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Valuation Techniques of the Three Categories of Premium Payment Options in Life Insurance Contract

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Abstract

We attempt to examine the three categories of premium payment options in life insurance contract. We will examine the value of these options in conjunction with participating life insurance contracts which include two standard options namely; an interest rate guarantee and a guaranteed annual surplus participation. In addition to these two standard options, life insurance contracts typically embed the right to stop premium payment during the term of the contract (paid up option),to resume payment later (resumption option), or to terminate the contract early (surrender option). Finally benefits adaptation techniques will be analyzed numerically.

1.0 Introduction

Life Insurance contracts often embed various type of implicit options. Due to an improper hedging of provided options, the British life insurer *equitable* had to stop taking new business. However it is not wise to consider a product as sufficiently priced, without analyzing specific options like elements embedded in the contract [1].

For valuation, we shall apply the standard principle of risk neutral evaluation and assume that the policy holder makes the optimal decision (timing the exercise) based on available information.

Premium payment options are traded in most life insurance contracts especially in the United State, and European Market. Premium payment are in three categories namely (a) paid – up – option (the right to stop premium payment annually until maturity of contracts), (b) resumption option (the right to resume payment after exercising paid – up – option) and (c) surrender option (the right to terminate the contract early). The benefits of these options when exercised on death, survival and surrender(if applicable) are adapted as decreased or increased depending on the option and the underlying contract policy [2].

We shall examine the value of these options in conjunction with life insurance contract which including two standard options; interest rate guarantee and a g surplus participation.

2.0 Basic Contract

Assuming we have a life insurance contract with periodic premium payment featuring the two standard options; an interest rate guarantee and annual surplus participation.

Let denote Basic contract with B; and let X be the age of the insured, we now introduce Mortality statistics for the insured and assumed that the financial risk and mortality risk are uncorrelated [3].

Premium payment defines as B_t - 1, t = 1, …., T, are paid annually at the beginning of the t^{th} policy year given that the insured remains alive until maturity T.

Normally $B_t \equiv B$ are constants, and mortality risk can be eliminated by taking sufficiently large numbers of contracts [3]. We derived death and survival probabilities from relevant mortality tables. Let t_p be the probability for an x – year- old policy holder surviving for the next t year and $t_q = (1 - t_p)$ be the probability that an x - year old will die within the next year. Guarantee considered in the basic contract are on benefits payable upon death and survival. We assumed that the basic contract are non surrendable and so there is no paid – up – option.

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3.0 Death Benefit

When there is death within the t^{th} year of the contract, the assigned policy holder's beneficiary is usually its heirs that received the death benefit. Let $\propto^{B}$ be the death benefit at the end of the year. For the purpose of this research, $\propto^{B}_{t} = \propto^{B}$ and is constant.

$\propto^{B}$ imply minimum benefit the insured will receive in the case of survival until maturity is calculated [4].

Given the annual premium B, the starting equation used in computing the death benefit only is

$$B\sum_{t=0}^{T-1} t_p \ (1+g)^{-t} = \propto^{B}\sum_{t=0}^{T-1} t_p \ q_{x+1}(1+g)^{t+1} + T_p \ (1+g)^{-T} \quad (1.1)$$

Where g is used as the annually compound interest rate for discounting future benefits and premiums. The LHS of (1.1) is the expected premium payments from the insured, while the RHS is the expected value of the payments to the insured. Also from (1.1)

$$\propto^{B} = \frac{B\sum_{t=0}^{T-1} t_p \ (1+g)^{-t}}{\sum_{t=0}^{T-1} t_p \ q_{x+1}(1+g)^{t+1} + T_p \ (1+g)^{-T}} \quad (1.2)$$

4.0 Survival Benefit:

The insurer payout the accumulated policy asset denoted by A^{B}_{T}, this include the guaranteed interest rate in the annual surplus of the life insure's investment portfolio. In this case, the annual premium payment B are split into a premium for life term insurance and a saving premium. Let B^{P}_{t-1} be the partial term life insurance, to cover the available policy assets, A^{B}_{t-1} which can be written as;

$$B^{P}_{t-1} = q_{x+t-1}\, m \quad (\propto^{B} - A^{B}_{t-1}, 0) \quad (1.3)$$

Where q_{x+t-1} is the death probability for an $x+t-1$ year old policy holder to die within the next year and the remaining saving premium.

$$B^{A}_{t-1} = B - B^{P}_{t-1} \quad (1.4)$$

The asset at the beginning of the t^{th} year, $A^{B}_{t-1}$and the premium payment B^{A}_{t-1} annually earn the greater of the guaranteed interest rate g of a fraction 6 of the annual surplus $\left(\frac{S_t}{S_{t-1}} - 1\right)$ of the insurer's investment portfolio. This follows a geometric Brownian motion given a complete market.

Suppose W^{f}_{t}, $t = 0, \ldots, T$ is a standard Brownian motion on a probability space (Ω, Φ, f) and Φ_t, t = 0, ….,T bethe filtration generated by the Brownian motion. In this case, we will define S_t as $dS_t = \mu S_t d + \delta S_t W^{f}_{t}$ with deterministic asset drift parameter μ and volatility δ, under the risk neutral unique equivalent martingale measure K, the drift changes to the risk free interest rate r, and the solution of the Stochastic Differential Equation (SDE) with K −Brownian motion W^{k} is given as

$$S_t = S_{t-1}e \quad \left(r - \frac{\delta^2}{2} + \delta(W^{k}_{t} - W^{k}_{t-1})\right) \quad (1.5)$$

With the initial condition S_0 [2]. The accumulated policy asset A^{B}_{T} are calculated as;

$$A^{B}_{t} = (A^{B}_{t-1} + t - 1p_x B^{A}_{t-1})\left(1 + \max\left(g, 6\left(\frac{S_t}{S_{t-1}} - 1\right)\right)\right)$$

$$= \left(A^{B}_{t-1} + t - 1p_x(B - q_{x+t-1}\max(\propto^{B} - A^{B}_{t-1}, 0))\right)$$

$$= X\left(1 + \max\left(g, 6\left(\frac{S_t}{S_{t-1}} - 1\right)\right)\right) \quad (1.6)$$

WithA^{B}_{0} = 0,

5.0 Contract payoff and Fairness Condition:

The accumulated payoff for basic contract denoted by P^{B}_{t} at maturity T is evaluated by the payment to the insured minus the premium payments, compounded with the risk free interest rate r. Mathematically it is given as;

$$P^{B}_{t} = \sum_{t=0}^{T-1} \propto^{B}_{t} p_x q_{x+t} e^{r(T-t-r)} + T p_x A^{B}_{T} - \sum_{t=0}^{T-1} B_t p_x e^{r(T-r)} \quad (1.7)$$

Equation (1.7) has three terms on the RHS, which represent the expected accumulated death benefit, the expected benefit payable at maturity in case of survival and the expected premium payment to the insurer, compounded to maturity T, respectively. If we denote the net present value of P^{B}_{t} at t=0, by$\Pi^{B}_{0}$and let E^{Q}_{t} be the conditional expected value with respect to the probability measure Q under the information available in t, we have that

$$\prod_0^{B} = E^{Q}_{t}(e^{-r} \ P^{B}_{t}) \quad (1.8)$$

If the value of the benefits under the risk neutral martingale measure is equal to the present value premium paid by the policy holder, then we have a fair contract [5].

$$\prod_0^{B} = 0 \quad (1.9)$$

If given all other parameters, it is possible to measure the annual surplus participation parameter λ to obtain fair contract satisfying (1.9) . This implies that the two options in basic contract are covered by the annual premium payment.

6.0 Paid – Up – Option

an option being a financial derivative, specifies contract between two parties for future transaction on an asset at a predetermine price called the strike price. When an option is not exercised by the expiration date, it becomes void and worthless [6].

The paid-up-option is the right to stop premium payment annually until maturity or expiration date of the contract, for instance if the expiration of a contract is 5years, rather than paying premium at the end of each year, the policy holder can decide not to pay any premium until 5years, at the end of the 5years,the policy holder paid up all the accrued premium. This option is an example of a Bermudan-style option.

Let us denote paid-up-option by P_U^u and let the contract setting be U. After exercising P_U^u at time $t = \tau, \tau = 1, \ldots, T-1$ denoted by $P_U^{u(\tau)}$, andgiven that the policy holder is still alive, the terminal benefit are adjusted according to the method described below.

(i) Death benefit;

Let $\propto^{u(\tau)}$ be the adjusted constant death benefit if P_U^u is exercise at time τ. We shall now calculate $\propto^{u(\tau)}$ by taking the accumulated policy asset A_τ^B present at the time the policy holder stop premium payment as single premium for a new contract, putting into consideration the current age of the insured, the adjusted benefit can be obtain as in (1.1) and (1.2).

Thus

$$\propto^{u(\tau)} = \frac{A_\tau^B(1+\propto_\tau^u)}{\sum_{t=\tau}^{T-1} t-\tau p_x + \tau q_x + t(1+g) - (t-\tau-1) + T-\tau\, p_x\, \tau\, (1+g)^{-(T-\tau)}} \quad (2.1)$$

With $\propto_\tau^u = \lambda$, model parameter assured to be 0 (default value). $\propto_\tau^u$ is also parameter that determine the flexibility of the insurer to adapt to the benefit after exercising the option [2].

If $\propto_\tau^u < 0$, it is a penalization parameter,

And if $\propto_\tau^u > 0$, it is an incentive parameter, however it is more favorable or profitable when this parameter is ≤ 0.

We also use $\propto_\tau^u$ in numerical analysis to asses the sensitivity of the option value on variation of the benefits.

(ii) Survival benefit;

The adjusted survival benefit can be estimated similarly to (1.6) and it is given by

$$A_t^{u(\tau)} = \left(A_{t-1}^{u(\tau)} - \tau - 1 p_x q_{x-\tau-1} \max\left(\propto^{u(\tau)} - A_{t-1}^{u(\tau)}, 0\right)\right)$$

$$= X\left(1 + \max\left(g, 6\left(\frac{S_t}{S_{t-1}} - 1\right)\right)\right) \quad (2.2)$$

With $A_\tau^{u(\tau)} = A_\tau^{u(\tau)}(1 + \propto_\tau^u)$

7.0 Payoff and paid-up-option value

The accumulated payoff in the case of the paid-up-option exercise in τ is given by

$$P_\tau^{u(\tau)} = \sum_{t=0}^{\tau-1} \propto^B + p_x\, q_{x+t} e^{r(\tau-t-1)} + \sum_{t=\tau}^{T-1} \propto^{u(\tau)} t p_x q_x + e^{r(\tau-t-1)} + \tau p_x A_\tau^{u(\tau)} - \sum_{t=0}^{\tau-1} B_t\, p_x e^{r(t-\tau)} \quad (2.3)$$

The first term in (2.3) is the original expected accumulated death benefit until the exercise date of the paid-up-option, the second term is the expected accumulated death benefit after exercising the paid-up-option, and the third term is the adjusted survival benefit at time T since the premium payments are stopped at time τ, while the last term contains expected premium until time τ , compounded to time T.

The value of the paid-up-option $P_U^{u(\tau)}$ exercise at time τ is obtained by the difference of the basic contract without the paid-up-option in (1.7) which can be written as;

$$\prod_0\left(P_U^{u(\tau)}\right) = E_0^Q\left(e^{-\tau}\left(P_T^{p(\tau)} - P_T^B\right)\right)$$

$$= E_0^Q\left(e^{-\tau}\ \tau\, p_x\ C_\tau P_U^{u(\tau)}\right) \quad (2.4)$$

Where

$$C_\tau P_U^{u(\tau)} = \sum_{t=\tau}^{\tau-1}\left(\propto^{p(\tau)} - \propto^B\right)t - \tau p_{x+\tau} q_{x+te^{-\tau(t-\tau+1)}} + \left(A_\tau^{p(\tau)} - A_\tau^B\right)_{t-\tau} p_{x+\tau e^{-\tau(t-\tau)}} + \sum_{t=\tau}^{T-1} B_\tau t p_{x+\tau e^{-\tau(t-\tau)}} \quad (2.5)$$

Now if given the fairness condition in (1.9), the net present value $\Pi_0^{p(\tau)}$ at time $\tau = 0$, will be equal to the exercise value of the option. This implies that

$$\Pi_0^{p(\tau)} = \Pi_0\left(P^{u(\tau)}\right) \quad (2.6)$$

8.0 Resumption Option

This is the right to resume payment after exercising the paid-up-option. In other word, there is no resumption option if the paid-up-option is not exercise. The resumption option can be exercised annually on dates $\varepsilon = \tau + 1, \dots, T - 1$. the resumption option arises when exercising the paid-up-option, hence the policy holder receives a combine option denoted by P^{u} . It will be necessary for us to examine the effect of combining the resumption option with paid-up-option.

9.0 Resumption and Paid-up-option.

Let us denote the resumption option by R. Then we can calculate the present value of the contract payoff assuming that the policy holder stops premium payment at time τ = 1, …, $T - 1$,and resume payment at time ε = τ+1, …,T-1 just as in the case of the P_0^{u}.

Given that the policy holder is still alive, let $P^{u}R^{(\tau,\varepsilon)}$ be the combine exercise of the two options at time τ and εrespectively, in this case the insurance benefits are adjusted as discuss below [4].

(i) Death benefit ;

Let$\propto^{R(\tau,\varepsilon)}$be the adjusted constant death benefit after the combined exercise $P^{u}R^{(\tau,\varepsilon)}$ from (2.1), we have

$$\propto^{R(\tau,\varepsilon)} = \frac{A_{\tau}^{E(t)}\left(1+\lambda_{\tau}^{R}\right)+B\sum_{t=\varepsilon}^{T-1} t\ P_{x+\varepsilon}\,(1+g)^{-(t-\varepsilon)}}{\sum_{t=\varepsilon}^{T-1} \tau-\varepsilon P_{x+\varepsilon}\, q_{x+\tau}(1+g)^{-(\tau-\varepsilon-1)}+T-\varepsilon P_{x+\varepsilon}(1+g)^{-(T-\varepsilon)}} \qquad (3.1)$$

With λ_{τ}^{R} a model parameter with default value 0, however if $\lambda_{\tau}^{R} \geq 0$, it will incentisize constant resumption. Also note that when τ = T -1, resumption is impossible, hence$P^{u}R^{(T-1)} = P^{u(T-1)}$.

(ii) Survival benefit:

Following from (2.2), we can evaluate the accumulated policy asset over time. $A_{t}^{R(\tau,\varepsilon)}$, t = $\varepsilon + 1$, - -- - T as follows;

$$A_{t}^{R(\tau,\varepsilon)} = A_{t-1}^{R(\tau,\varepsilon)} + t\text{-}1P_{x}\left(B - q_{x+t-1}\, m\left(\propto^{R(\tau,\varepsilon)} A_{t}^{R(\tau,\varepsilon)},\ B\right)\right)$$

$$= X\left(1+\max\left(g,\, \delta\left({}^{S_t}/_{S_{t-1}} - 1\right)\right)\right) \qquad (3.2)$$

With $A_{\varepsilon}^{R(\tau,\varepsilon)} = A_{\varepsilon}^{u(\tau)}(1 + \lambda_{\tau}^{R})$.

The adjusted survival benefit is given by $A_{T}^{R(\tau,\varepsilon)}$.

10.0 Contract payoff and option value

The accumulated payoff of the contract including the paid-up and resumption options at maturity is given by ;

$$P_{T}^{R(\tau,\varepsilon)} = \sum_{t=0}^{\tau-1} \propto^{B} t\ {}_{x}q_{x+t}\, e^{\tau(T-t-1)} + \sum_{t=\tau}^{\varepsilon-1} \propto^{u(\tau)} t\ {}_{x+t}\, e^{\tau(T-\tau-1)} + \sum_{t=\varepsilon}^{T-1} \propto^{R(\tau,\varepsilon)} t\ {}_{x}q_{x+t}\, e^{\tau(T-t-1)} + T\ {}_{x}A_{T}^{R(\tau,\varepsilon)} - \sum_{t=0}^{\tau-1} B_{t}\, p_{x}\, e^{\tau(T-t)} - \sum_{t=\varepsilon}^{T-1} B_{t}\, p_{x}\, e^{\tau(T-1)} \qquad (3.3)$$

The value of the exercise at time t =0 of the combined paid- up and resumption options $P^{u\ (\tau,\varepsilon)}$at time τ and ε respectively can be calculated as;

$$\prod\nolimits_0\left(P^{uR(\tau,\varepsilon)}\right) = E_0^{Q}\left(e^{-\tau}\tau p_{x}\, c_{\tau}\left(P^{u\ (\tau,\varepsilon)}\right)\right) \qquad (3.4)$$

Where $c_{\tau}\left(P^{u\ (\tau,\varepsilon)}\right) = \sum_{t=\tau}^{\varepsilon-1}\left(\propto^{u(\tau)} - \propto^{B}\right)t - \tau\ {}_{x} + \tau\ {}_{x+t}\, e^{-\tau(t-\tau+1)} + \sum_{t=\varepsilon}^{T-1}\left(\propto^{R(\tau,\varepsilon)} - \propto^{B}\right)t - \tau\ {}_{x} + \tau\ {}_{x}\, e^{-\tau(T-\tau)} + \sum_{t=\tau}^{\varepsilon-1} B_{t} - \tau\ {}_{x} + \tau\ {}_{x+\tau}\ {}^{-\tau(t-\tau)}$ (3.5)

11.0 Surrender Option

This is the right to terminate the contract early before the maturity date. In this section, we will only consider the basic contract B and include the Bermudan-style of surrender option in which the policy holder can exercise annually until maturity.

We shall denote the surrender option by S. S will be exercise at time t = ή**,** ή =1 , - - -,T-1 which we will also denote by $S^{(ή)}$. In the case of the surrender option, the death benefit is eliminated while the surrender benefit is paid out at time ή. This is because the policy holder is still alive, but the contract was only terminated.

4.1 Surrender benefit:

Let $A_{ή}^{S(ή)}$ be the surrender benefit paid out at time ή when exercising S at time t = ή. We assume that the surrender benefit correspond to the accumulated assets until time ή,

hence for ή = 1,- - - -, T-1, $A_{ή}^{S(ή)} = A_{ή}^{B}\left(1 + \lambda_{ή}^{S}\right)$ (4.1)

Where $A_{ή}^{B}$ is given by (1.6), and $\lambda_{ή}^{S}$ is the parameter initially set to zero as shown in (3.1).

12.0 Contract payoff and surrender option value:

The contract payoff at the time of surrender t = ή can be expressed as;

$$P_{ή}^{S(ή)} = \sum_{t=0}^{ή-1} \propto^{B} t \; {}_{x}q_{x+t} \; {}^{ή(ή-t-1)}{}_{+ή}p_{x} A_{ή}^{S(ή)} - \sum_{t=0}^{ή-1} B_t P_{xt} {}^{ή(ή-t)} \quad (4.2)$$

The exercise value at time t = 0 of the $S^{(ή)}$ is given by

$$\prod_0 \left(S^{(ή)}\right) = E_0^{Q}\left(e^{-ή(ή)}\left(P_0^{S(ή)} - e^{-ή(T-ή)}\right)P_T^{B}\right) = E_0^{Q}\left(e^{-ή(ή)} ή P_x c_{ή} S^{(ή)}\right) \quad (4.3)$$

Where $C_{ή} S^{(ή)} = \sum_{t=0}^{T-1} \propto_t^{B} f - ήp_x + ήq_{x+t}e^{-ή(t-ή-1)} + \left(A_{ή}^{S(ή)} - A_{ή}^{S(ή)}{}_{T-ή}p_x + ήe^{-ή(t-ή)}\right) + \sum_{t=L}^{T-1} B_t - ήp_x + ήe^{-(t-ή)}$ (4.4).

13.0 Numerical Result and Discussion

We shall analyze numerically the impact of the various level of the guaranteed interest rate, the annual surplus participation and the policies regarding the exercise and in particular, the timing of the three options described above. Let us considered a 30years old man policy holder who sign a contract with a term of 10years. The contract are taken out in year 2012, this imply that the policy holder was born in 1982. We shall also consider a contract term of 20 and 30years, and the case of a 50years old man with a contract of 10years.

Here, Monte-Carlo simulation is used with antithetic variables to obtain numerical solution. This method has to do with variance reduction by generating negatively correlated variables such that large output are accompanied and counter balance with small output [1].In this regard, let assume k different Monte-Carlo paths are simulated. We define $k = 1000{,}000$ paths. In the numerical analysis, the risk-free interest rate $r = 4\%$ and the annual premium payments are $B = 1200$ currency units. In the first step, the parameter g (guaranteed interest rate) and α (fraction of the annual return of the investment portfolio) are marked in order to get a fair contract condition see (1.9).

A standard bisection method is used to determine $'\alpha'$ for a given $'g'$. We will illustrate the basic contracts B and consider a reference example with the following: $T = 10$, $B = 1200$, $r = 4\%$, $g = 3\%$ a $= 0.20$. Both cases of an $x = 30$ a $= 50$, the policy holder taking out reference contract are considered for both cases, When $x = 30$ a $x = 50$, w ha $= 24.1\%$ and 29.3% respectively. The parameter α is determined such that the contract is fair for different contract length. The parameter α is stable under variations of the time to maturity. Note that if α = 0, there is no financial risk, and a fair contract should provide rate of return equal to riskless rate r.

14.0 Summary and Conclusion

We have developed a model framework which deals with the fair valuation of premium payment options within participating life insurance. In our work, option values are highly depended on the conversion techniques of the guaranteed benefits. The valuation of payment option is connected to the policy holder behavior which may depend on:

(i) Maximum of expected option payoff for given exercise time(s)
(ii) Optional admissible exercise strategy.
(iii) Maximum expected value of the option payoff over any exercise method.

The result of the second technique is regarded as the option value in this paper. The first and the third method assumes that the policy holder knows the future, hence can choose optional time of exercise which is not in practice [3]. Note also that the last method does not give an acceptable value for the option but a maximum value for it. One way of evaluating the option value is to consider a policy holder who follows an exercise strategy that maximizes his expected discounted payoff(under the risk-neutral measure) given available information at exercise date.

This method leads to an optional stopping problem that can be solved using Monte-Carlo simulation. In this case, the random variable, τ in the contract P which describes when to exercise the option is an admissible strategy as it only uses information known as the present time. This imply that 'τ' has to be adapted to the filtration of time.

In conclusion, this paper have developed a model framework which deals with the fair valuation of premium payment options within participating life insurance. In this framework, option values are highly depended on the conversion techniques of the guaranteed benefits upon exercise of the option. We also provide option values as well as the maximum value for the inherited risk potential for presented contracts. This paper focuses on premium payment and surrender options offered on top of a contract that already include a guaranteed interest rate and annual surplus participation.

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