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# Reliability Analysis of Four Redundant Communication Networks

***Ibrahim Yusuf[1] and U.A. Ali[2]***

**[1]Department of Mathematical Sciences, Bayero University, Kano, Nigeria**
**[2]Department of Mathematics, Federal University, Dutse, Nigeria.**

## *Abstract*

***In this paper, probabilistic models for four redundant communication networks have been developed to analyze and compare their reliability with respect to mean time to system failure (MTSF) and busy period of repairman. A procedure for computing reliability measures (Mean time to system failure and busy period) in this study is based on the first order linear differential equations. Explicit expressions for mean time to system failure and steady-state busy period for the four redundant communication networks are developed. Furthermore, comparisons are made between the four redundant communication networks based on their mean time to system failure and busy period.***

***Mathematics Subject Classification: 90B25.***

**Keywords:** Reliability, MTSF, busy period, communication network, redundancy.

## 1.0 Introduction

Reliability connection between networks can be usually achieved through a number of redundant paths/units, thus making the connection reliable. The reliability of these network systems is of increasing importance since the failure of some components may lead to disastrous results. Example of such systems include water distribution, oil and gas supply, power generation and transmission, transport by rail and by road, communication system consisting of a transmitter, relay stations and a receiver, where a signal from transmitter is received by two consecutive relay and distributed to other relay stations before it finally arrived at the receiver for consumptions. Reliability of an industrial system is becoming an increasingly important issue. High system reliability and availability plays a vital role towards industrial growth as the profit is directly dependent on production volume which depends upon system performance. Because of their prevalence in power plants, manufacturing systems, and industrial systems, many researchers have studied reliability and availability problem of different communication systems, a great number of models have been introduced to describe the behaviour and performance of a communication network that is subject to failure. Vasar et al. [1] proposed Markov models for the reliability analysis of the wireless sensor networks with an emphasis on fault tolerant systems. Hassan [2] performed reliability evaluation of network with quickest path and capacity constraint. Evaluation of reliability of network flows with stochastic capacity and cost constraint was studied by Fathabadi and Khodaei [3]. Ali [4] investigated the reliability of wireless body area networks used for ambulatory monitoring and health care. The study of System reliability of a limited-flow network in multi commodity case was presented by Lin [5]. Rocco and Zio [6] presented cellular automata and Monte Carlo sampling in Solving advanced network reliability problems. Assessment of network reliability using a cellular automata approach was studied by Rocco and Moreno [7]. Yusuf [8] presented reliability characteristics comparison between redundant systems requiring supporting units for their operation. Wang and Kuo [9] investigated the cost and probabilistic analysis of series systems with mixed standby components. Wang and co-workers have also performed comparative analysis of availability between three systems with general repair times, reboot delay and switching failures. Ke and Chu [10] performed comparative of availability for a redundant repairable system. Wang et. al [11] performed comparative analysis of availability between two systems with warm standby units and different imperfect coverage. The problem considered in this paper is different from the work of the authors above. This paper is devoted to deal with mean time to system failure and busy period modelling of four redundant network communication systems. The contributions of this paper are twofold. First, a procedure for computing reliability measures in this study is based on the first order linear differential equations to develop explicit expressions of mean time to system

Corresponding author: Ibrahim Yusuf, E-mail: iyusuf.mth@buk.edu.ng, Tel.: +2348065281616

failure and busy period for the four configurations. Secondly, we used maple software to perform comparisons of mean time to system failure and busy period for four configurations analytically.

The rest of the paper is organized as follows. Section 2 gives the description of the systems/ configurations. Section 3 deals with mean time to system failure models formulation. Section 4 deals with busy period models formulation. Analytical comparisons of the configurations are presented in Section 4. The paper is concluded in Section 5.

## 2.0 Description of the Networks

We consider four dissimilar redundant network flow systems as follows. The first system consists of two subsystems A and B arranged in parallel. Each subsystem has two units as shown in Figure 1. The system works whenever two units are working. Signal from the source is received by units $A_1$ and $A_2$. When one of the primary units ($A_1$ or $A_2$) fails, the standby unit in subsystem B ($B_1$ or $B_2$) is switched on to assume the role of the failed primary unit. The system failed when more than two units have failed. The second system consists of three subsystems A, B and C as shown in Figure 2. With subsystems A and B in series and parallel to subsystem C. Subsystem A has one unit $A_1$ while subsystems B and C two units each. Initially signal is received from the source by unit $A_1$ which is distributed to one of the primary units in subsystem B ($B_1$ or $B_2$). When one of the primary units ($B_1$ or $B_2$) fails, the standby unit in subsystem B ($B_1$ or $B_2$) is switched on to assume the role of the failed primary unit. At the failure of unit $A_1$, subsystem B ceased to work and one of the primary unit in subsystem C ($C_1$ or $C_2$) is switched on to assume the role of $A_1$. The system failed when both units $A_1$, $C_1$ and $C_2$ have failed. The third system consists of three subsystems A, B and C with subsystem A and B in parallel and series to subsystem C as shown in Figure 3. Each subsystem has two units. Signal is received by primary units in subsystem A and is distributed any of the primary unit in subsystem C. Whenever of the primary unit in subsystem C fails, the standby unit in subsystem C is switched on to assume the role of failed primary unit. At the failure of a unit in subsystem A, units in subsystem B are switched on to assume the role of units in subsystem A. The system fails when both units in subsystems A and B and units in subsystem C have failed. The fourth system is parallel series system with units $A_1$ and $A_2$ in series and parallel to subsystems B and C as shown in Figure 4. Subsystems B and C are in series and have two units each.Signal is received initially by unit $A_1$ and is distributed to $A_2$. At the failure of $A_1$ or $A_2$, both subsystems B and C are switched on. Signal is then received by one of the primary unit in subsystem B and is then distributed to one of the primary unit in subsystem C. At the failure of either primary unit in subsystem B and C, the standby unit is switched to assume the role of failed primary unit. The system failed when unit $A_1$ or $A_2$, and any of subsystem B and C have failed. We assume that switching is perfect and instantaneous. We also assume that two or more units cannot fail simultaneously. Whenever a unit fails with failure rate ~ , it is immediately sent to service station for repair with service rate and the standby unit/subsystem is immediately switched into operation.

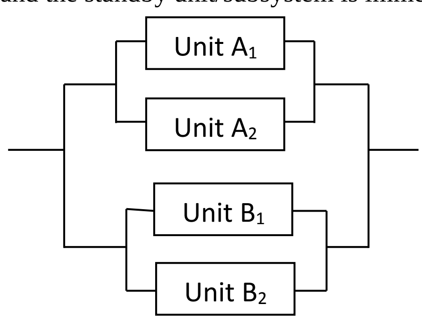


**Figure 1**: Reliability block diagram of configuration I

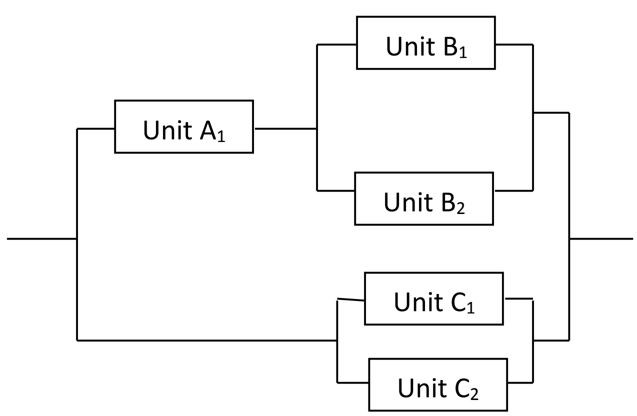


**Figure 2:** Reliability block diagram of configuration II

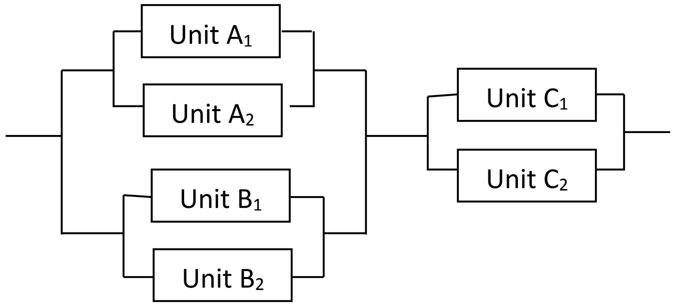


**Figure 3:** Reliability block diagram of configuration III

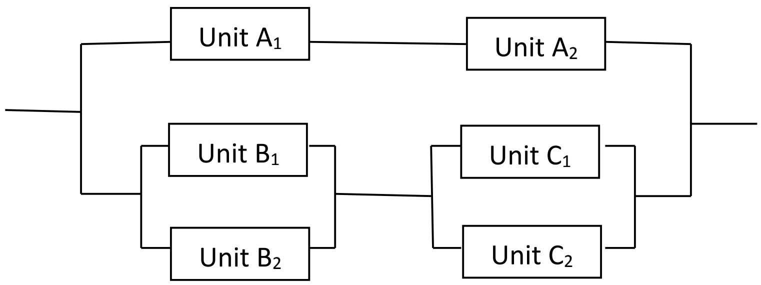


**Figure 4:** Reliability block diagram of configuration IV

## 3.0 Mean time to System Failure Models Formulation

## 4.0 MTSF Formulation for Configuration I

For the analysis of mean time to system failure case of configuration I, we define $P_i(t)$ to be the probability that the system at time $t \geq 0$ is in state $S_i$. Also let $P(t)$ be the probability row vector at time $t$. The initial condition for this problem is:

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), ..., P_{11}(0)] = [1,0,0,0,0,0,0,0,0,0,0,0]$

We obtain the following differential equations:

$$\dot{P}_0(t) = -2\sim P_0(t) + \quad P_1(t) + \quad P_2(t)$$
$$\dot{P}_1(t) = -(2\sim + \quad )P_1(t) + \sim P_0(t) + \quad P_3(t) + \quad P_4(t)$$
$$\dot{P}_2(t) = -(2\sim + \quad )P_2(t) + \sim P_0(t) + \quad P_5(t) + \quad P_6(t)$$
$$\dot{P}_3(t) = -(\sim + \quad )P_3(t) + \sim P_1(t) + \quad P_7(t)$$
$$\dot{P}_4(t) = -(\sim + \quad )P_4(t) + \sim P_1(t) + \quad P_8(t) + \quad P_9(t)$$
$$\dot{P}_5(t) = -(\sim + \quad )P_5(t) + \sim P_2(t) + \quad P_{10}(t)$$
$$\dot{P}_6(t) = -(2\sim + \quad )P_6(t) + \sim P_2(t) + \quad P_{10}(t) + \quad P_{11}(t)$$
$$\dot{P}_7(t) = - \quad P_7(t) + \sim P_3(t)$$
$$\dot{P}_8(t) = - \quad P_8(t) + \sim P_4(t)$$
$$\dot{P}_9(t) = - \quad P_9(t) + \sim P_4(t)$$
$$\dot{P}_{10}(t) = -2 \quad P_{10}(t) + \sim P_5(t) + \sim P_6(t)$$
$$\dot{P}_{11}(t) = - \quad P_{11}(t) + \sim P_6(t) \qquad (1)$$

This can be written in the matrix form as

$$\dot{P} = T_1 P \qquad (2)$$

where

$$T_1 = \begin{pmatrix} -2\sim & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sim & -X & 0 & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sim & 0 & -X & 0 & 0 & & & 0 & 0 & 0 & 0 & 0 \\ 0 & \sim & 0 & -Y & 0 & 0 & 0 & & 0 & 0 & 0 & 0 \\ 0 & \sim & 0 & 0 & -X & 0 & 0 & 0 & & & 0 & 0 \\ 0 & 0 & \sim & 0 & 0 & -Y & 0 & 0 & 0 & 0 & & 0 \\ 0 & 0 & \sim & 0 & 0 & 0 & -X & 0 & 0 & 0 & & \\ 0 & 0 & 0 & \sim & 0 & 0 & 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sim & 0 & 0 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sim & 0 & 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sim & \sim & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sim & 0 & 0 & 0 & 0 & - \end{pmatrix}$$

$X = (2\sim + \ )$, $Y = (\sim + \ )$

It is difficult to evaluate the transient solutions, hence using the concept of Wang and Kuo [9] to develop the explicit for MTSF. The procedures require deleting rows and columns of absorbing states of matrix $T_1$ and take the transpose to produce a new matrix, say $M_1$. The expected time to reach an absorbing state is obtained from the relation:

$$E[T_{P(0)\to P(absorbing)}] = MTSF_1 = P(0)(-M_1^{-1})\left[1,1,1,1,1,1,1\right]^T$$

$$= \frac{\ ^3 + 5\sim\ ^2 + 15\sim^2\ + 14\sim^3}{2\sim^3(4\sim + 3\ )} \qquad (3a)$$

$$M_1 = \begin{pmatrix} -2\sim & \sim & \sim & 0 & 0 & 0 & 0 \\ & -(2\sim+\ ) & 0 & \sim & \sim & 0 & 0 \\ & 0 & -(2\sim+\ ) & 0 & 0 & \sim & \sim \\ 0 & & 0 & -(\sim+\ ) & 0 & 0 & 0 \\ 0 & & 0 & 0 & -(2\sim+\ ) & 0 & 0 \\ 0 & 0 & & 0 & 0 & -(\sim+\ ) & 0 \\ 0 & 0 & & 0 & 0 & 0 & -(2\sim+\ ) \end{pmatrix} \qquad (3b)$$

## 5.0 MTSF formulation for Configuration II

For configuration II, we define $P_i(t)$ to be the probability that the System at time $t \geq 0$ is in state $S_i$. Also let $P(t)$ be the probability row vector at time $t$, we have the following initial condition: $P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \ldots, P_{10}(0)] = \left[1,0,0,0,0,0,0,0,0,0,0\right]$.

The differential equations are expressed in the form

$$\dot{P} = T_2 P \qquad (4)$$

where

$$T_2 = \begin{pmatrix} -2\sim & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sim & -Y & 0 & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sim & 0 & -X & 0 & 0 & & & 0 & 0 & 0 & 0 \\ 0 & \sim & 0 & -Y & & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sim & - & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sim & 0 & 0 & -Y & 0 & & 0 & 0 & 0 \\ 0 & 0 & \sim & 0 & 0 & 0 & -Y & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sim & 0 & -Y & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sim & 0 & -Y & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sim & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sim & 0 & - \end{pmatrix}$$

Using the procedure described insubsection 3.1, the expected time to reach an absorbing state is

$$E[T_{P(0)\to P(absorbing)}] = MTSF_2 = P(0)(-M_2^{-1})\left[1,1,1,1,1,1,1,1\right]^T =$$

$$= \frac{10\sim^5 + 14\sim^4 \quad + 15\sim^3 \quad ^2 + 10\sim^2 \quad ^3 + 4\sim \quad ^4 + \quad ^5}{\sim^3\left(4\sim^3 + 3\sim^2 \quad + 3\sim \quad ^2 + \quad ^3\right)} \tag{5a}$$

where

$$M_2 = \begin{pmatrix} -2\sim & \sim & \sim & 0 & 0 & 0 & 0 & 0 \\ & -(\sim + \ ) & 0 & \sim & 0 & 0 & 0 & 0 \\ & 0 & -(2\sim + \ ) & 0 & \sim & \sim & 0 & 0 \\ 0 & & 0 & -(\sim + \ ) & 0 & 0 & 0 & 0 \\ 0 & 0 & & 0 & -(\sim + \ ) & 0 & \sim & 0 \\ 0 & 0 & & 0 & 0 & -(\sim + \ ) & 0 & \sim \\ 0 & 0 & 0 & 0 & & 0 & -(\sim + \ ) & 0 \\ 0 & 0 & 0 & 0 & 0 & & 0 & -(\sim + \ ) \end{pmatrix} \tag{5b}$$

## 6.0 MTSF formulation for Configuration III

For the analysis of mean time to system failure case of configuration III, the initial conditions are

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \ldots, P_{10}(0)] = \left[1,0,0,0,0,0,0,0,0,0,0\right]$$

The differential equations are expressed in the form

$$\dot{P} = T_3 P \tag{6}$$

Where

$$T_3 = \begin{pmatrix} -2\sim & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sim & -X & 0 & & & 0 & 0 & 0 & 0 & 0 & 0 \\ \sim & 0 & -X & 0 & 0 & 0 & & & & 0 & 0 \\ 0 & \sim & 0 & -X & 0 & & 0 & 0 & 0 & 0 & 0 \\ 0 & \sim & 0 & 0 & - & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sim & 0 & - & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sim & 0 & 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & \sim & 0 & 0 & 0 & 0 & -X & 0 & & \\ 0 & 0 & \sim & 0 & 0 & 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sim & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sim & 0 & 0 & - \end{pmatrix}$$

Using the procedure described insubsection 3.1, the expected time to reach an absorbing state is

$$E[T_{P(0)\to P(absorbing)}] = MTSF_3 = P(0)(-M_3^{-1})\left[1,1,1,1,1\right]^T$$

$$= \frac{^2 + 5\sim \quad + 10\sim^2}{2\sim^2(4\sim + \ )} \tag{7a}$$

where

$$M_3 = \begin{pmatrix} -2\sim & \sim & \sim & 0 & 0 \\ & -(2\sim + \ ) & 0 & \sim & 0 \\ & 0 & -(2\sim + \ ) & 0 & \sim \\ 0 & & 0 & -(2\sim + \ ) & 0 \\ 0 & 0 & & 0 & -(2\sim + \ ) \end{pmatrix} \tag{7b}$$

## 7.0 MTSF formulation for Configuration IV

For the analysis of mean time to system failure case of configuration IV, the initial conditions are

$$P(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),...,P_{11}(0)]\ =[1,0,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P}=T_4P \tag{8}$$

Where

$$T_4=\begin{pmatrix} -\sim & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sim & -X & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sim & -X & 0 & & 0 & & 0 & 0 & 0 & 0 & 0 \\ 0 & \sim & 0 & -X & 0 & & 0 & & 0 & 0 & 0 & 0 \\ 0 & 0 & \sim & 0 & -X & 0 & 0 & 0 & & & 0 & 0 \\ 0 & 0 & 0 & \sim & 0 & -X & 0 & 0 & 0 & 0 & & \\ 0 & 0 & \sim & 0 & 0 & 0 & - & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sim & 0 & 0 & 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sim & 0 & 0 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sim & 0 & 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sim & 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & 0 & \sim & 0 & 0 & 0 & 0 & 0 & - \end{pmatrix}$$

Using the procedure described insubsection 3.1, the expected time to reach an absorbing state is

$$E[T_{P(0)\to P(absorbing)}]=MTSF_4=P(0)(-M_3^{-1})[1,1,1,1,1,1]^T$$

$$=\frac{18\sim^3+11\sim^2\quad+4\sim\quad^2+\quad^3}{2\sim^3(4\sim+\quad)} \tag{9a}$$

where

$$M_4=\begin{pmatrix} -\sim & \sim & 0 & 0 & 0 & 0 \\ & -(2\sim+\quad) & \sim & \sim & 0 & 0 \\ 0 & & -(2\sim+\quad) & 0 & \sim & 0 \\ 0 & & 0 & -(2\sim+\quad) & 0 & \sim \\ 0 & 0 & & 0 & -(2\sim+\quad) & 0 \\ 0 & 0 & 0 & & 0 & -(2\sim+\quad) \end{pmatrix} \tag{9b}$$

## 8.0 Comparison of The Configurations

## 9.0 Comparison of Mean time to System Failure Using MAPLE Software

$$MTSF_2-MTSF_1=\frac{24\sim^6+70\sim^5\quad+97\sim^4\quad^2+92\sim^3\quad^3+59\sim^2\quad^4+24\sim\quad^4+5\quad^6}{2\sim^3\left(4\sim^3+3\sim^2\quad+3\sim\quad^2+\quad^3\right)\left(4\sim+3\quad\right)} \tag{10}$$

$$MTSF_1-MTSF_3=\frac{16\sim^4+24\sim^3\quad+16\sim^2\quad^2+6\sim\quad^3+\quad^4}{2\sim^3\left(4\sim+3\quad\right)\left(4\sim+\quad\right)} \tag{11}$$

$$MTSF_4-MTSF_1=\frac{16\sim^4+24\sim^3\quad+14\sim^2\quad^2+7\sim\quad^3+2\quad^4}{2\sim^3\left(4\sim+3\quad\right)\left(4\sim+\quad\right)} \tag{12}$$

$$MTSF_2-MTSF_4=\frac{8\sim^6+34\sim^5\quad+45\sim^4\quad^2+43\sim^3\quad^3+26\sim^2\quad^4+9\sim\quad^5+\quad^6}{2\sim^3\left(4\sim^3+3\sim^2\quad+3\sim\quad^2+\quad^3\right)\left(4\sim+\quad\right)} \tag{13}$$

## 10.0 Comparison of Busy Period of the Repairman

$$B_{T2}-B_{T1}=\frac{\sim^2\quad^3(2\sim^3-2\sim\quad-\quad^2)}{(2\sim^4+3\sim^3\quad+3\sim^2\quad^2+2\sim\quad^3+\quad^4)(5\sim^3+4\sim^2\quad+2\sim\quad^3+\quad^3)} \tag{14}$$

Let $f(\sim, \ ) = 2\sim^2 - 2\sim \ - \ ^2$ (15)

The sign of (27) depend on the sign of (28). The roots of (28) are $\sim_1 = \left(\frac{1+\sqrt{3}}{2}\right)$ and $\sim_2 = \left(\frac{1-\sqrt{3}}{2}\right)$ . The second root lies on the left hand of the plane for all positive values of . To determine the sign of (28), we need to test points either side of $\sim_1$. Consider a positive number £ , $0 < £ < \sim_1$. Without loss of generality we assume £ = . Replacing $\sim$ with $\sim_1 -$ in (15) yield

$$f(\sim_1 - \ , \ ) = 2 \ ^2\left(1-\sqrt{3}\right) < 0 \tag{16}$$

On the other hand, replacing $\sim$ with $\sim_1 +$ in (15) yield

$$f(\sim_1 + \ , \ ) = 2 \ ^2\left(1+\sqrt{3}\right) > 0 \tag{17}$$

In a nutshell

$$f(\sim, \ ) = \begin{cases} negative \ (B_{T2} < B_{T1}),\ if \sim < \sim_1 \\ positive \ (B_{T2} > B_{T1}),\ if \sim > \sim_1 \\ 0 \quad (B_{T2} = B_{T1}),\ if \sim = \sim_1 \end{cases} \tag{18}$$

$$B_{T1} - B_{T3} = \frac{\sim^3 \ ^3}{(5\sim^3 + 4\sim^2 \ + 2\sim \ ^2 + \ ^3)(4\sim^3 + 4\sim^2 \ + 2\sim \ ^2 + \ ^3)} \tag{19}$$

From (32), it can be seen that for all positive values of $\sim$ and . Thus, $B_{T1} > B_{T3}$

$$B_{T4} - B_{T1} = \frac{\sim \ ^3(\sim - \ )(4\sim^2 + 3\sim \ + \ ^2)}{(5\sim^3 + 4\sim^2 \ + 2\sim \ ^2 + \ ^3)(4\sim^4 + 4\sim^3 \ + 2\sim^2 \ ^2 + \sim \ ^3 + \ ^4)} \tag{20}$$

From (33) , it can be seen that $B_{T4} > B_{T1}$ if $\sim >$ .

$$B_{T2} - B_{T3} = \frac{\sim^2 \ ^3(\sim - \ )(2\sim + \ )}{(2\sim^4 + 3\sim^3 \ + 3\sim^2 \ ^2 + 2\sim \ ^3 + \ ^4)(4\sim^3 + 4\sim^2 \ + 2\sim \ ^2 + \ ^3)} \tag{21}$$

From (34) , it can be seen that $B_{T2} > B_{T3}$ if $\sim >$ .

$$B_{T4} - B_{T2} = \frac{\sim \ ^4(2\sim^3 + \sim^2 \ - \sim \ ^2 - \ ^3)}{(2\sim^4 + 3\sim^3 \ + 3\sim^2 \ ^2 + 2\sim \ ^3 + \ ^4)(4\sim^4 + 4\sim^3 \ + 2\sim^2 \ ^2 + \sim \ ^3 + \ ^4)} \tag{22}$$

From (35),

let $h(\sim, \ ) = 2\sim^3 + \sim^2 \ - \sim \ ^2 - \ ^3$ (23)

Equation (36) is a cubic with one real root and two imaginary roots with negative real parts. The root is

$$\sim_2 = \frac{1}{6}\left(C^{\frac{1}{3}} + 7C^{-\frac{1}{3}} - 1\right) \ \approx \frac{754}{909} \tag{24}$$

Where $C = \left(44 + 3\sqrt{177}\right)$.To determine the sign of equation (23), it suffices to test points in the neighbourhood of $\sim_2$. Substituting $\sim = \sim_2 -$ and $\sim = \sim_2 +$ in (23) to obtained

$$h(\sim_2 - \ , \ ) = \frac{-608624899}{751089429} \ ^3 < 0 \tag{25}$$

and $h(\sim_2 + \ , \ ) = \frac{9586989983}{751089429} \ ^3 > 0$ respectively. Thus,

$$h(\sim, \ ) = \begin{cases} positive\,(B_{T4} > B_{T2}),\ if \ \sim > \sim_2 \\ negative \ (B_{T4} < B_{T2}),\ if \ \sim < \sim_2 \end{cases} \tag{26}$$

$$B_{T4} - B_{T3} = \frac{\sim \ ^3(4\sim^3 - 2\sim \ ^2 - \ ^3)}{(4\sim^3 + 4\sim^2 \ + 2\sim \ ^2 + \ ^3)(4\sim^4 + 4\sim^3 \ + 2\sim^2 \ ^2 + \sim \ ^3 + \ ^4)} \tag{27}$$

From (40), let

$$g(\sim,\ \ ) = 4\sim^{3} - 2\sim\ ^{2} - \ ^{3} \tag{28}$$

The sign of (27) depends on the sign of (28) which is cubic in $\sim,$ with one real root and two imaginary roots with negative real parts. The real root is

$$\sim_{3} = \frac{1}{6}\left(B^{\frac{1}{3}} + 6B^{\frac{-1}{3}}\right) \approx \frac{1112}{1257} \tag{29}$$

where $B = \left(27 + 3\sqrt{57}\right)$. Thus,

$$g(\sim,\ \ ) = \begin{cases} positive\ (B_{T4} > B_{T3}),\ if \sim\ > \sim_{3} \\ negative\,(B_{T4} < B_{T3}),\ if \sim\ < \sim_{3} \end{cases} \tag{30}$$

Thus, from equations (10) - (13) we have

$MTSF_2 > MTSF_4 > MTSF_1 > MTSF_3\ ,\ \ \forall \sim,\ \ > 0$

and from (14) to (30), the optimal configuration depend on the value of $\sim$ .

## 11.0 Conclusion

In this paper, we studied the mean time to system failure and busy period of repairman of four dissimilar redundant communication networks. We developed the explicit expressions for mean time to system failure and busy period for each configuration and performed comparison analytically using MAPLE software to determine the optimal configuration. It is evident from the analysis that configuration II is optimal configuration using mean time to system failure while using busy period, the optimal configuration depend on the value of $\sim$ . The present study will help the maintenance managers, reliability analyst, engineers and system designers to develop sophisticated models and to design more critical system in interest of human kind. The study will also assist engineers, decision makers and plant management to avoid an incorrect reliability assessment and consequent erroneous decision making which may lead to unnecessary expenditures, incorrect maintenance scheduling and reduction of safety standards.

## 12.0 References

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