

Two-Factor DP Model Based on Recruitment and Wastage

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Abstract

The problem of assessing future manpower requirement in terms of number, level of skills, and competence as well as formulating plans to meet those requirements has remained a challenge to researchers in human resources management. We proposed a DP model of backward recursive approach that is based on wastage and recruitment costs with an algorithm for obtaining planned periodic wastages and recruitments which can result in maximum net accruable revenue. We have also applied data obtained from an institution of higher learning to illustrate our proposed two-factor DP model based on wastage and recruitment costs. It is observed that based on present records of periodic staff salaries with known initial and final manpower requirements, the DP model can be used to determine future periodic optimal recruitment and wastage schedules for a given time horizon.

Keywords: recruitment, wastage, dynamic programming, manpower

1.0 Introduction

Bontis et al [1] view manpower as the human factor in an organization, the combined intelligence, skills and expertise that give the organization its distinctive character. Bulla and Scott [2] stated that manpower planning is a process of ensuring that the human resource requirement of an organization are identified and plans are made for satisfying those requirements.

The two major questions usually asked in manpower planning [3,4] are: (i) How many people are needed? and (ii) what sort of people are needed? It has been reported that the three factors responsible for staff transition or migration are recruitment, promotion and wastage [5-7].

2.0 Recruitment/Promotion

Recruitment is a process of absorbing an employee into a manpower system of an organization. There are two sources of manpower supply namely; external and internal supply [8]. External supply has to do with recruitment of staff from outside the organization while internal manpower supply sources include transfer and redeployment of employees within the organization. Promotion is a process whereby a staff in an organization is moved from a lower grade to a higher one [9].

Wastage: This refers to staff who leave an organization for various reasons such as resignation, retirement, retrenchment, dismissal, death etc. [6,3,9].

Dynamic programming (DP) is a mathematical technique in which a given problem is decomposed into a number of sub-problems called stages whereby lower dimensional optimization takes place[1].

We first state the assumptions and notations as follows:

Assumptions

The following are the assumptions of the DP model in LP form for manpower planning based on recruitment and wastage.

- Recruitment and wastage at a particular grade are considered
- Periodic recruitment (c'_j) and wastage (c_j) costs are known and fixed.
- Number of staff of the organization at initial and end of time-horizon interval are known.
- Both overstaffing and understaffing are allowed.

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Notations

$x_j =$ number of staff that are on wastage in period j .

$y_j =$ number of staff that are recruited in period j .

$c_j =$ average accrued revenue to the organization from each wastage staff in period j by virtue of their exit from the system.

$c'_j =$ average salary per recruited staff in period j .

$h =$ initial number of staff on ground in the organization at the beginning of the time horizon.

$H =$ total number of staff at the end of the time horizon under consideration.

3.0 Model Formulation

Let $y_j(t+u)$ be the number of staff recruited at time $(t+u)$ in period j where u is the very small time difference between recruitment and assumption of duty so that the recruited staff arrive at time $(t+u)$ for work. Let $x_j(t+u)$ and $c_j(t+u)$ be the number of staff on wastage and the average accrued revenue to the organization from each wastage staff in period j by virtue of their exit from the organization. Let $c'_j(t+u)$ be the average salary per recruited staff at time $(t+u)$ in period j when the recruitment was done. As $u \rightarrow 0$, the above notations become $x_j(t)$, $y_j(t)$, $c_j(t)$ and $c'_j(t)$ or simply x_j , y_j , c_j and c'_j . Given h , H , $c_j(t)$ and $c'_j(t)$ of a manpower planning problem, it is required to determine the optimal quantities x_j and y_j so that the accruable net revenue is a maximum. As we are dealing here with a dynamic situation, we divide the time span of interest into time intervals, which we shall assume to be sufficiently small so that we can consider $x_j(t)$, $y_j(t)$, $c_j(t)$ and $c'_j(t)$ to be constant during the time intervals but discontinuous from one time interval to the next.

The problem of the manpower planning is to maximize the periodic additional revenue accruable to the organization from the wastage staff wage bill less the periodic salary of recruited staff i.e. $\sum_{j=1}^n (c_j x_j - c'_j y_j)$.

The objective function can be written as:

$$\text{Maximize } z = \sum_{j=1}^n (c_j x_j - c'_j y_j) \quad (1)$$

There are two sets of staffing constraints and two sets of nonnegativity constraints in this manpower planning problem.

(i) The overstaffing constraints:

The constraints of overstaffing state that the total number of overstaffing staff of the first i periods should not exceed the available vacancies $(H - h)$ in the establishment, i.e.

$$\sum_{j=1}^i (y_j - x_j) = - \sum_{j=1}^i x_j + \sum_{j=1}^i y_j \leq H - h, \quad i = 1(1)n \quad (2)$$

Where $(y_j - x_j) > 0$ is the number of staff by which the organization is overstaffed in period j

The LHS of equation (2) can also be called the net increase in manpower in the first i periods.

(ii) The understaffing constraints:

The constraints of understaffing represent the number of staff by which the organization is understaffed for the first $(i-1)$ periods plus wastages at period i and this should not exceed h the number of staff originally in the organization. If it does, it means the organization has only material resources which is not the case in practical situation as existence of an organization is based on the contribution of human and material resources. Mathematically this is expressed as:

$$\sum_{j=1}^{i-1} (x_j - y_j) + x_i = \sum_{j=1}^i x_j - \sum_{j=1}^{i-1} y_j \leq h, \quad i = 1(1)n \quad (3)$$

Where $(x_j - y_j) > 0$ is the number of staff by which the organization is understaffed in period j

The L.H.S of equations (3) can also be called the net increase in manpower subtracted from wastage staff in the first $(i-1)$ periods plus the wastage manpower in period i .

Note that the second summation in equation (3) does not exist for $i=1$ (iii) Nonnegativity constraints:

The nonnegativity constraints are

$$x_j, y_j \geq 0, \quad j = 1(1)n \quad (4)$$

Equation (1) stated above constitutes the total manpower planning cost from all the n periods while equations (1)-(4) constitute a DP problem which is stated thus:

Primal DP Problem

$$\left. \begin{array}{l} \text{Maximize } z = \sum_{j=1}^n (c_j x_j - c'_j y_j) \\ \text{s.t.} \\ -\sum_{j=1}^i x_j + \sum_{j=1}^i y_j \leq H - h, \quad i = 1(1)n \\ \text{and } \sum_{j=1}^i x_j - \sum_{j=1}^{i-1} y_j \leq h, \quad i = 1(1)n \\ x_j, y_j \geq 0, \quad j = 1(1)n \end{array} \right\} \quad (5)$$

The system (5) is the DP model of the manpower planning problem which makes use of both recruitment and wastage factors. The DP model in system (5) has $2n$ linear constraints, $2n$ nonnegativity constraints in $2n$ variables.

Let d_1, d_2, \dots, d_n be the first n dual variables for the first n constraints in system (5) and e_1, e_2, \dots, e_n be the last n dual variables for dual DP model of the manpower planning problem:

Dual DP Problem

$$\text{Minimize } w = (H - h) \sum_{i=1}^n d_i + h \sum_{i=1}^n e_i \quad (6)$$

s.t.

$$-\sum_{i=k}^n d_i + \sum_{i=k}^n e_i \geq c_k, \quad k = 1(1)n \quad (7)$$

$$\sum_{i=k}^n d_i - \sum_{i=k+1}^n e_i \geq -c'_k, \quad k = 1(1)n \quad (8)$$

$$d_i, e_i \geq 0, \quad i = 1(1)n \quad (9)$$

It is understood that the second summation in equation (8) does not exist if $k=n$. We define new variables D_k and E_k as follows:

$$D_k = \sum_{i=k}^n d_i, \quad k = 1(1)n \quad (10)$$

$$E_k = \sum_{i=k}^n e_i, \quad k = 1(1)n \quad (11)$$

Since by the dual DP problem, d_i and e_i are nonnegative, D_k and E_k must be nonnegative. However, non negativity of D_k and E_k does not imply that $d_i \geq 0$ and $e_i \geq 0, \forall i$. In view of the definition of D_k and E_k , we see that non negativity of d_i and e_i will be ensured if we augment the dual LP problem, expressed in terms of D_k and E_k by the constraints:

$$D_k \geq D_{k+1}, \quad k = 1(1) n - 1 \quad (12)$$

$$E_k \geq E_{k+1}, \quad k = 1(1) n - 1 \quad (13)$$

Note that the constraints in equations (12) and (13) do not exist when $k=n$ because $D_{n+1} = E_{n+1} = 0$. Hence, we have $2(n-1)$ additional constraints in equations (12) and (13)

Substituting for $\sum_{i=1}^n d_i$ and $\sum_{i=1}^n e_i$ in the dual DP problem in equations (6)-(9) and incorporating constraints (12) and (13),

we obtain the dual in system (14).

$$\left. \begin{aligned} \text{Min } w &= (H - h)D_1 + hE_1 \\ \text{s.t.} \\ D_k &\geq E_{k+1} - c'_k, k = 1(1)n \\ D_k &\geq D_{k+1}, k = 1(1)n - 1 \\ D_k &\geq 0, k = 1(1)n \\ E_k &\geq D_k + c_k, k = 1(1)n \\ E_k &\geq E_{k+1}, k = 1(1)n - 1 \\ E_k &\geq 0, k = 1(1)n \end{aligned} \right\} \tag{14}$$

This is the dual DP problem starting with period 1 while D_1 and E_1 are the smallest values in their solution set.

The dual DP problem in system (14) has n sub problems each involving the pairs $(D_k, E_k), k = 1(1)n$. In particular the sub problem involving D_1 and E_1 is:

$$\left. \begin{aligned} \text{Min } w &= (H - h)D_1 + hE_1 \\ \text{s.t.} \\ D_1 &\geq D_2 - c'_1 \\ D_1 &\geq D_2 \\ D_1 &\geq 0 \\ E_1 &\geq D_1 + c_1 \\ E_1 &\geq E_2 \\ E_1 &\geq 0 \end{aligned} \right\} \tag{15}$$

The constraints in system (15) imply that the subproblem in (15) cannot be solved except we know D_2 and E_2 . This suggests that the procedure should be by backward recursive approach to first obtain D_2 and E_2 . Continuing this way, we certainly have to solve for n suboptimal solutions for the pairs $(D_k, E_k), k = 1(1)n$. We start from the last (nth) pair (D_n, E_n) with subproblem given as:

$$\left. \begin{aligned} \text{Min } w &= (H - h)D_n + hE_n \\ \text{s.t.} \\ D_n &\geq -c'_n \\ D_n &\geq 0 \\ E_n &\geq D_n + c_n \\ E_n &\geq 0 \end{aligned} \right\} \tag{16}$$

We now proceed to use the backward recursive approach to determine (D_1, E_1) .

Backward Recursive Approach for the Determination of D_1 and E_1

The backward recursive approach is used to determine the n optimal pairs $(D_n, E_n), (D_{n-1}, E_{n-1}), \dots, (D_1, E_1)$ as suboptimal solutions to the n subproblems. We start from the last subproblem in system (16)

nth Sub-problem

The constraints are:

$$\left. \begin{aligned} D_n &\geq -c'_n \\ D_n &\geq 0 \end{aligned} \right\} \Rightarrow \text{solution set is } D_n \geq 0, \text{ minimizing } w, \text{ means } D_n \text{ should be the smallest value in the solution}$$

set.

$$\text{i.e. } D_n = \max(-c'_n, 0) \tag{17}$$

$$\begin{aligned}
 D_n &= \max(-c'_n, 0) = 0 \\
 \left. \begin{aligned} E_n &\geq D_n + c_n \\ E_n &\geq 0 \end{aligned} \right\} & \text{solution set is } E_n \geq (D_n + c_n) \text{ and } E_n = \max(D_n + c_n, 0) \\
 E_n &= \max(D_n + c_n, 0) = D_n + c_n = c_n
 \end{aligned} \tag{18}$$

kth Sub-problem ($k = (n - 1), (n - 2), \dots, 3, 2, 1$)

This is the general case and the subproblem is stated as follows:

$$\begin{aligned}
 \text{Min } w &= (H - h)D_k + hE_k \\
 \text{s.t.} & \\
 \left. \begin{aligned} D_k &\geq E_{k+1} - c'_k \\ D_k &\geq D_{k+1} \\ D_k &\geq 0 \\ E_k &\geq D_k + c_k \\ E_k &\geq E_{k+1} \\ E_k &\geq 0 \end{aligned} \right\} & \tag{19}
 \end{aligned}$$

The optimal values of D_k and E_k are obtained as follows:

$$\begin{aligned}
 \left. \begin{aligned} D_k &\geq E_{k+1} - c'_k \\ D_k &\geq D_{k+1} \\ D_k &\geq 0 \end{aligned} \right\} & \text{solution set is } D_k \geq \max(E_{k+1} - c'_k, D_{k+1}, 0) \\
 \text{i.e. } D_k &= \max(E_{k+1} - c'_k, D_{k+1}, 0), \quad k = (n - 1), (n - 2), \dots, 3, 2, 1 \\
 \left. \begin{aligned} E_k &\geq D_k + c_k \\ E_k &\geq E_{k+1} \\ E_k &\geq 0 \end{aligned} \right\} & \text{solution set is } E_k \geq \max(D_k + c_k, E_{k+1}, 0) \\
 \text{i.e. } E_k &= \max(D_k + c_k, E_{k+1}, 0), \quad k = (n - 1), (n - 2), \dots, 3, 2, 1 \\
 \left. \begin{aligned} D_k &= \max(E_{k+1} - c'_k, D_{k+1}, 0), \\ E_k &= \max(D_k + c_k, E_{k+1}, 0) \end{aligned} \right\} & k = (n - 1), (n - 2), \dots, 3, 2, 1 \tag{20}
 \end{aligned}$$

In each subproblem the D_k are obtained before the E_k . The last of these are D_1 and E_1 which are partial sums of c_k and c'_k . D_1 and E_1 are then substituted into equation (6) to yield the minimum value of the objective function of the dual DP problem which by Duality Theorem for symmetric duals is equal to the maximum value of the objective function of the primal DP problem for manpower planning.

4.0 Numerical Illustration

The data in Table 1 shows the average monthly salary (c_j) on wastage and the average monthly salary (c'_j) of recruited junior staff for the year 2001 to 2012 in Delta State College of Physical Education Mosogar which had 162 junior staff in 2012. Based on the present salary trend, determine the optimal annual number of staff on wastage and recruitment that will maximize total accruable revenue to the institution in the next 12 years (i.e by the year 2024) when the junior staff strength is planned to be 393.

Table 1: Average monthly salary for junior staff on wastage and recruitment

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
c_j	33286	32045	35770	35918	36637	37552	38437	39126	33065	32281	38084	40124
c'_j	30148	32281	33665	34305	37545	34305	37894	36157	32981	30467	37688	36645

Solution

Initial number of staff in 2012 is $h = 162$, number of staff by the year 2024 i.e $H = 393$.

We have 12 periods in this example, $n = 12$ and we proceed to evaluate the D_k and E_k ($k = 12, 11, 10, \dots, 2, 1$) using system (20)

$$\left. \begin{aligned} D_k &= \max(E_{k+1} - c'_k, D_{k+1}, 0) \\ E_k &= \max(D_k + c_k, E_{k+1}, 0) \end{aligned} \right\}, (k = 12, 11, 10, \dots, 2, 1) \quad (21)$$

$$D_{12} = \max(-c'_{12}, 0) = 0$$

$$E_{12} = \max(D_{12} + c_{12}, 0) = D_{12} + c_{12} = c_{12}$$

$$D_{11} = \max(E_{12} - c'_{11}, D_{12}, 0) = E_{12} - c'_{11} = c_{12} - c'_{11}$$

$$E_{11} = \max(D_{11} + c_{11}, E_{12}, 0) = D_{11} + c_{11} = c_{12} - c'_{11} + c_{11}$$

$$D_{10} = \max(E_{11} - c'_{10}, D_{11}, 0) = E_{11} - c'_{10} = c_{12} - c'_{11} + c_{11} - c'_{10}$$

$$E_{10} = \max(D_{10} + c_{10}, E_{11}, 0) = D_{10} + c_{10} = c_{12} - c'_{11} + c_{11} - c'_{10} + c_{10}$$

$$D_9 = \max(E_{10} - c'_9, D_{10}, 0) = D_{10} = c_{12} - c'_{11} + c_{11} - c'_{10}$$

$$E_9 = \max(D_9 + c_9, E_{10}, 0) = D_9 + c_9 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_9$$

$$D_8 = \max(E_9 - c'_8, D_9, 0) = D_9 = c_{12} - c'_{11} + c_{11} - c'_{10}$$

$$E_8 = \max(D_8 + c_8, E_9, 0) = D_8 + c_8 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8$$

$$D_7 = \max(E_8 - c'_7, D_8, 0) = E_8 - c'_7 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7$$

$$E_7 = \max(D_7 + c_7, E_8, 0) = D_7 + c_7 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7$$

$$D_6 = \max(E_7 - c'_6, D_7, 0) = E_7 - c'_6 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6$$

$$E_6 = \max(D_6 + c_6, E_7, 0) = D_6 + c_6 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6$$

$$D_5 = \max(E_6 - c'_5, D_6, 0) = E_6 - c'_5 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6 - c'_5$$

$$E_5 = \max(D_5 + c_5, E_6, 0) = E_6 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6$$

$$D_4 = \max(E_5 - c'_4, D_5, 0) = E_5 - c'_4 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6 - c'_4$$

$$E_4 = \max(D_4 + c_4, E_5, 0) = D_4 + c_4 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6 - c'_4 + c_4$$

$$D_3 = \max(E_4 - c'_3, D_4, 0) = E_4 - c'_3 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6 - c'_4 + c_4 - c'_3$$

$$E_3 = \max(D_3 + c_3, E_4, 0) = D_3 + c_3 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6 - c'_4 + c_4 - c'_3 + c_3$$

$$D_2 = \max(E_3 - c'_2, D_3, 0) = E_3 - c'_2 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6 - c'_4 + c_4 - c'_3 + c_3 - c'_2$$

$$E_2 = \max(D_2 + c_2, E_3, 0) = E_3 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6 - c'_4 + c_4 - c'_3 + c_3$$

$$D_1 = \max(E_2 - c'_1, D_2, 0) = E_2 - c'_1 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6 - c'_4 + c_4 - c'_3 + c_3 - c'_1$$

$$E_1 = \max(D_1 + c_1, E_2, 0) = D_1 + c_1 = c_{12} - c'_{11} + c_{11} - c'_{10} + c_8 - c'_7 + c_7 - c'_6 + c_6 - c'_4 + c_4 - c'_3 + c_3 - c'_1 + c_1$$

$$D_1 = (c_3 + c_4 + c_6 + c_7 + c_8 + c_{11} + c_{12}) - (c'_1 + c'_3 + c'_4 + c'_6 + c'_7 + c'_{10} + c'_{11})$$

$$= 265011 - 238472$$

$$= \mathbf{26,539}$$

$$E_1 = (c_1 + c_3 + c_4 + c_6 + c_7 + c_8 + c_{11} + c_{12}) - (c'_1 + c'_3 + c'_4 + c'_6 + c'_7 + c'_{10} + c'_{11})$$

$$= 298297 - 238472$$

$$= \mathbf{59825}$$

The dual objective function value is given as:

$$(H - h)D_1 + hE_1 = 231(26539) + 162(59825)$$

$$= 6130509 + 9691650$$

= ₦15,822,159 which is the recruitment/wastage policy cost of the manpower planning problem obtained from the DP model.

To obtain the solution to the primal DP problem we proceed as follows:

$$(H - h)D_1 + hE_1 = \sum_{j=1}^{12} (c_j x_j - c'_j y_j) \quad (21)$$

$$\begin{aligned} \text{LHS} &= 231(c_3 + c_4 + c_6 + c_7 + c_8 + c_{11} + c_{12}) - 231(c'_1 + c'_3 + c'_4 + c'_6 + c'_7 + c'_{10} + c'_{11}) \\ &+ 162(c_1 + c_3 + c_4 + c_6 + c_7 + c_8 + c_{11} + c_{12}) - 162(c'_1 + c'_3 + c'_4 + c'_6 + c'_7 + c'_{10} + c'_{11}) \\ &= 162c_1 + 0c_2 + 393c_3 + 393c_4 + 0c_5 + 393c_6 + 393c_7 + 393c_8 + 0c_9 + 0c_{10} + 393c_{11} + 393c_{12} \\ &- 393c'_1 - 0c'_2 - 393c'_3 - 393c'_4 - 0c'_5 - 393c'_6 - 393c'_7 - 0c'_8 - 0c'_9 - 393c'_{10} - 393c'_{11} - 0c'_{12} \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= x_1 c_1 + x_2 c_2 + x_3 c_3 + x_4 c_4 + x_5 c_5 + x_6 c_6 + x_7 c_7 + x_8 c_8 + x_9 c_9 + x_{10} c_{10} + x_{11} c_{11} + x_{12} c_{12} \\ &- y_1 c'_1 - y_2 c'_2 - y_3 c'_3 - y_4 c'_4 - y_5 c'_5 - y_6 c'_6 - y_7 c'_7 - y_8 c'_8 - y_9 c'_9 - y_{10} c'_{10} - y_{11} c'_{11} - y_{12} c'_{12} \end{aligned}$$

$$\Rightarrow x_1 = 162, x_2 = 0, x_3 = 393, x_4 = 393, x_5 = 0, x_6 = 393, x_7 = 393, x_8 = 393,$$

$$x_9 = 0, x_{10} = 0, x_{11} = 393, x_{12} = 393,$$

$$y_1 (i.e. x_{13}) = 393, y_2 (i.e. x_{14}) = 0, y_3 (i.e. x_{15}) = 393, y_4 (i.e. x_{16}) = 393, y_5 (i.e. x_{17}) = 0,$$

$$y_6 (i.e. x_{18}) = 393, y_7 (i.e. x_{19}) = 393, y_8 (i.e. x_{20}) = 0, y_9 (i.e. x_{21}) = 0, y_{10} (i.e. x_{22}) = 393,$$

$$y_{11} (i.e. x_{23}) = 393, y_{12} (i.e. x_{24}) = 0$$

The primal objective function value is:

$$162(33286) + 393(35770 + 35918 + 37552 + 38437 + 39126 + 38084 + 40124)$$

$$- 393(30148 + 33665 + 34305 + 34305 + 37894 + 30467 + 37688)$$

$$+ 5392332 + 393(265011) - 393(238472)$$

$$= ₦15,822,159$$

The dual objective function value (*i.e.* $(H - h)D_1 + hE_1$) and that of the primal (*i.e.* $\sum_{j=1}^{12} (c_j x_j - c'_j y_j)$) are equal in this solution. This is in agreement with the Duality Theorem for symmetric duals.

5.0 Discussion of Results

The optimal solution to the junior staff problems from the proposed two-factor DP model algorithm reveals that there should be no staff wastage in periods 2,5,9, and 10 and no recruitment in periods 2,5,8,9 and 12 if the total accruable revenue from human resources to the institution is to be maximized. In the manpower planning problem for the junior staff, it is observed that wastage (x_j) and recruitment (y_j) in many periods are equal to the expected capacity (H) of the institution.

6.0 Conclusion

In this paper, we have developed a backward recursive DP model based on recruitment and wastage factors. The model algorithm has been applied to solve practical manpower problems using data from Delta State college of Physical Education, Mosogar. It is observed that based on present record of periodic staff salaries, initial and final manpower needs, we can from our two-factor DP model determine periodic optimal recruitment and wastage schedules for a given time horizon. This encourages business organization to plan ahead and this is an extension of the preceding paper.

7.0 References

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