

DP Model in LP Form for Manpower Recruitment

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Abstract

Dynamic programming (DP) models which are enumeration techniques are currently being applied to manpower planning. However, such dynamic programming models have not been incorporating cost factors in their model formulation. This paper is aimed at developing a dynamic programming model in linear programming (LP) form based on only recruitment cost factor and can be solved by using backward recursive method of dynamic programming subject to the condition that the overstaffing cost at any period must be lower than that of its preceding period. We observe that when this condition is not satisfied it is advisable to solve the primal DP problem using a computer program for large size problems. The model has been numerical illustrated using program full simplex in Pascal.

Keywords: Recruitment, Manpower Wastage, promotion

1.0 Introduction

Manpower are people in their various roles as contributors to the production of goods and services in an organization [1]. Bontis et al [2] views manpower as the human factor in an organization, the combined intelligence, skills and expertise that give the organization its distinctive character.

The two major questions usually asked in manpower planning [3] are: (i) How many people are needed? and (ii) What sort of people are needed? The principal objective of manpower planning is to model the migration of staff from one grade to another in discrete time which could be as a result of recruitment, promotion or retirement [4]. Gregoriades [5] reported that the three factors responsible for staff transition or migration is recruitment, promotion and wastage.

2.0 Recruitment/Promotion

Recruitment is a process of absorbing an employee into a manpower system of an organization. Promotion is a process whereby a staff in an organization is moved from a lower grade to a higher one, [6]. **Wastage** are staff who leave an organization for various reasons such as resignation, retirement dismissal, death etc, [3, 6]

Dynamic programming (DP) is a mathematical technique in which a given problem is decomposed into a number of sub-problems called stages whereby lower dimensional optimization takes place [7,8]. The objective in such problems is to find a combination of decisions that will optimize some appropriate measure of effectiveness. The problem addressed using dynamic programming approach require series of interrelated decisions.

3.0 The DP Model Formulation Based on Recruitment Cost

We make the following assumptions while formulating the manpower planning problem to determine optimal recruitment policies

- (a) The requirement size is known and fixed.
- (b) Recruitment at a particular grade is considered.
- (c) Recruitment and overstaffing costs are known and fixed
- (d) Understaffing is not allowed.

Notations

R_j = requirement in period j

k_j = fixed recruitment cost in period j

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- $l_j =$ cost of overstaffing per recruited staff per period
- $p_j =$ number of people recruited in period j
- $y_j =$ number of people recruited in an earlier period for the requirement of period j
- $v_j =$ recruitment cost per recruited employee in period j .
- $n =$ Number of periods

The model makes use of Table 1 which has been earlier used in [9].

Table 1: Requirements and Recruitment costs

Period	No. of staff required (R_j)	Fixed recruitment cost k_j (₦)	Unit overstaffing cost l_j (₦)
1	R_1	k_1	l_1
2	R_2	k_2	l_2
3	R_3	k_3	l_3
\vdots	\vdots	\vdots	\vdots
n	R_n	k_n	l_n

Rao [9] proposed a DP model in LP form which is stated as follows:

$$\left. \begin{aligned}
 & \text{Minimize } z = \sum_{j=1}^n [k_j \cup p_j + v_j p_j + l_j y_j] \quad (\text{total cost of recruitment}) \\
 & \text{s.t.} \\
 & \sum_{j=1}^i y_j = \sum_{j=1}^i R_j, \quad i = 1, 2, \dots, n \quad (\text{constraint s of the DP model}) \\
 & y_j \geq 0, \quad j = 1(1)n \quad (\text{nonnegativity constraint s})
 \end{aligned} \right\} \tag{1}$$

Where $\cup p_j = \begin{cases} 0 & \text{if } p_j = 0 \\ 1 & \text{if } p_j > 0 \end{cases}$

$l_j y_j$ is the overstaffing cost

We take $y_0 = y_n = 0$ without loss of generality.

The types of linear constraints in systems (1) are typical of dynamic systems hence they are DP models. However, some of the variables ($p_j, j = 1(1)n$) in the objective functions in system (1) are not in the constraints hence the model cannot be solved by any known linear programming method. This is possibly why the model in system (1) was not solved in [9]. This paper is aimed at formulating a DP model in LP form which is solvable by either back-ward recursive method of DP or LP method.

Model Formulation

Let us consider recruitment in a cadre. The mathematical formulation of this model begins from the objective function in Rao’s model now restated as follows:

$$\text{Minimize } z = \sum_{j=1}^n [k_j \cup p_j + v_j p_j + l_j y_j] \tag{2}$$

The variable cost of recruitment is constant and we have that $\sum_{j=1}^n v_j p_j$ is a constant in equation (2). Whereas $\sum_{j=1}^n l_j y_j$ is not a constant because the point of its application depends on the earlier period at which recruitment took place and not period j . Furthermore $p_j y_j = 0, \forall j$.

Hence, the objective function in equation (9) becomes:

$$\text{Minimize } z = \sum_{j=1}^n [k_j + l_j y_j]$$

i.e. *Minimize* $z = K + \sum_{j=1}^n l_j y_j$ (3)

where $K = \sum_{j=1}^n k_j$ (4)

In equation (3), K is a fixed known cost for all periods which we shall show how it can be numerically determined in section 4.

Hence, equation (3) becomes:

Minimize $z = \sum_{j=1}^n l_j y_j$

Besides, we have the constraints:

$$\sum_{j=1}^i y_j \geq \sum_{j=1}^i R_j, \quad i = 1, 2, \dots, n$$

The choice of the inequality ' \geq ' is based on the assumption that overstaffing is allowed.

Thus the proposed DP model in LP form for determining the periodic recruitments (y_j) when R_j ($j = 1(1)n$) are known is stated as follows:

Minimize $z = \sum_{j=1}^n l_j y_j$ (5)

$$\sum_{j=1}^i y_j \geq \sum_{j=1}^i R_j, \quad i = 1, 2, \dots, n$$
 (6)

$$y_j \geq 0, \quad j = 1, 2, \dots, n$$
 (7)

Equation (5) is the objective function which is the total recruitment cost while equation (6) is the set of linear constraints with equation (7) as set of nonnegativity constraints. It can be seen in the proposed DP model in equations (5)-(7) that all the objective function variables are in the constraints of the problem, hence the problem is solvable, provided a feasible region exists.

The DP model in equations (5)–(7) is further transformed to the system (8) as primal DP model which also makes use of

Table 1: Primal DP Model

<i>Min</i> $z = l_1 y_1 + l_2 y_2 + l_3 y_3 + \dots + l_n y_n$	}	(8)
<i>s.t.</i>		
$y_1 \geq R_1$		
$y_1 + y_2 \geq R_1 + R_2$		
$y_1 + y_2 + y_3 \geq R_1 + R_2 + R_3$		
$y_1 + y_2 + y_3 + y_4 \geq R_1 + R_2 + R_3 + R_4$		
.....		
$y_1 + y_2 + y_3 + \dots + y_n \geq R_1 + R_2 + R_3 + \dots + R_n$		
$y_1, y_2, \dots, y_n \geq 0$		

In quest for a DP model solution, we decide to formulate the dual of the DP model in system (8). The corresponding dual of the DP model in (8) is given in system (9).

Dual DP Model

$$\left. \begin{aligned}
 \text{Max } w &= R_1 \sum_{i=1}^n d_i + R_2 \sum_{i=2}^n d_i + R_3 \sum_{i=3}^n d_i + \dots + R_n \sum_{i=n}^n d_i \\
 \text{s.t.} \\
 d_1 + d_2 + d_3 + \dots + d_n &\leq l_1 \\
 d_2 + d_3 + \dots + d_n &\leq l_2 \\
 d_3 + \dots + d_n &\leq l_3 \\
 &\dots\dots\dots \\
 d_n &\leq l_n \\
 d_i &\geq 0, \quad i = 1(1)n
 \end{aligned} \right\} \tag{9}$$

Where the d_i 's are the dual variables.

The system (9) is transformed to system (10) as follows:

$$\left. \begin{aligned}
 \text{Max } w &= R_1 \sum_{i=1}^n d_i + R_2 \sum_{i=2}^n d_i + R_3 \sum_{i=3}^n d_i + \dots + R_n \sum_{i=n}^n d_i \\
 \text{s.t.} \\
 \sum_{i=k}^n d_i &\leq l_k, \quad k = 1(1)n \\
 d_i &\geq 0, \quad i = 1(1)n
 \end{aligned} \right\} \tag{10}$$

By deleting the first constraint/period (i.e. starting from period 2), we obtain a primal sub problem of (8) with a corresponding dual sub problem also obtained by deleting the first column in system (9). Continuing this way, we have n subproblems for n-periods manpower horizon. By backward recursive approach of DP we start to determine by enumeration the dual suboptimal solution of the last nth period sub problem and continue to the suboptimal solution of the first period which is the dual DP problem of the original primal DP problem.

In order to solve any of the dual sub problems starting from the nth sub problem, we re-denote the dual variables as follows:

$$\text{Let } D_k = \sum_{i=k}^n d_i, \quad k = 1(1)n$$

This ensures the non negativity of the D_k since $d_i \geq 0, \quad i = 1(1)n$. However, non negativity of D_k does not imply that $d_i \geq 0, \quad \forall i$, hence we impose additional constraints in equation (11).

$$D_k \geq D_{k+1}, \quad k = 1(1)n - 1 \tag{11}$$

Note $D_n \geq D_{n+1}$ is the same as $D_n \geq 0$ because $D_{n+1} = 0$ as period $(n + 1)$ does not exist.

The dual DP problem in (11) is now updated as follows:

$$\left. \begin{aligned}
 \text{Max } w &= \sum_{k=1}^n R_k D_k \\
 \text{s.t.} \\
 D_k &\leq l_k, \quad k = 1(1)n \\
 D_k &\geq D_{k+1}, \quad k = 1(1)n - 1 \\
 D_k &\geq 0, \quad k = 1(1)n
 \end{aligned} \right\} \tag{12}$$

The dual DP problem in system (12) is broken into n separate sub problems and we start from the nth dual sub problem using backward recursive approach. The last sub problem is given as:

$$\left. \begin{aligned}
 \text{Max } w &= R_n D_n \\
 \text{s.t.} \\
 D_n &\leq l_n \\
 D_n &\geq 0
 \end{aligned} \right\} \text{ This gives the solution set } 0 \leq D_n \leq l_n \tag{13}$$

Since we are maximizing w , and R_k are known in Table 1,

$$D_n = l_n \text{ or simply } D_n = \max(l_n, 0) = l_n$$

Similarly, the $(n - 1)^{th}$ dual subproblem is:

$$\left. \begin{array}{l} \text{Max } w = R_{n-1}D_{n-1} \\ \text{s.t.} \\ D_{n-1} \leq l_{n-1} \\ D_{n-1} \geq D_n = l_n \\ D_{n-1} \geq 0 \end{array} \right\} \quad (14)$$

The constraints in system (14) can produce solution set iff $D_n = l_n \leq l_{n-1}$ and $l_n \leq D_{n-1} \leq l_{n-1}$ i.e. $D_{n-1} = l_{n-1}$. In general if

$$l_k \leq l_{k-1}, \quad (k = 2(1)n) \quad (15)$$

$$\text{then } D_k = l_k \quad (k = 1(1)n) \quad (16)$$

Substituting for D_k in the dual objective function, we have:

$$\begin{aligned} w &= l_1R_1 + l_2R_2 + l_3R_3 + \dots + l_nR_n \\ &= l_1y_1 + l_2y_2 + l_3y_3 + \dots + l_ny_n \end{aligned}$$

of the primal by Duality Theorem.

When the condition (15) is satisfied the solution is automatically $y_j = R_j$ (and R_j are given in Table 1). When the condition in equation (15) is not satisfied, it is advisable to solve the primal DP problem using a computer program for large size problems.

4.0 Numerical Illustration

Given that recruitment process attracts additional costs, how should the labour force be maintained throughout the planning period of the organization in order to minimize total recruitment cost given the data in Table 2.

Table 2:Hypothetical data for recruitment and overstaffing costs

Year N	No. of Staff required R	Fixed Recruitment Cost k (₦)	Overstaffing cost i (₦)
1	74	718	13
2	35	707	11
3	47	688	14
4	62	716	15
5	20	698	14
6	90	741	16
7	51	685	13
8	30	706	10
9	43	679	11
10	35	714	15

The DP model is as stated in system (5)–(7)

$$\left. \begin{array}{l} \text{Minimize } z = \sum_{j=1}^n l_j y_j \\ \text{s.t.} \\ \sum_{j=1}^i y_j \geq \sum_{j=1}^i R_j, \quad i = 1, 2, \dots, n \\ y_j \geq 0, \quad j = 1, 2, \dots, n \end{array} \right\}$$

Consequently, the LP problem is now formulated as a DP problem:

$$\text{Minimize } z = 13y_1 + 11y_2 + 14y_3 + 15y_4 + 14y_5 + 16y_6 + 13y_7 + 10y_8 + 11y_9 + 15y_{10}$$

such that

$$\begin{aligned}
 y_1 &\geq 74 \\
 y_1 + y_2 &\geq 109 \\
 y_1 + y_2 + y_3 &\geq 156 \\
 y_1 + y_2 + y_3 + y_4 &\geq 218 \\
 y_1 + y_2 + y_3 + y_4 + y_5 &\geq 238 \\
 y_1 + y_2 + y_3 + y_4 + y_5 + y_6 &\geq 328 \\
 y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 &\geq 379 \\
 y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 &\geq 409 \\
 y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 &\geq 452 \\
 y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} &\geq 487 \\
 y_j &\geq 0, \quad j = 1(1)10
 \end{aligned}$$

Since the unit overstaffing costs l_k do not satisfy the condition

$$l_k \leq l_{k-1}, \quad k = 2(1)n \tag{15}$$

Which is a limitation earlier stated the model cannot be solved by backward recursive approach of DP technique. Consequently, we use the Program FullSimplex[10].

Note that in the solution process, the decision variables are denoted as follows:

$$y_1 = x_1, y_2 = x_2, \dots, y_{10} = x_{10}$$

Output

After compiling and running the program, the optimal solution is obtained at the 17th iteration

The initial and optimal tableau are:

INITIAL TABLEAU (ITERATION 0)

Table 3: Iteration 0

BASE VAR.	VALUE	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	X21	X22	X23	X24	X25	X26	X27			
X28	74.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X22	109.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X23	156.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X24	218.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X25	238.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X26	328.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X27	379.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X28	409.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X29	452.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X30	487.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
z	0.00	13.00	11.00	14.00	15.00	14.00	16.00	13.00	10.00	11.00	15.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	-2850.00	-10.00	-9.00	-8.00	-7.00	-6.00	-5.00	-4.00	-3.00	-2.00	-1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	

Table 4: OPTIMAL TABLEAU (ITERATION 17)

BASE VAR.	VALUE	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	
X1	74.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
X2	305.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	
X12	270.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	
X13	223.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	
X14	161.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	-1.00	0.00	0.00	0.00	
X15	141.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	-1.00	0.00	0.00	0.00	
X16	51.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-1.00	0.00	0.00
X8	108.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	-1.00
X18	78.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	-1.00	
X19	35.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	-1.00	
z	-5397.00	0.00	0.00	3.00	4.00	3.00	5.00	2.00	0.00	1.00	5.00	2.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	10.00	

5.0 Analysis of the Results

The reason for using the computer program to solve the practical problem is that apart from the high speed and accuracy (for a sparse LP), the DP has up to ten linear constraints and thirty variables in each of its tableau. It is even too complicated to solved manually.

From iteration 17 (Table 4) the optimal solution in terms of the original variables is:

$$y_1 = \mathbf{74}, y_2 = \mathbf{305}, y_{12} = 270, y_{13} = 223, y_{14} = 161, y_{15} = 141, y_{16} = 51$$

$$y_8 = \mathbf{108}, y_{18} = 78, y_{19} = 35 \text{ and } z = \mathbf{N5,397} =$$

The bolded values are the three decision variables (y_1, y_2 and y_8) and the objective function value. The total number of staff recruited in periods 1, 2, and 8 is $y_1 + y_2 + y_8 = \mathbf{487}$ and using the objective function we obtain $l_1 y_1 + l_2 y_2 + l_8 y_8 = \mathbf{N74 + N305 + N108 = N5,397}$ which is equal to the objective function value in the optimal tableau

The optimal solution to the proposed DP model in LP form for the given example reveals that out of the ten basic variables in the optimal tableau (iteration 17), seven of them which are surplus are non decision variables while the remaining three variables are decision variables that contribute to the objective function value. It is interesting to note that the three decision variables constitute a total staff recruitment of 487. This is equal to the 487 recruited staff obtained from Rao's model when it was solved recursively. This shows that although the proposed model is better than that of Rao based on sensitivity analysis, they both yield equal number of total staff recruitment. Furthermore, while Rao's DP model yielded $\mathbf{N5,757}$ as minimum total recruitment cost, that of our proposed DP model in LP form for the same problem yielded $\mathbf{N5,397}$. The difference of $\mathbf{N360}$ is certainly the constant cost K which is part of the objective function. The proposed DP model in LP form has the computational advantages of quick and accurate solutions over that of Rao's model because if the condition $l_k \leq l_{k-1}$ is satisfied then $y_j = R_j$ (R_j are given in Table 1) $\forall j$.

6.0 Conclusion

We have been able to formulate a manpower planning problem based on only recruitment factor as a DP problem in LP form which has the advantage of quick and accurate solution. This DP model is sparse with computation implementation advantage using program Full simplex. If a numerical manpower planning problem satisfies the periodic unit overstaffing cost condition (*i.e* $I_k \leq I_{k-1}$) then the periodic recruitments are equated to their corresponding periodic requirements. Furthermore, we have for the first time incorporated cost factor into the DP model formulation for manpower planning resulting in more meaningful practical interpretation of the results.

7.0 References

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