Journal of the Nigerian Association of Mathematical Physics
***Volume 29,* (*March, 2015*), pp 139 – 146**
© J. of NAMP

Optimization of Flexural and Compressive Strengths of Fibre Reinforced Concrete Beams Using Predictive Models –A Review

[1]Orie O. U. and [2]Ijeh I. P

[1]Civil Engineering Department, University of Benin, Benin City, Nigeria
[2]Institute for Energy Systems, School of Engineering, University of Edinburgh, Scotland, United Kingdom.

Abstract

The mechanical properties of fibre reinforced concrete beams was investigated with a view to determining the performance of fibre reinforcement in concrete beams, using coconut coir fibres as a reinforcing material and ascertaining the structural stability of the material for reinforcing cement composites in construction. The flexural and compressive strengths of coconut coir fibre have been compared with the performance of other natural fibres. The procedure for obtaining the percentage fibre volume and sand to cement ratio that yields maximum flexural and compressive strengths were established. The paper concluded that the blending of coconut coir fibre with concrete improves its flexural strength.

Keywords: Concrete, Construction, Fibre, Beams and Flexural strength

1.0 Introduction

Apart from food, the most basic problem facing man is shelter. The problem of shelter is as old as man and it is unfortunate that no ideal solution to this problem has evolved over the many centuries of man's existence. This problem is acute in Nigeria with increasing cost of construction materials [1]. There is therefore need to develop cost-effective materials that are also structurally viable [2]. Fibre reinforced concrete (FRC) is concrete that contains relatively high tensile strength fibres [3]. Fibre reinforced concretes have been categorized into, small amounts (<2 vol. %) of fibre and large amounts (>2 vol. %) of fibre [4]. FRC composite materials has also been classified as particle-reinforced and fibre-reinforced [5]. When the nature of the matrix is used as a criterion, composite materials are classified as natural, synthetic and inorganic composites [6, 7]. The mechanical properties of the matrix can be improved by controlling certain properties of fibre such as fibre type, fibre length, fibre volume fraction, dispersion and orientation of the fibres and the fibre matrix adhesion. The mechanical properties of a fibre reinforced composite has been predicted using the 'rule of mixtures' equation [8],

$$E_1 = (1 - f)E_m + fE_f \tag{1}$$

Where E_1 = composite axial elastic modulus, f = fibre volume fraction, E_m = matrix elastic modulus, E_f = fibre elastic modulus

A mathematical model showing the relationship between the compressive strength of cement composite and its fibre volume percentage have been developed [9],

$$\sigma_c = \frac{P(1 - \lambda_f)}{3bd(1 - 2v)} \tag{2}$$

Where; λ_f = volume fraction of coconut fibres in the composite, v = Poisson's ratio for concrete, P = Compressive load at failure of concrete cube, b = width of concrete cube d = depth of concrete cube

Research interests have been directed toward coconut fibre as a construction material because of its high ductility [10, 11]. A study on the effect of coconut fibres on the torsional, compressive and tensile strengths of concrete, showed that on the average there was 15% increase in the tensile strength with 0.5% fibre content and 3.2% tensile strength increase for 0.75% fibre content. The torsional strength was also found to increase by up to 25% with increasing fibre content [12]. Stress-strain curves for coconut fibre in comparison with other natural fibres were produced by researchers to illustrate its superiority in

Corresponding author: Orie O. U., E-mail: ogheneale@yahoo.com, Tel.: +2348023395190

ductility over them. Figure 1, show the relative ductility of coconut fibre [13].

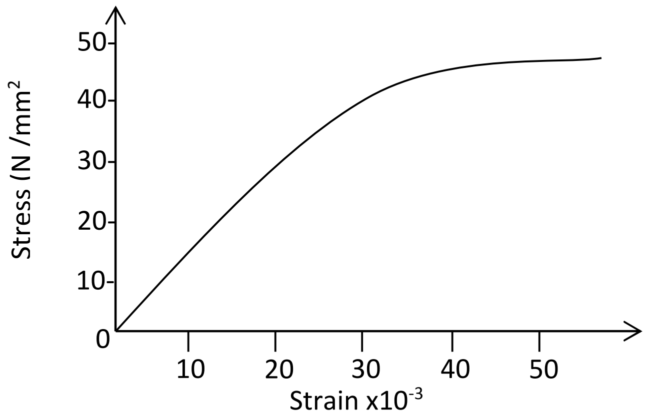

Fig.1: Mean stress-strain curve for coconut fibre [13]

This is not the case for other natural fibres with much steeper stress-strain [10, 11]. Figure 2 presents the relative ductility of some natural fibres and it showed that coconut fibre had the most ductile property. The high ductility of coconut fibre relative to other natural fibres was also indicated by the stress-strain curves of a group of natural fibres including coconut fibre shown in Figure 3. The curves further emphasized the superior ductile nature of coconut fibre over other natural fibres. The flexural strength of raffia palm fibre-cement composites has been investigated and determined as [14];

$$F_s = \frac{3Pl}{2bd^2} \tag{3}$$

Where; F_s= the flexural strength in N/mm^2, P = maximum load in N, l = length of the beam specimen in mm, b = width of the tested specimen in mm, d = depth of the tested specimen in mm
The results from the test were fit to a statistical model,

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_1x_2 + a_5x_1x_3 + a_6x_2x_2 + a_7x_1^2 + a_8x_2^2 + a_9x_3^2 \tag{4}$$

Where x_1, x_2 and x_3 represented sand–to–cement ratio, water cement ratio and fibre volume respectively and y represented the flexural strength.

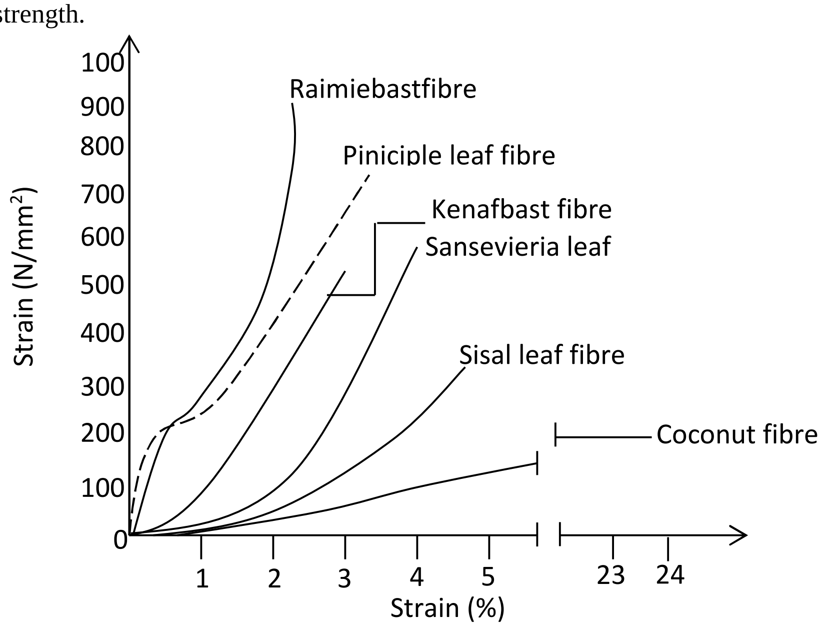

Fig.2: Typical stress –strain curves for the non-wood plant fibre [10]

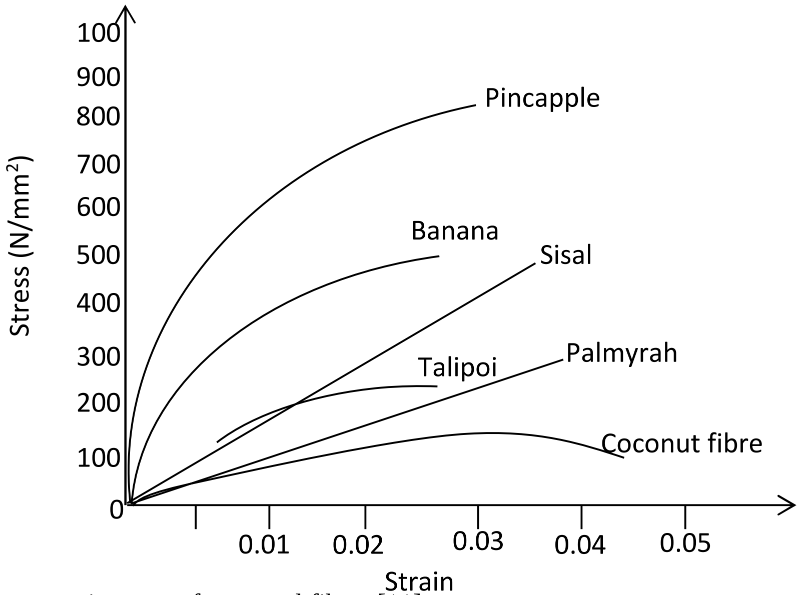

Fig. 3: Stress strain curves for natural fibres [11]

The statistical model was then developed and optimized using the tool box MATLAB 7.0 and optimal values of factors that maximize flexural strength were determined.

The present work investigated the effect of varying fibre volume fractions using natural bandage fibres (made from coconut coir) as a reinforcing material.

2.0 Materials and Methods

The work applies statistical analysis approach to generate a model expressing the relationship between the relevant variables and using a computer based optimization method. The optimal sand to cement ratio and fibre volume percentage at a constant water-cement ratio that yields maximum flexural and compressive strengths can be determined. A range of samples varying in s/c ratio and fibre volume are tested for flexure and compression and a statistical model generated and fitted, to establish a relationship between the dependent and regressor variables. Optimization is carried out using the MATLAB computer application to determine values of regressor variables that yield maximum values of the dependent variables (flexural strength).

3.0 Background theory

Linear models play a central part in modern statistical methods. On one hand, these models are able to approximate a large amount of metric data structures in their entire range of definition or at least piecewise [15].

4.0 Linear Models and Regression Analysis

Suppose the outcome of any process denoted by a random variable *y*, called a dependent (or study) variable, depends on *k* independent (or explanatory) variables denoted by $X_1, X_2,\ldots,X_k$. Suppose the behavior of *y* can be explained by a relationship given as;

$$y = f(X_1, X_2, ..., X_K, \ _1, \ _2, ..., \ _k) + £ \qquad (5)$$

Where *f* is some well-defined function and $\beta_1, \beta_2, \ldots \beta_k$ are the parameters which characterize the role and contribution of X_1, $X_2,\ldots X_k$, respectively. The term ε reflects the stochastic nature of the relationship between *y* and $X_1, X_2,\ldots,X_k$ and indicates that such a relationship is not exact in nature. When ε = 0, then the relationship is called the mathematical model otherwise the statistical model. The term "model" is broadly used to represent any phenomenon in a mathematical framework.

A model or relationship is termed as linear if it is linear in parameters and nonlinear, if it is not linear in parameters. In other words, if all the partial derivatives of *y* with respect to each of the parameters $\beta_1, \beta_2,\ldots,\beta_k$ are independent of the parameters, then the model is called as a linear model. If any of the partial derivatives of *y* with respect to any of the $\beta_1, \beta_2,\ldots,\beta_k$ is not independent of the parameters, the model is called as nonlinear. Note that the linearity or non- linearity of the mode l is not described by the linearity or non-linearity of explanatory variables in the model [15]. For example;

$$y = \ _1 X_1^{\ 2} + \ _2\sqrt{X_2} + \ _3 \log X_3 + £ \qquad (6)$$

is a linear model because $\delta y/\delta\beta_1$, are independent of the parameters β_i, (i = 1,2,3).

On the other hand,

$$y = {}_1^2 X_1 + {}_2 X_2 + {}_3 \log X + £ \qquad (7)$$

Is a non-linear model because $\delta y/\delta\beta_1 = 2\beta_1 X_1$ depends on β_1 although $\delta y/\delta\beta_2$ and $\delta y/\delta\beta_3$, are independent of any of the β_1, β_2 or β_3.

When the function *f* is linear in parameters, then;

$$y = f(X_1, X_2, \ldots X_K, {}_1, {}_2, \ldots, {}_K) + £ \qquad (8)$$

is called a linear model and when the function, *f* is nonlinear in parameters, then it is called a nonlinear model. In general, the function *f* is chosen as [15];

$$f(X_1, X_2, \ldots X_K, {}_1, {}_2, \ldots, {}_K) = {}_1 X_1 + \ldots + {}_K X_K \qquad (9)$$

to describe a linear model. Since, X_1, X_2,...X_K are pre-determined variables and y is the outcome, so both are known. Thus the knowledge of the model depends on the knowledge of the parameters $\beta_1, \beta_2, \ldots \beta_K$. The statistical linear modeling essentially consists of developing approaches and tools to determine $\beta_1, \beta_2, \ldots \beta_K$ in the linear model,

$$y = {}_1 X_1 + \ldots {}_2 X_2 + {}_K X_K + £ \qquad (10)$$

Consider a simple example to understand the meaning of "regression". Suppose the yield of crop (y) depends linearly on two explanatory variables, viz., the quantity of a fertilizer (X_1) and level of irrigation (X_2), we can write;

$$y = {}_1 X_1 + \ldots + {}_2 X_2 + £ \qquad (11)$$

5.0 Multiple linear Regression

In linear regression, there exist one dependent variable, y and more than one regressor variable say k-1. The relevant relationship has been given as [15];

$$y_i = {}_o + {}_1 X_{i1} + {}_2 X_{i2} + \ldots + {}_{k-1} X_{i,k-1} + £_i \qquad (12)$$

Given i = 1(1)$_n$

Where *y* is the dependent variable, while β and *X* represent the model parameters and regressor variables respectively. For a given set of data, these variables can be represented in matrix form,

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \qquad (13a)$$

$$= \begin{pmatrix} {}_o \\ {}_1 \\ \vdots \\ {}_n \end{pmatrix} \qquad (13b)$$

$$£ = \begin{pmatrix} £_1 \\ £_2 \\ \vdots \\ £_n \end{pmatrix} \qquad (13c)$$

$$X = \begin{pmatrix} 1 & X_{11} & X_{12} \ldots & X_{1,k-1} \\ 1 & X_{21} & X_{22} \ldots & X_{2,k-1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & X_{n1} & X_{n2} \ldots & X_{n,k-1} \end{pmatrix} \qquad (13d)$$

The model expressed in equation (12) can then be represented in its matrix form as;

$$Y = X \quad + \varepsilon. \qquad (14)$$

6.0 Estimation of Model Parameters

The model parameters would be estimated using the least square method by minimizing;

$SS_{res} = \sum_1^n ei^2 = \sum_i^n (Y_i - \hat{Y}_i)^2$, Where SS_{res} is the residual sum of squares.

Suppose $\hat{Y} = \beta_0 + \beta_1 X_1 + \cdots + \beta_{k-1} X_{k-1}$ (15)

$SS_{res} = \sum (Y_i - \beta_0 - \beta_1 X_{i1} - \beta_2 X_{i2} \ldots \beta_{k-1} X_{k-1})2$ (16)

$SS_{res} = \sum_1^n ei^2 = e^1 e = (Y - \hat{Y})(Y - \hat{Y}) = (Y - X \ )'(Y - X \ )$ (17)

$SS_{res} = Y'Y - Y'X\beta - \beta'X'Y + \beta'X'X\beta$ (18)

Using the least squares method, the SS_{res} has been minimized thus [15];

$\frac{\partial}{\partial} = 0$ (19)

$\frac{\partial}{\partial} = -2X'Y + 2X'X \ = 0$ (20)

$\beta = (X'X)^{-1} X'Y$ (21)

The model parameters are estimated using the relationship given in equation (18), after which the model is fit to establish the relationship between the dependent variables and the regressor variables. The statistical significance of the model is tested using the global test in the form of Analysis of Variance (ANOVA). The significance of each regressor variable is tested to determine its contribution to the model. This is carried out using the t-test.

7.0 Estimation of Percentage Fibre Volume

Percentage fibre volume is estimated using the weight fraction principle and the mixture rule.

$V_p + V_m + V_f = 1$ (22)

Where V_p is the fraction of porosity, V_m is the volume of matrix while V_f is the volume of fibre and is determined as;

$V_f = \frac{w_f}{\rho_f V_c}$ (23)

Where, W_f = weight of fibre in given beam specimen, ρ_f is the density of the fibre and V_c is the volume of the concrete specimen which is determined as,

$V_c = t_c b_c I_c$ (24)

Where, t_c = thickness of the beam specimen, b_c = width of the beam specimen and l_c = length of the beam specimen. In obtaining the specified fibre percentage fraction in each beam specimen, the density of the fibre is determined as well as volume of the concrete specimen. After which, the values obtained including the specified volume percentage is substituted into equation (16) and the corresponding fibre weight is determined. This weight is used in the laboratory instead of the direct use of the volume percentage fraction.

8.0 Sample Preparation and Testing

Mixing of the composite material is done in batches conforming to standards [16]. Firstly, dry sand and cement is mixed together in a mixing machine to specified mix ratios after which the specified amount of fibre is added. Mixing is continued until a uniform matrix is achieved and the moulds are filled. The samples are transferred to the vibration table where proper vibration is carried out to eliminate voids and achieve appropriate compaction. The specimens are demoulded after 24 hours of casting and stored in curing tanks. The beams are tested for flexure at 28 days while the cube specimens are tested for compression at 7, 14 and 28 days.

Beam specimens measuring 150mm x150mm x750mm are subjected to flexural testing with the use of the Universal Testing Machine, with point loading at mid-span (See Figure 5). The deflection values are measured with the aid of a dial gauge attached to the bottom of the beam during stressing. Loading is incremental by 1kN until ultimate failure.

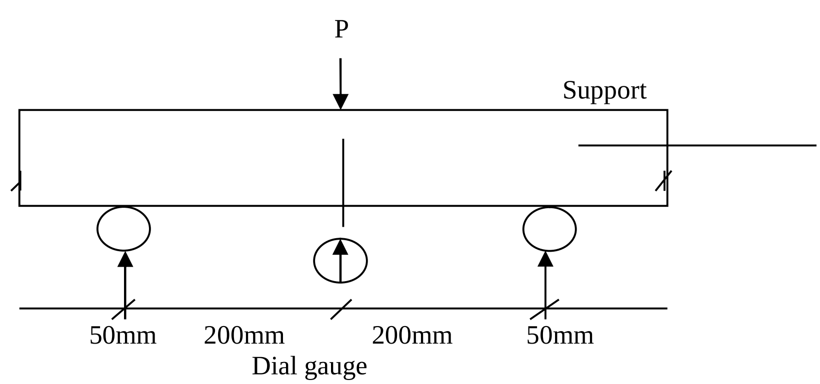

Fig.5. Simply supported beam specimen subjected to point load [17]

9.0 Results

The presentation of experimental results is shown in Table 1. The sand/cement ratio has been used to categorize the beams into types A-E.

Table 1: Flexural and compressive strengths for varying sand/cement and fibre volumes

Beam type (sand to cement ratio) X_1	**Fibre percentage volume** X_2	**Flexural strength** Y	**Compressive strength** P
	0	$Y_{1,0}$	$P_{1,0}$
Type A (s/c ratio = 1)	1	$Y_{1,1}$	$P_{1,1}$
	.	.	.
	.	.	.
	6	$Y_{1,6}$	$P_{1,6}$
	0	$Y_{2,0}$	$P_{2,0}$
Type C(s/c ratio = 2)	1	$Y_{2,1}$	$P_{2,1}$
	.	.	.
	.	.	.
	6	$Y_{2,6}$	$P_{2,6}$
	0	$Y_{3,0}$	$P_{3,0}$
Type D(s/c ratio = 3)	1	$Y_{3,1}$	$P_{3,1}$
	.	.	.
	.	.	.
	6	$Y_{3,6}$	$P_{3,6}$
	0	$Y_{3,0}$	$P_{4,0}$
Type D (s/c ratio = 4)	1	$Y_{3,1}$	$P_{4,1}$
	.	.	.
	.	.	.
	6	$Y_{3,6}$	$P_{4,6}$
	0	$Y_{4,0}$	$P_{5,0}$
Type E (s/c ratio = 5)	1	$Y_{4,1}$	$P_{5,1}$
	.	.	.
	.	.	.
	6	$Y_{4,6}$	$P_{5,6}$

10.0 Conclusion

Much has been achieved by the use of both synthetic and natural fibre as reinforcement in concrete. The procedure of obtaining an optimized compressive and flexural strengths of coconut coir fibre concrete has been established, The inclusion of coconut coir fibre to concrete have been shown to improve its flexural and compressive strength.

11.0 References

[1] Orie, O. U. (2007), Recycled waste concrete: an alternative to natural mineral aggregate. J. Sc. and Tech. Research. Vol. 7, No. 7, pp 57-61.

[2] Orie, O. U. and Osadebe, N. N. (2008), Cost effectiveness of Scheffe's optimized five-component-concrete with mound soil as a mix component. J. Advances in Materials and Systems Technologies. Vol. 367, pp 33-40.

[3] Marijan, S., Development and application of hybrid fibre reinforced concrete. 2nd International Conference on Sustainable Construction Materials and technologies, (2010).

[4] Skazlic, M. (2003), High performance hybrid fibre reinforced concretes. MSc. *Thesis,* Faculty of Civil Engineering, University of Zagreb, Zagreb.

[5] Clyne T.W. and Hull, D. (1996), Introduction to Composite material, 2nd ed.

[6] Rogar, M., Speleter, H., Arola, R., Davis P., Fibers, T., Hemingway, R., Rials, T., Luneke, D., Narayan, R., John, S. and White. D. (1993), Opportunities for composites from recycled waste wood–based resources; a problem analysis and research plan. Forest Products J. Vol. 43, No. 1, pp 55-63.

[7] John, A. (1999), Wood-based composites and panel products: In Forest Product Laboratory Wood Handbook – wood as an engineering material, Gen. Tech. Rep. FPL – GTR – 113. Madison, WI: U.S. Dept. of Africa, Forest service, Forest Product Laboratory, pp 463.

[8] Chou, T.W., (1993), "Polymer – matrix Composites", VCH, New York.

[9] Sule, S. (2013), Structural models for the prediction of the compressive strength of coconut fibre-reinforced concrete. International Journal of Engineering and Advanced Technology. Vol. 2, No. 4, pp

[10] Munawar, S. S., Umemura, K. and Kawai, S. (2007), Characterization of the morphological, physical and mechanical properties of seven non-wood plant fibre bundles. Journal of Wood Sciences, Vol. 53 No. 2, pp 108 – 113.

[11] Satyanarayana, K. G., Sukumaran, K., Mukherjee, P. S., Parithran, C. and Pillai, S.G.K. (1990), Natural fibre- polymer composites. Cement and Concrete Composites, Vol. 12, No. 2, pp 117 - 136.

[12] Yalley P.P and Kwan, A.S.K. (2013) "Use of Coconut fibres as an Enhancement of Concrete", KNUST Paper Publications.

[13] Paramasivam, P., Nathan, G. K. and Das Gupta, N. C. (1984), Coconut fibre-reinforced corrugated slabs. International Journal of Cement Composites, Vol. 6, No.1, pp 19 -27.

[14] Odera, R. S., Onukwuli, O. D. and Osoka, E. C. (2011), Optimization of the flexural strength of raffia palm fibre–cement composites, J. Emerging Trends in Eng. and Applied Sc. Vol. 2, No.2. pp 294 – 297.

[15] Shalabh, L. (2010), Linear regression models. Department of Mathematics and Statistics, Indian Institute of Technology, Kanpur. http://nptelonlinecourses.iitm.ac.in/syllabus/syllabus.php?subjectId=111104074, (29/01/2015).

[16] British Standards International BS 1881-125 (1986), Testing concrete -Methods for mixing and sampling fresh concrete in the laboratory. British Standards House, London.

[17] Orie, O. U. and Alutu, O. E. (2007), Flexural characteristics of reinforced hollow cored concrete beams point loaded at midspan. Nig. J. of Tech. Dev. Vol. 5, No. 1 and 2, pp 30-38.