Journal of the Nigerian Association of Mathematical Physics
Volume 29, (March, 2015), pp 223 – 230
© J. of NAMP

Statistical Validation of the Disaggregated Cobb-Douglas Production Function

Omoregie, M. J.

Department of Production Engineering, University of Benin, Benin City.

Abstract

Production functions have been widely used in modeling aggregated inputs and output in various organizations. Cobb-Douglasaggregate model has found prominence among these functions. Lack of specificity has however limited the application of the aggregate production function as an effective management tool. In this paper, a disaggregated six-input structure has been used to model inputs/output of a local oil production company. The output elasticities obtained from the analysis have been statistically tested and found to be valid and dependable within the 95% confidence interval. Thus the six-factor disaggregated Cobb-Douglas production function has been presented as a veritable management tool for productivity improvement through optimal utilisation of input resources.

Keywords: Return to scale, Output elasticity, Production function, Bootstrapping, Aggregation

1.0 Introduction

Every organization, be it producer of goods or provider of services is characterized by the use of a group of inputs (resources) which are transformed vide one form of transformation system or the other into desired outputs. These inputs which are largely materials, capital, labour, plant and machinery must be well proportioned in the transformation process. In other words, we must have a means of specifying the output of the organization for all combinations of inputs. A production function serves this purpose by relating inputs to outputs in an orderly manner. Production functions are generally classified as fixed or flexible production functions. They can also be classified according to the type of returns to scale, elasticity of substitution; and whether or not it is constant across output levels.

Several production functions have been developed and applied over time. Klacek et al.[1] employed a three-factor translog production function to model the gross output and input relationship for the Czech national economy, using 40 quarterly time series data for the period 1995 - 2004. Arrow et al. [2] developed the constant elasticity of substitution (CES) production function. Berndt and Khaled [3] employed the generalized Leontiff (GL), Translog and Square-root quadratic production functions in modeling the producer behavior for the United States of America manufacturing sector for the 1947 – 1971 period. They identified the need for caution in assuming little technological progress in modeling the production function.

Cobb-Douglas is perhaps the most widely used of all production functions for modeling the relationship of an output to inputs. It was pioneered by Knut Wicksel (1857 – 1976) and statistically tested by Charles Cobb and Paul Douglas in 1928 in modeling the growth of the American economy during the period 1899 – 1922 [4]. The Cobb-Douglas production function has the functional form:

$$P(L,K) = AL^{\alpha}K^{\beta} \quad (1)$$

where P = total production (the monetary value of goods produced in a year)

L = Labour input (the total number of man – hour used for production in a year)

K = Capital input (the monetary worth of all machinery, equipment and buildings).

A = Total factor productivity; α and β are the output elasticities of labour and capital respectively. These values are constant and determined by available technology [5].

The total factor productivity A, represents the aspect of total output not caused by stated inputs. Due to its empirical flexibility, complex production functions often converge to the Cobb-Douglas function. For example the constant marginal share (CMS) function.

Corresponding author: Omoregie, M. J., E-mail: , Tel.: +2348023412612.

$V = \alpha K^{\alpha} L^{1-\alpha} - mL$ (2)

reduces to the Cobb-Douglas production function when m = 0 [6].
Similarly, Zellner – Revanker production function,

$$V \exp(\bar{\ } V) = ¥ K^{\sim(1-¢)} L^{\sim ¢} \;:\; 0 < ¢ < 1 : ¥ > 0 : \sim > 0 \quad (3)$$

reduces to the Cobb-Douglas function when $\theta = 0$ [7].
The production function, particularly the Cobb-Douglas function, is universally applicable whenever inputs give rise to an output, even in domestic affairs. For example, Graham and Green [8] employed the Cobb-Douglas principle to develop a house-hold production model as an activity function to relate effective leisure of husband and wife to family output in the form

$U = U(C_L M_L L_L . M_w L_w)$ (4)

where C = goods obtained in the market or produced at home, and
$M_L L_L$ and $M_w L_w$ are effective leisure of husband and wife respectively

$C = X_m + Z$ (5)

where X_m represents goods purchased in the market and Z represents goods produced at home (and measured in the same units as market – purchased goods).

2.0 Constraints of the Cobb-Doudglas Production Function

The Cobb-Douglas production function is a linear – homogenous function that lends itself to regression analysis. However it has the disadvantage of not allowing identification of the nature of the technological progress. Furthermore and like all previous functions, aggregates all inputs in its computation. This results in non-specificity of outcome.
We need to find some means of relaxing these constraints and still derive the benefit of this important function.
Welfens [9] used a quassi – Cobb-Douglas model to decompose the output function in an ICT environment into elemental variables of capital labour and a residual variable. He disaggregated the capital input into ICT – capital and non – ICT capital to come up with a linear – homogenous production function.

$Y = [\ B (K^1/K) K^1]^{\beta 1} \ [K^{11}]^{\beta 11} \ [AL]^{(1-\beta 1-\beta 11)}$ (6)

where A and B represent progress parameters.
Omoregie[10], developed a six-factor disaggregated Cobb-Douglas function, which he found to be a superior management control tool, when compared with the traditional two-factor aggregate model.
This paper therefore seeks to throw new light on the disaggregated Cobb-Douglas function and statistically establish its validity.

3.0 Method

Ten-year historical data were obtained from a local producer of oil and adapted to fit into the proposed model. From these data, nine other sets of data were simulated using the Bootstrap method (random sampling with replacement) with the aid of a suitably selected urn. On the basis of these data, a six-factor Cobb-Douglas production function has been formulated in the form:

$$\begin{aligned} Q &= f\left(K_1, K_2, L_1, L_2, M, E\right) \\ &= A K_1^{\sim} . K_2 L_1^{¥} L_2^{«} M^{¨} E^{\mu} \end{aligned} \quad (7)$$

where
A = total factor productivity, K_1 = Fixed Capital, K_2= Working Capital, L_1 = Direct Labour, L_2 = Indirect Labour, M = Machinery Cost (Annual depreciation charge or depletion charges) E = energy cost (public power and in-house power source)
It is expected that the inherent variabilities of input elements in the input/output equation will become more explicit, with the new disaggregated input structure. Thus aspects of the inputs contributing positively to productivity growth can be emphasized while those responsible for declining performance become visible for management consideration for necessary remedial action.

4.0 Equation Formulation and Computation

The input and output data obtained are presented in Table 1. Q is the total monetary value of annual production, while K_1, K_2, L_1, L_2, M and E are annual monetary values of input resources of Fixed Capital, Working Capital, Direct Labour, Indirect Labour, Machinery Cost and Energy Cost expended in the course of producing Q. Least Square Regression method was used to evaluate the data. Table 2 contains the computations in the regression process. y is the logarithmic transformation of the total output, while x_1, ……, x_6 are logarithmic transformations of various disaggregated inputs of capital, labour, machinery and energy earlier described.

Table1: Total Output and Input data (Ten-year period).

Period	**Q** x10^6(₦)	**K$_1$** x10^6(₦)	**K$_2$** x10^6(₦)	**L$_1$** x10^6(₦)	**L$_2$** x10^6(₦)	**M** x10^6(₦)	**E** x10^6(₦)
1	150.4	12.98	8.179	2.726	1.2	2.982	1.238
2	183.6	7.862	8.693	4.914	1.31	2.965	1.982
3	107.1	1.095	6.482	5.094	1.227	1.055	1.844
4	179.3	9.46	9.054	5.294	1.362	1.093	1.353
5	133.5	7.038	9.838	5.294	1.389	1.072	1.677
6	134.2	1.451	11.110	5.294	1.025	1.089	1.237
7	192.8	21.82	1.732	5.294	1.221	1.048	1.119
8	187.4	14.57	13.214	5.294	1.197	1.067	1.119
9	108.8	3.782	6.127	5.294	1.218	1.216	1.342
10	106.2	21.5	8.934	5.294	1.142	1.096	1.092
Total	1483	101.6	83.362	49.79	12.29	14.68	14

Table 2 : Regression Computation table

Y x10^6	x_1 x10^6	x_2 x10^6	x_3 x10^6	x_4 x10^6	x_5 x10^6	x_6 x10^6	x_1^2 x10^6	x_2^2 x10^6	x_3^2 x10^6	x_4^2 x10^6	x_5^2 x10^6	x_6^2 x10^6	$x_1.x_2$ x10^6	$x_1.x_3$ x10^6	$x_1.x_4$ x10^6	$x_1.x_5$ x10^6	$x_1.x_6$ x10^6
2.1772	1.1132	0.9127	0.4355	0.0792	0.4745	0.0927	1.2392	0.833	0.1897	0.0063	0.2252	0.0086	1.016	0.4848	0.0881	0.5282	0.1032
2.2639	0.8955	0.9392	0.6914	0.1173	0.472	0.2971	0.802	0.882	0.4781	0.0138	0.2228	0.0883	0.841	0.6192	0.105	0.4227	0.2661
2.0299	0.0394	0.8117	0.7071	0.0888	0.0233	0.2658	0.0016	0.6588	0.4999	0.0079	0.0005	0.0706	0.032	0.0279	0.0035	0.0009	0.0105
2.2535	0.9759	0.9568	0.7238	0.1342	0.0386	0.1313	0.9524	0.9155	0.5239	0.018	0.0015	0.0172	0.9338	0.7063	0.1309	0.0377	0.1281
2.1255	0.8474	0.9929	0.7238	0.1427	0.0302	0.2245	0.7182	0.9858	0.5239	0.0204	0.0009	0.0504	0.8414	0.6134	0.1209	0.0256	0.1903
2.1277	0.1617	1.0457	0.7238	0.0107	0.037	0.0924	0.0261	1.0935	0.5239	0.0001	0.0014	0.0085	0.1691	0.117	0.0017	0.006	0.0149
2.2852	1.3389	0.2385	0.7238	0.0867	0.0204	0.0489	1.7927	0.0569	0.5239	0.0075	0.0004	0.0024	0.3194	0.9691	0.1161	0.0273	0.0655
2.2727	1.1635	1.121	0.7238	0.0781	0.0282	0.0489	1.3536	1.2567	0.5239	0.0061	0.0008	0.0024	1.3043	0.8421	0.0909	0.0328	0.0569
2.0367	0.5777	0.7872	0.7238	0.0856	0.0851	0.1276	0.3338	0.6197	0.5239	0.0073	0.0072	0.0163	0.4548	0.4181	0.0495	0.0492	0.0737
2.0262	1.3325	0.9511	0.7238	0.0577	0.0397	0.0382	1.7756	0.9045	0.5239	0.0033	0.0016	0.0015	1.2673	0.9644	0.0768	0.0529	0.0509
21.598	8.4458	8.7569	6.9004	0.881	1.249	1.3674	8.9951	8.2066	4.8346	0.0907	0.4623	0.2662	7.1791	5.7623	0.7836	1.1832	0.9601

Table2 continued

$x_2.x_3$ x10^6	$x_2.x_4$ x10^6	$x_2.x_5$ x10^6	$x_2.x_6$ x10^6	$x_3.x_4$ x10^6	$x_3.x_5$ x10^6	$x_3.x_6$ x10^6	$x_4.x_5$ x10^6	$x_4.x_6$ x10^6	$x_5.x_6$ x10^6	$y.x_1$ x10^6	$y.x_2$ x10^6	$y.x_3$ x10^6	$y.x_4$ x10^6	$y.x_5$ x10^6	$y.x_6$ x10^6
0.3975	0.0723	0.4331	0.0846	0.0345	0.2066	0.0404	0.0376	0.0073	0.044	2.4236	1.9871	0.9481	0.1724	1.0331	0.2019
0.6494	0.1101	0.4433	0.279	0.0811	0.3264	0.2054	0.0554	0.0348	0.1402	2.0274	2.1261	1.5653	0.2655	1.0686	0.6726
0.5739	0.0721	0.0189	0.2157	0.0628	0.0164	0.1879	0.0021	0.0236	0.0062	0.08	1.6476	1.4352	0.1803	0.0472	0.5395
0.6925	0.1284	0.037	0.1256	0.0971	0.028	0.095	0.0052	0.0176	0.0051	2.1991	2.1562	1.631	0.3024	0.087	0.2958
0.7186	0.1417	0.03	0.2229	0.1033	0.0219	0.1625	0.0043	0.032	0.0068	1.8013	2.1104	1.5384	0.3033	0.0642	0.4772
0.7569	0.0112	0.0387	0.0966	0.0078	0.0268	0.0669	0.0004	0.001	0.0034	0.344	2.225	1.54	0.0228	0.0788	0.1965
0.1727	0.0207	0.0049	0.0117	0.0628	0.0147	0.0354	0.0018	0.0042	0.001	3.0597	0.5451	1.654	0.1982	0.0465	0.1118
0.8114	0.0875	0.0316	0.0548	0.0565	0.0204	0.0354	0.0022	0.0038	0.0014	2.6442	2.5477	1.6449	0.1775	0.064	0.1112
0.5698	0.0674	0.067	0.1004	0.062	0.0616	0.0923	0.0073	0.0109	0.0109	1.1767	1.6033	1.4741	0.1744	0.1733	0.2599
0.6884	0.0548	0.0378	0.0364	0.0417	0.0288	0.0277	0.0023	0.0022	0.0015	2.7	1.9271	1.4665	0.1168	0.0805	0.0774
6.0309	0.7663	1.1421	1.2278	0.6096	0.7515	0.9489	0.1184	0.1376	0.2204	18.456	18.876	14.898	1.9136	2.7432	2.9438

Applying the Cobb-Douglas function, we have:

$$Q = AK_1^{\sim}\,.\,K_2\,.\,L_1^{¥}\,.\,L_2^{«}\,.\,M^{¨}\,.\,E^{\mu} \qquad (8)$$

Taking logarithms

$$\log\ Q = \log A + \sim \log K_1 + \quad \log K_2 + ¥ \log L_1 + « \log L_2 + ¨ \log M + \mu \log E \qquad (9)$$

This is of the form

$$y = \ _0 + \sim x_1 + \quad x_2 + ¥ x_3 + « x_4 + ¨ x_5 + \mu x_6 \qquad (10)$$

where $y = \log Q, \quad _0 = \log A, x_1 = \log K_1, x_2 = \log K_2, x_3 = \log L_1, x_4 = \log L_2, x_5 = \log M, x_6 = \log E$

From multivariate linear regression, we have the following seven normal equations

$$\Sigma y \ = \Sigma \ _0 + \sim \Sigma x_1 + \quad \Sigma x_2 + ¥ \Sigma x_3 + « \Sigma x_4 + ¨ \Sigma x_5 + \mu \Sigma x_6 \qquad (11)$$

$$\Sigma x_1 y = \ _0 \Sigma x_1 + \sim \Sigma x_1^2 + \quad \Sigma x_1 x_2 + ¥ \Sigma x_1 x_3 + « \Sigma x_1 x_4 + ¨ \Sigma x_1 x_5 + \mu \Sigma x_1 x_6 \qquad (12)$$

$$\Sigma x_2 y = \ _0 \Sigma x_2 + \sim \Sigma x_1 x_2 + \quad \Sigma x_2^2 + ¥ \Sigma x_2 x_3 + « \Sigma x_2 x_4 + ¨ \Sigma x_2 x_5 + \mu \Sigma x_2 x_6 \qquad (13)$$

$$\Sigma x_3 y = \ _0 \Sigma x_3 + \sim \Sigma x_1 x_3 + \quad \Sigma x_2 x_3 + ¥ \Sigma x_3^2 + « \Sigma x_3 x_4 + ¨ \Sigma x_3 x_5 + \mu \Sigma x_3 x_6 \qquad (14)$$

$$\Sigma x_4 y = \ _0 \Sigma x_4 + \sim \Sigma x_1 x_4 + \quad \Sigma x_2 x_4 + ¥ \Sigma x_3 x_4 + « \Sigma x_4^2 + ¨ \Sigma x_4 x_5 + \mu \Sigma x_4 x_6 \qquad (15)$$

$$\Sigma x_5 y = \ _0 \Sigma x_5 + \sim \Sigma x_1 x_5 + \ _0 \Sigma x_2 x_5 + ¥ \Sigma x_3 x_5 + « \Sigma x_4 x_5 + ¨ \Sigma x_5^2 + \mu \Sigma x_5 x_6 \qquad (16)$$

$$\Sigma x_6 y = \ _0 \Sigma x_6 + \sim \Sigma x_1 x_6 + \quad \Sigma x_2 x_6 + ¥ \Sigma x_3 x_6 + « \Sigma x_4 x_6 + ¨ \Sigma x_5 x_6 + \mu \Sigma x_6^2 \qquad (17)$$

We solve the seven equations by relating them to the data in Table 2 thus,

$$11.59847 = 10 \ _0 - 1.60463\sim + 8.756854 \quad + 6.701888¥ - 6.48107« - 8.303434 - .60705\mu \qquad (18)$$

$$1.64025 = 1.60463 \ _0 - 2.202966\sim + 1.61869 \quad + 1.19621¥ - 0.943024« - 2.1112134 + .09278\mu \qquad (19)$$

$$10.11892 = 8.756854 \ _0 - 1.61869\sim + 8.206625 \quad + 5.849747¥ - 5.68453« - 7.24008¨ - 0.14787\mu \qquad (20)$$

$$7.763548 = 6.701888 \ _0 - 1.19621\sim + 5.84974 \quad + 4.701102¥ - 4.40791« - 5.715934 - .47919\mu \qquad (21)$$

$$7.47297 = 6.48107 \ _0 - .943024\sim + 5.68453 \quad + 4.40791¥ - 4.525079« - 5.1920464 - .355108\mu \qquad (22)$$

$$9.73289 = 8.30343 \ _0 - 2.111213\sim + 7.24008 + 5.71593¥ - 5.192046« - 8.356794¨ - .629417\mu \qquad (23)$$

$$0.81961 = .60705 \ _0 + .09278\sim + .14787 \quad + .47919¥ + .355108« - .629417¨ - .51157\mu \qquad (24)$$

From the above equations we have the matrix:

$$M_0 = \begin{pmatrix} 10 & -1.60463 & 8.756854 & 6.701888 & -6.48107 & -8.303434 & -60705 & : 11.59847 \\ 1.60463 & -2.202966 & 1.61869 & 1.19621 & -0.943024 & -2.111213 & 0.99278 & : 1.64025 \\ 8.756854 & -1.61869 & 8.206625 & 5.849747 & -5.68453 & -7.24008 & -.14787 & : 10.11892 \\ 6.701888 & -1.19621 & 5.84974 & 4.701102 & -4.40791 & -5.71593 & .47919 & : 7.763548 \\ 6.48107 & -.943024 & 5.68453 & 4.40791 & -4.525079 & -5.192046 & -0.355108 & : 7.47297 \\ 8.30343 & -2.111213 & 7.24008 & 5.71593 & -5.192046 & -8.35679 & -.629417 & : 9.73289 \\ 0.60705 & .09278 & .14787 & .47919 & .355108 & -.629417 & -.51157 & : 0.81961 \end{pmatrix}$$

We will use MATLAB to solve the matrix M_0 to obtain the values of A and input exponents thus:

$_0 = 1.2831$

$= \log A$

$\therefore \quad A = 19.19111 \times 10^6$

$\sim = 0.1653; \quad = 0.0457$

$¥ = 0.0864;\ « = -0.3136$

$¨ = 0.0403;\ \mu = 0.3843$

5.0 Statistical Validation

With the aid of the bootstrap resampling technique and using the ten-year data of input and output presented in Table 1, thirty sets of input and output data were generated with the use of a suitably selected urn(A vessel capable of concealing its contents from the sampler in order to eliminate bias in the different resamples). This is a process of resampling with replacement. Thereafter, the input exponents (output elasticities) were computed. The results presented in Table3 are output elasticities of the two capital and two labour input components, machinery input and energy input. These are values obtained in the thirty different iterations in the bootstrapping analysis.

Table3: Output Elasticities obtained from Bootstrapping Analysis

Data Set	**A**	**β**	¥	«	¨	ω
1	1.1216	0.6046	1.1787	2.9129	1.3643	-5.7248
2	-0.0889	-0.1616	0.3645	-0.3629	0.0921	0.5872
3	0.905	0.8366	0.3612	1.4672	0.7868	-3.5744
4	0.8245	0.1556	-2.0992	0.5487	-0.3305	0.8271
5	3.339	0.274	-2.5273	1.9826	0.4984	-4.7686
6	-0.4279	0.4867	0.9042	-1.6514	-0.1105	1.4552
7	0.02	-1.6998	2.9014	0.3169	0.1325	1.1453
8	0.4522	0.2187	-0.7566	-0.3672	-0.1468	0.7957
9	2.9903	1.2619	-0.0025	2.0792	1.3564	-7.8114
10	-0.2907	-0.3479	0.5346	-0.2201	0.1451	0.5928
11	1.5294	1.3211	-0.4913	1.7297	0.6774	-4.0814
12	1.8189	1.4546	-0.6167	2.2776	0.8377	-5.07
13	0.4341	0.1884	-0.7102	-0.3345	-0.1253	0.7608
14	0.3986	0.2826	-0.5725	-0.4489	-0.2156	0.8756
15	-1.2236	1.3062	1.7025	-2.1198	-0.4521	1.9491
16	0.0694	-0.0608	-0.508	-0.1161	0.2448	0.1749
17	0.2218	1.8177	-1.8857	2.3911	-1.1945	-0.3272
18	0.6117	0.0598	-1.1146	0.5784	-0.1083	0.1892
19	1.8941	1.0524	1.6516	-1.9567	0.1418	-2.6724
20	-0.2316	0.2014	0.1746	-1.0633	-0.0987	1.4057
21	-1.4923	-0.6628	0.987	-0.0192	-0.1212	2.3385
22	1.7844	1.9553	0.8056	2.3036	1.361	-7.0419
23	0.8343	-0.3449	-2.6078	1.8807	0.2991	-0.577
24	-1.4929	-0.6628	0.987	-0.0192	2.3385	2.3385
25	0.1649	0.0456	0.0826	-0.3155	0.039	0.3907
26	1.2273	0.5297	0.3689	-0.3689	0.3231	-1.7354
27	1.5012	0.6205	0.6113	-0.9838	0.233	-2.0624
28	-1.4923	-0.6628	0.987	-0.0192	-0.1212	2.3365
29	-1.4923	-0.6628	0.987	-0.0192	-0.1212	2.3385
30	1.5294	0.5142	0.2877	-0.661	0.3998	-2.3163
Mean	0.514653	0.330713	0.066167	0.314057	0.27083	-0.90873

In order to validate the parameters, they were statistically tested to establish the confidence limit for 95% confidence interval Using the relationship:

$$UCL = \bar{x} + 1.96\frac{\pm}{\sqrt{n}} \tag{25}$$

and

$$LCL = \bar{x} - 1.96\frac{\pm}{\sqrt{n}} \tag{26}$$

where UCL is the Upper 95% Confidence Limit and LCL is the Lower 95% Confidence Limit,
$\bar{x}$ = Sample mean of each output elasticity of the Cobb-Douglas model using the thirty sets of data.

$\pm$ = population standard deviation which was estimated from the sample standard deviation; valid for n $\geq$30 [11].
The population standard deviation σ, computed from the relationship

$$\pm = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}} \tag{27}$$

where x =parameter value,
$\bar{x}$ = sample mean
n = sample size
Using Microsoft Excel package, we obtained means and standard deviations for each of the six output elasticities of the production function. Using these values in Equations (25) and (26), UCL and LCL values were computed. These values are presented in Table 4.

Table 4: Result of statistical Validation

	α	**β**	**γ**	**μ**	**φ**	**ω**
σ	1.236383	0.807686	1.24528	1.340056	0.665538	2.8945
$\bar{x}$	0.5147	0.3307	0.0662	0.3141	0.2708	-0.909
UCL	0.9571	0.6197	0.5118	0.7936	0.509	0.1271
LCL	0.0722	0.0417	-0.379	-0.165	0.0327	-1.945

6.0 Discussion of Results

The normal equations derived from the data set were solved using **Matlab software**, to obtain values of the input exponents as presented in Table3. As shown in Table 4, estimates of the mean values of the six input exponents were found to be within acceptable limits.The strength of this new approach is derived from the disaggregated input structure which is a positive departure from the traditional method of lumping inputs as two aggregate resources of capital and labour respectively.

7.0 Conclusion

A novel approach has been introduced in input/output modeling in the application of the Cobb-Douglas production function. The output elasticities obtained have been statistically validated and shown to be dependable; thus a veritable management tool has been introduced in organizational management.

8.0 References

[1] Klacek J., Miloslav, V. and Stefan, S. (2007) “KLE Translog Production Function and Total Factor Productivity” *Statistical*, 4/2007, pp.281-294.

[2] Arrow, K. J., Chenery B. H., Minhas B. S. and Solow R. M. (1961) “Capital-Labour Substitution and Economics Efficiency, “ *The Review of Economics and Statistics*, 43(3) pp.225-250.

[3] Berndt, E. R. and Khaled M. (1979) “Parametric Productivity Measurement and Choice among Flexible Functional Forms”, *Journal of Political Economy*, vol. 87, No 6, pp1220-1245.

[4] Border, K. C. (2004) “On the Cobb-Douglas Production Function”, *Publication of the Division of the Humanities and Social Sciences, Califonian Institute of Technology.* Available at http://www.hss.caltech.edu~k.cobb (Accessed 20th March 2010)

[5] Bao Hong, T. (2008) “Cobb-Douglas Production Function”, Available at. http://docentes.fe.unl.pt/~/amador/macro. (Accessed 6th Nov. 2011)

[6] Revanker N. S. (1971) “A Class of Substitution Production Functions”. E*conometrica*, vol.39, Issue 1, pp.61-71.

[7] Mishra, S. K. (2007) 'Estimation of Zellner - Revankar Production Function Revisited' *Economics Bulletin,* Vol. 3, No 14, pp 1 – 7.

[8] Graham, J. W. and Green, C. (1984) " Estimating the Parameters of a Household Production Function with Joint Products" *The Review of Economics and Statistics*, vol. 66, No 2, pp.277-282.

[9] Welfens, P.J.J (2005) "A Quasi-Cobb-Douglas Production Function with Sectoral Progress: Theory and Application to the New Economy". *Discussion Paper No.132, European Economy and International Economic Relations Conference.* Available at https://ideas.repec.org/p/bwu/eiiwdp/disbei132.html(Accessed 17th June 2012)

[10] Omoregie, M. J. (2014) "Comparative Analysis of Aggregate and Disaggregated Cobb-Douglas Production Functions', *Journal of the Nigerian Association of Mathematical Physics,* Vol.28, No. 2, pp

[11] Mendenhall, W. and Reinmuth, J. E. (1982) "Point Estimate of a Population Mean"; *Statistics for Management and Economics*, pp.257-261, Duxbury Press, Boston, Massachusetts.