \tag{23}$$

We substitute (23) in any one of the redundant rows in (22) to obtain

$$\begin{bmatrix} -(\alpha_{12}+\alpha_{13}+\alpha_{14}) & \beta_{21} & 0 & \beta_{41} \\ \alpha_{12} & -(\alpha_{23}+\beta_{21}) & \beta_{32} & 0 \\ \alpha_{13} & \alpha_{23} & -(\alpha_{34}+\beta_{32}) & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{24}$$

The solution of (24) provides the steady-state probabilities and the explicit expression for availability is given by

$$AV_2(\infty) = \frac{N_5}{D_5} \tag{25}$$

Where,

$$N_5 = \alpha_{23}\alpha_{34}\beta_{41} + \alpha_{34}\beta_{21}\beta_{41} + \beta_{21}\beta_{32}\beta_{41} + \alpha_{12}\alpha_{34}\beta_{41} + \alpha_{12}\beta_{32}\beta_{41} + \alpha_{13}\beta_{32}\beta_{41} + \alpha_{12}\alpha_{23}\beta_{41} + \alpha_{13}\alpha_{23}\beta_{41} + \alpha_{13}\beta_{21}\beta_{41}$$

$$D_5 = \alpha_{12}\alpha_{23}\alpha_{34} + \alpha_{13}\alpha_{23}\alpha_{34} + \alpha_{14}\alpha_{13}\alpha_{13} + \alpha_{13}\alpha_{34}\beta_{21} + \alpha_{14}\alpha_{34}\beta_{21} + \alpha_{12}\alpha_{23}\beta_{41} + \alpha_{13}\alpha_{23}\beta_{41} + \alpha_{12}\alpha_{34}\beta_{41} + \alpha_{23}\alpha_{34}\beta_{41}$$
$$+ \alpha_{14}\beta_{21}\beta_{32} + \alpha_{13}\beta_{21}\beta_{41} + \alpha_{12}\beta_{32}\beta_{41} + \alpha_{13}\beta_{32}\beta_{41} + \alpha_{34}\beta_{21}\beta_{41} + \beta_{21}\beta_{32}\beta_{41}$$

14.0 Availability Calculations for Configuration 3

For the availability of configuration 3, we use the same initial conditions (11) and the differential equations (12). The differential equations (12) can be expressed in matrix form as

$$\begin{bmatrix} P'_1(t) \\ P'_2(t) \\ P'_3(t) \\ P'_4(t) \\ P'_5(t) \end{bmatrix} = \begin{bmatrix} -(\alpha_{12}+\alpha_{13}+\alpha_{14}+\alpha_{15}) & \beta_{21} & 0 & 0 & \beta_{51} \\ \alpha_{12} & -(\alpha_{23}+\beta_{21}) & \beta_{32} & 0 & 0 \\ \alpha_{13} & \alpha_{23} & -(\alpha_{34}+\beta_{32}) & \beta_{43} & 0 \\ \alpha_{14} & 0 & \alpha_{34} & -(\alpha_{45}+\beta_{43}) & 0 \\ \alpha_{15} & 0 & 0 & \alpha_{45} & -\beta_{51} \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \end{bmatrix}$$

The steady- state probability can be obtained using the following procedure. In the steady-state, the derivatives of the state probabilities become zero which allows us to calculate the steady -state probabilities. The states S_1 , S_2 , S_3 and S_4 , are the only working states of the system. The steady-state availability is the sum of the probability of operational states. Thus,

$$AV_3(\infty) = P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) \tag{26}$$

and

$$AP = 0,$$

or in matrix form

$$\begin{bmatrix} -(a_{12}+a_{13}+a_{14}+a_{15}) & «_{21} & 0 & 0 & «_{51} \\ a_{12} & -(a_{23}+«_{21}) & «_{32} & 0 & 0 \\ a_{13} & a_{23} & -(a_{34}+«_{32}) & «_{43} & 0 \\ a_{14} & 0 & a_{34} & -(a_{45}+«_{43}) & 0 \\ a_{15} & 0 & 0 & a_{45} & -«_{51} \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (27)$$

Using the following normalizing condition:

$$P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)=1 \quad (28)$$

We substitute (28) in any one of the redundant rows in (27) to obtain

$$\begin{bmatrix} -(a_{12}+a_{13}+a_{14}+a_{15}) & «_{21} & 0 & 0 & «_{51} \\ a_{12} & -(a_{23}+«_{21}) & «_{32} & 0 & 0 \\ a_{13} & a_{23} & -(a_{34}+«_{32}) & «_{43} & 0 \\ a_{14} & 0 & a_{34} & -(a_{45}+«_{43}) & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (29)$$

The solution of (29) provides the steady-state probabilities and the explicit expression for availability is given by

$$AV_3(\infty)=\frac{N_6}{D_6} \quad (30)$$

Where,

$$N_6 = a_{23}a_{34}a_{45}«_{51} + a_{34}a_{45}«_{21}«_{51} + a_{45}«_{21}«_{32}«_{51} + «_{21}«_{32}«_{43}«_{51} + a_{12}a_{34}a_{45}«_{51} + a_{12}a_{45}«_{32}«_{51}$$
$$+a_{13}a_{45}«_{32}«_{51} + a_{12}«_{32}«_{43}«_{51} + a_{13}«_{32}«_{43}«_{51} + a_{14}«_{32}«_{43}«_{51} + a_{12}a_{23}a_{45}«_{51} + a_{13}a_{23}a_{45}«_{51}$$
$$+a_{12}a_{23}«_{43}«_{51} + a_{13}a_{23}«_{43}«_{51} + a_{13}a_{45}«_{21}«_{51} + a_{14}a_{23}«_{43}«_{51} + a_{13}«_{21}«_{43}«_{51} + a_{13}«_{21}«_{43}«_{51}$$
$$+a_{12}a_{23}a_{34}«_{51} + a_{13}a_{23}a_{34}«_{51} + a_{14}a_{23}a_{34}«_{51} + a_{13}a_{34}«_{21}«_{51} + a_{14}a_{34}«_{21}«_{51} + a_{14}«_{21}«_{32}«_{51}$$

$$D_6 = a_{14}a_{45}«_{21}«_{32} + a_{15}a_{45}«_{21}«_{32} + a_{13}a_{34}«_{21}«_{51} + a_{14}a_{34}«_{21}«_{51} + a_{12}a_{23}«_{43}«_{51} + a_{13}a_{23}«_{43}«_{51}$$
$$+a_{13}a_{45}«_{21}«_{51} + a_{14}a_{23}«_{43}«_{51} + a_{12}a_{45}«_{32}«_{51} + a_{13}a_{45}«_{32}«_{51} + a_{34}a_{45}«_{21}«_{51} + a_{15}«_{21}«_{32}«_{43}$$
$$+a_{14}«_{21}«_{32}«_{51} + a_{13}«_{21}«_{43}«_{51} + a_{14}«_{21}«_{43}«_{51} + a_{12}«_{32}«_{43}«_{51} + a_{13}«_{32}«_{43}«_{51} + a_{14}«_{32}«_{43}«_{51}$$
$$+a_{45}«_{21}«_{32}«_{51} + «_{21}«_{32}«_{43}«_{51} + a_{12}a_{23}a_{34}a_{45} + a_{13}a_{23}a_{34}a_{45} + a_{14}a_{23}a_{34}a_{45} + a_{15}a_{23}a_{34}a_{45}$$
$$+a_{13}a_{34}a_{45}«_{21} + a_{14}a_{34}a_{45}«_{21} + a_{15}a_{34}a_{45}«_{21} + a_{12}a_{23}a_{34}«_{51} + a_{13}a_{23}a_{34}«_{51} + a_{14}a_{23}a_{34}«_{51}$$
$$+a_{12}a_{23}a_{45}«_{51} + a_{13}a_{23}a_{45}«_{21} + a_{12}a_{34}a_{45}«_{51} + a_{23}a_{34}a_{45}«_{51}$$

15.0 Results and Discussions

In this section, We use Matlab to compare the results for Mean time to system failure and system availability for the three configurations using the following set of parameter values

$a_{12}=0.1$, $a_{13}=0.2$, $a_{14}=0.4$, $a_{15}=0.5$, $a_{23}=0.2,$ $a_{34}=0.25,$ $a_{45}=0.3,$ $«_{21}=0.2,$ $«_{31}=0.4,$ $«_{32}=0.2,$ $«_{41}=0.4,$ $«_{43}=0.15,$ $«_{51}=0.45.$

We show the following results:

$$MTSF_3 > MTSF_1 > MTSF_2 \quad (31)$$

$$AV_3(\infty) > AV_1(\infty) > AV_2(\infty) \quad (32)$$

It can be seen from (31) and (32) that the optimal configuration using both Mean time to system failure and Steady-state availability is configuration 3.

16.0 Conclusion

In this paper, we developed three dissimilar deteriorating systems configurations to study the effectiveness of each model. Configurations 1,2 and 3 have one, two and three deteriorating state(s) respectively. Explicit expression for Mean time to system failure and Steady-state availability for three configurations were derived and comparative analysis was performed numerically. The optimal configuration is configuration 3 using MTSF and AV.

17.0 References

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