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Vibration of Timoshenko Beam subjected to Partially Distributed Moving Load

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Abstract

This paper examines the vibration of Timonshenko beam subjected to partially distributed moving load. The governing partial differential equation were analysed to determine the behavior of the system under consideration. The methods of series solution and numerical method were used to solve the governing equation. Result revealed that the amplitude increases as the fixed length of the beam increases. It was also found that there sponse amplitude increases as the foundation moduli increases at the fixed length of the beam increases

Keywords: Timoshenko Beam, Partially Distributed Moving Load, Vibration, Amplitude, Moving Load.

1.0 Introduction

There has always been a description through a system of second order differential equations, in which the vibration amplitude and the angle due to pure bending were the searched functions. Boundary conditions related to the initial-boundary value problem under consideration were described by a proper second order differential equation of both or only one of these functions. However, there is a remark arises from the fact that in the case of the simply supported beam, the boundary conditions are described by the same differential equations as in the case of the Euler-Bernoulli beam. In Euler-Bernoulli beam theory, sheared formation and rotation effects are neglected, and palne sections remain plane and normal to the longitudinal axis, while in Timoshenko beam theory, plane sections still remain plane but no longer normal to the longitudinal axis.
The Timonshenko model is an extension of the Euler-Bernoulli model by taking into account two additional effects: shearing force effect and rotator motion effect.
In this paper, a simple and practical analytical-numerical method to determine the response of Timoshenko beam subjected to partially distributed moving load is discussed.

Definition (1.1):
Beam is a piece of horizontal or vertical structure that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of external load, own weight, span and external reaction. Beams are traditionally description of building or civil engineering structural elements, but smaller structures such as truck or automobiles frames, machine frames and other mechanical structural systems contains beam structures that are designed and analysed in a similar fashion. There are three basic types of beam which include: (i) Simple span, supported at both ends
(ii) Continuous, supported at more than two points
(iii) Cantilever, supported at one end with the other end over hanging and free
It was recognized by the early researchers that the bending effect is the single most important factor in a transversely vibrating beam[1-12]. Introducing the mass by the Dirac delta function, Chen[6] solved analytically the problem of a simply supported beam carrying a concentrated mass. Cha[4] obtained the natural frequencies of a continuous structure with spring mass attachments by using the classical assumed-modes method in conjunction with Langrange sequation.

2.0 The Governing Equation

The governing equations describing the vibration behavior of aTimoshenko beam subjected to partially distributed moving load are

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$$EI\frac{\partial^4 w(x,t)}{\partial t^4}+\rho A\frac{\partial^2 w(x,t)}{\partial t^2}+K\,w(x,t)=P_f(x,t) \quad (1)$$

Where E is Young's modulus, I is the constant moment of inertia of the beam's cross section about the axis, A is the area, $w(x,t)$ is the deflection of the beam, K is the Winkler foundation, ρ is the density, t is the time, x is the spatial coordinate and $P_f(x,t)$ is the applied force (i.e. the resultant concentrated force caused by the moving mass).

The applied force per unit length $P_f(x,t)$ is the uniform partially distributed moving load which is defined as

$$P_f(x,t)=\frac{1}{\varepsilon}\left|-M \quad -M\frac{d^2 w}{d\ ^2}\right|\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right] \quad (2)$$

Where M is the mass of the load, g is the acceleration due to gravity ξ is the fixed length of load, is the length of the beam.

The differential operator $\frac{d^2 w(x,t)}{dx^2}$ is defined as

$$\frac{d^2 w(x,t)}{dx^2}=\frac{\partial^2 w(x,t)}{\partial t^2}+2V\frac{\partial^2 w(x,t)}{\partial x\partial t}+V^2\frac{\partial^2 w(x,t)}{\partial x^2} \quad (3)$$

H is the heavy-side function such that

$$H\left(x-\xi+\frac{\varepsilon}{2}\right)=H\left(x-\left(\xi-\frac{\varepsilon}{2}\right)\right)=\begin{cases}0 & x<\xi-\frac{\varepsilon}{2}\\ 1 & x>\xi-\frac{\varepsilon}{2}\end{cases}$$

$$H\left(x-\xi+\frac{\varepsilon}{2}\right)=H\left(x-\left(\xi-\frac{\varepsilon}{2}\right)\right)=\begin{cases}0 & x<\xi+\frac{\varepsilon}{2}\\ 1 & x>\xi+\frac{\varepsilon}{2}\end{cases} \quad (4)$$

Hence, the governing equation becomes

$$E\frac{\partial^4 w(x,t)}{\partial t^4}+\rho A\frac{\partial^2 w(x,t)}{\partial t^2}K\quad(x,t)\frac{1}{\varepsilon}\left[-M\quad\left(M\frac{\partial^2 w(x,t)}{\partial t^2}+2M\frac{\partial^2 w(x,t)}{\partial}+MV^2\frac{\partial^2 w(x,t)}{\partial x^2}\right)\right]\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right] \quad (5)$$

With the boundary conditions

$$w(0,t)=0=w(l,t) \quad (6)$$

$$\frac{\partial^2 w(0,t)}{\partial x^2}=0=\frac{\partial^2 w(l,t)}{\partial x^2} \quad (7)$$

Without loss of generality,one can consider the initial conditions of the form

$$\mathbf{w(x,0)=0}\ \frac{\partial\ \ (x,0)}{\partial} \quad \mathbf{(8)}$$

3.0 Method of Solution

Inthissection,weproceedtosolvetheboundary-initialvalueproblemscomprisingofequations(1)−(8).
The transverse displacement and external applied force may be expressed as

$$w(x,t) = \sum_{i=1}^{\omega} X_i(x)\gamma_j(t) \tag{9}$$

Substituting(9)into(5),we have

$$E\sum_{i=1}^{m} X_i^{iv}(x)\gamma_j(t) + \rho\sum_{i=1}^{m} X_i(x)\gamma_j''(t) + K\sum_{i=1}^{m} X_i(x)\gamma_j(t)$$
$$= \left[-\frac{M}{\varepsilon} - \frac{M}{\varepsilon}\sum_{i=1}^{m}\gamma_j''(t)X_i(x) - \frac{2M}{\varepsilon}\sum_{i=1}^{m}\gamma_j'(t)X_j'(x) - \frac{V^2M}{\varepsilon}\sum_{i=1}^{m}\gamma_j(t)X_j''(x)\right]$$
$$\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\} \tag{10}$$

Furthermore, we assume that

$$P_f\,(x,t) = \sum_{i=1}^{m}\gamma_f\,(t)X_i(x) \tag{11}$$

Substituting(11)into(10),wehave

$$\sum_{i=1}^{m}\gamma_f\,(t)X_i(x) = -\frac{M}{\varepsilon}\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\} - \frac{M}{\varepsilon}\sum_{i=1}^{m}\gamma_j''(t)X_i(x)$$
$$\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\} - \frac{2M}{\varepsilon}\sum_{i=1}^{m}\gamma_j'(t)X_j'(x)$$
$$\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\} - \frac{V^2M}{\varepsilon}\sum_{i=1}^{m}\gamma_i(t)X_i''(x)$$
$$\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\} \tag{12}$$

Tonormalizeequation(12),wemultiplyallthroughby*Xj*(*x*)toobtain

$$\sum_{i=1}^{m}\gamma_f\,(t)X_i(x)X_j(x) = -\frac{M}{\varepsilon}X_j(x)\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}$$
$$-\frac{M}{\varepsilon}\sum_{i=1}^{m}\gamma_j''(t)X_i(x)\,X_j(x)\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}$$
$$-\frac{2M}{\varepsilon}\sum_{i=1}^{m}\gamma_j'(t)X_i'(x)\,X_j(x)\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}$$
$$-\frac{V^2M}{\varepsilon}\sum_{i=1}^{m}\gamma_i(t)X_i''(x)\,X_j(x)\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\} \tag{13}$$

IntegratingbothsideofL(13)withrespecttoxalongthelengthLof thebeam,wehave

$$\sum_{i=1}^{m}\gamma_f\,(t)\int_0^L X_i(x)X_j(x)\,d \;= --\frac{M}{\varepsilon}\int_0^L X_j(x)\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d$$
$$-\frac{M}{\varepsilon}\sum_{i=1}^{m}\gamma_i''(t)\int_0^L X_i(x)X_j(x)\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d$$

$$-\frac{2M}{\varepsilon}\sum_{i=1}^{\omega}\gamma_i'(t)\int_L^L X_i'(x)X_j(x)\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d$$
$$-\frac{V^2M}{\varepsilon}\sum_{i=1}^{\omega}\gamma_i(t)\int_L^L X_i''(x)X_j(x)\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d \quad (14)$$

From(14),weassumethefollowing

$$0=-\frac{M}{\varepsilon}\int_L^L X_i(x)\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d \quad (15)$$

Integratingbypart,

$$H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\int_L^L X_j(X)d$$
$$-\int_L^L\int_L^L X_j(x)H'\left[\left(x-\xi+\frac{\varepsilon}{2}\right)-H'\left(x-\xi-\frac{\varepsilon}{2}\right)\right]d \quad (16)$$

Assume

$F(x)=H(x)$ where $H'(x)-H(x)=\delta(x)$ (17)

Such that we get,

$$X_j\left(\xi+\frac{\varepsilon}{2}\right)-X_j\left(\xi-\frac{\varepsilon}{2}\right) \quad (18)$$

Furthermore, expanding using Taylorseries,weobtain,

$$X_j\left(\xi+\frac{\varepsilon}{2}\right)=X_j(\xi)+\frac{\left(\frac{\varepsilon}{2}\right)}{1!}X_j'(\xi)+\frac{\left(\frac{\varepsilon}{2}\right)^2}{2!}+X_j''(\xi)+\frac{\left(\frac{\varepsilon}{2}\right)^3}{3!}X_j'''(\xi)+\cdots \quad (19)$$

Also,

$$X_j\left(\xi-\frac{\varepsilon}{2}\right)=X_j(\xi)-\frac{\left(\frac{\varepsilon}{2}\right)}{1!}X_j'(\xi)+\frac{\left(\frac{\varepsilon}{2}\right)^2}{2!}X_j''(\xi)-\frac{\left(\frac{\varepsilon}{2}\right)^3}{3!}X_j'''(\xi)+\cdots \quad (20)$$

Bysubstituting(19)−(20)into(18),we have

$$X_i(\xi)+\frac{\left(\frac{\varepsilon}{2}\right)}{1!}X_j'(\xi)+\frac{\left(\frac{\varepsilon}{2}\right)^2}{2!}X_j''(\xi)+\frac{\left(\frac{\varepsilon}{2}\right)^3}{3!}X_j'''(\xi)-X_i(\xi)+\frac{\left(\frac{\varepsilon}{2}\right)}{1!}X_j'(\xi)$$
$$-\frac{\left(\frac{\varepsilon}{2}\right)^2}{2!}X_j''(\xi)+\frac{\left(\frac{\varepsilon}{2}\right)^3}{3!}X_j'''(\xi) \quad (21)$$

We get,

$$\varepsilon X_j'(\xi)+\frac{\varepsilon^3}{2}X_j'''(\xi) \quad (22)$$

Substituting(22)into(16)and having satisfied the condition (4), we have,

$$0=-M\left[X_i(\xi)+\frac{\varepsilon^2}{2}X_j''(\xi)\right] \quad (23)$$

Similarargumentsisapplicabletosecond,thirdandfourthdefiniteintegralin(14),hence,evaluating theintegralsusingTaylor'sseriesexpansionandapplyingorthogonalitypropertiesofthecharacteristics function$\gamma fi(t)$thelefthandsideof(14),wefinallyobtain

$$\gamma(t)=-M\left[X_i(\xi)+\frac{\varepsilon^2}{2}X_j''(\xi)\right]$$
$$-M\sum_{j=1}^{\omega}\gamma_i''(t)\left[X_j(\xi)X_i(\xi)+\frac{\varepsilon^2}{2}\left(X_j(\xi)X_j''(\xi)+2X_j'(\xi)X_j'(\xi)+X_j''(\xi)X_j(\xi)\right)\right]$$
$$-2M\sum_{j=1}^{\omega}\gamma_i'(t)\left[X_j(\xi)X_i'(\xi)+\frac{\varepsilon^2}{2}\left(X_j(\xi)X_i'''(\xi)+2X_j'(\xi)X_i''(\xi)+X_j''(\xi)X_i'(\xi)\right)\right]$$
$$-V^2M\sum_{j=1}^{\omega}\gamma_i'(t)\left[X_j(\xi)X_i''(\xi)+\frac{\varepsilon^2}{2}\left(X_j(\xi)X_i^{iv}(\xi)+2X_j'(\xi)X_i'''(\xi)+X_j''(\xi)X_i''(\xi)\right)\right] \quad (24)$$

Furthermore,from(10)wehavethat

$$E\sum_{i=1}^{\omega}X_i^{iv}(x)\gamma_j(t)+\rho\sum_{i=1}^{\omega}X_i(x)\gamma_j''(t)+K\sum_{i=1}^{\omega}X_i(x)\gamma_j(t)=\sum_{i=1}^{\omega}\gamma_j(t)X_i(x) \quad (25)$$

substituting(24)into(25)becomes

$$E\sum_{i=1}^{\omega}X_i^{iv}(x)\gamma_j(t)+\rho\sum_{i=1}^{\omega}X_i(x)\gamma_j''(t)+K\sum_{i=1}^{\omega}X_i(x)\gamma_j(t)=-M\left[X_j(\xi)+\frac{\varepsilon^2}{24}X_j''(\xi)\right]$$

$$-M\sum_{j=1}^{\infty}\gamma_i''(t)\left[X_j(\xi)X_i(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i''(\xi)+2X_j'(\xi)X_i'(\xi)+X_j''(\xi)X_i(\xi)\right)\right]$$
$$-2M\sum_{i=1}^{\infty}\gamma_i'(t)\left[X_j(\xi)X_i'(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i'''(\xi)+2X_j'(\xi)X_i''(\xi)+X_j''(\xi)X_i'(\xi)\right)\right]$$
$$-V^2M\sum_{i=1}^{\infty}\gamma_i(t)\left[X_j(\xi)X_i''(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i^{iv}(\xi)+2X_j'(\xi)X_i'''(\xi)+X_j''(\xi)X_i''(\xi)\right)\right] \quad (26)$$

so that(26)becomes

$$E\sum_{i=1}^{\infty}X_i^{iv}(x)\gamma_j(t)+\rho\sum_{i=1}^{\infty}X_i(x)\gamma_j''(t)+K\sum_{i=1}^{\infty}X_i(x)\gamma_j(t)+M\left[X_j(\xi)+\frac{\varepsilon^2}{24}X_j''(\xi)\right]$$
$$+M\sum_{j=1}^{\infty}\gamma_i''(t)\left[X_j(\xi)X_i(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i''(\xi)+2X_j'(\xi)X_i'(\xi)+X_j''(\xi)X_i(\xi)\right)\right] \quad (27)$$
$$+2M\sum_{i=1}^{\infty}\gamma_i'(t)\left[X_j(\xi)X_i'(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i'''(\xi)+2X_j'(\xi)X_i''(\xi)+X_j''(\xi)X_i'(\xi)\right)\right]$$
$$+V^2M\sum_{i=1}^{\infty}\gamma_i(t)\left[X_j(\xi)X_i''(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i^{iv}(\xi)+2X_j'(\xi)X_i'''(\xi)+X_j''(\xi)X_i''(\xi)\right)\right]=0$$

Next,weconsiderthefreevibrationofaTimoshenkobeamunderconsideration,thuswehave

$$X_i^{iv}(x)-\beta_i^4\,\alpha(t)=0 \quad (28)$$
$$\alpha_i^2=\frac{\beta^4 E}{M} \quad (29)$$

Where$\alpha 2$ isthecircularfrequencyof thebeam.
Considering(28)and(29),(27)yields

$$\sum_{i=1}^{\infty}X_i^{iv}\left[M\alpha_i^2(t)\gamma_j+\rho\,\gamma_j''(t)+K\gamma_j\right]+M\left[X_j(\xi)+\frac{\varepsilon^4}{2}X_j''(\xi)\right]$$
$$+M\sum_{j=1}^{\infty}\gamma_i''(t)\left[X_j(\xi)X_i(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i''(\xi)+2X_j'(\xi)X_i'(\xi)+X_j''(\xi)X_i(\xi)\right)\right] \quad (30)$$
$$+2M\sum_{i=1}^{\infty}\gamma_i'(t)\left[X_j(\xi)X_i'(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i'''(\xi)+2X_j'(\xi)X_i''(\xi)+X_j''(\xi)X_i'(\xi)\right)\right]$$
$$+V^2M\sum_{i=1}^{\infty}\gamma_i(t)\left[X_j(\xi)X_i''(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i^{iv}(\xi)+2X_j'(\xi)X_i'''(\xi)+X_j''(\xi)X_i''(\xi)\right)\right]=0$$

Equation(30)abovemustbe satisfiedforarbitrary*Xi*(*x*)andthisispossibleonlywhentheexpression inthecurlybracketisequaltozero.Wethereforeobtain

$$M\alpha_i^2(t)\gamma_j+\rho\,\gamma_j''(t)+K\gamma_j=-M\left[X_j(\xi)+\frac{\varepsilon^4}{2}X_j''(\xi)\right]$$
$$-M\sum_{j=1}^{\infty}\gamma_i''(t)\left[X_j(\xi)X_i(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i''(\xi)+2X_j'(\xi)X_i'(\xi)+X_j''(\xi)X_i(\xi)\right)\right] \quad (31)$$
$$-2M\sum_{i=1}^{\infty}\gamma_i'(t)\left[X_j(\xi)X_i'(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i'''(\xi)+2X_j'(\xi)X_i''(\xi)+X_j''(\xi)X_i'(\xi)\right)\right]$$
$$-V^2M\sum_{i=1}^{\infty}\gamma_i(t)\left[X_j(\xi)X_i''(\xi)+\frac{\varepsilon^4}{2}\left(X_j(\xi)X_i^{iv}(\xi)+2X_j'(\xi)X_i'''(\xi)+X_j''(\xi)X_i''(\xi)\right)\right]=0$$

Fortheboundaryconditionsgivenunderthegoverningequation

$$X_i(x)\sqrt{\frac{2}{L}}S\left[\frac{i}{L}\right] \quad (32)$$

Toobtainasetofexactgoverningdifferentialequationforthesimplysupportedbeamunderconsid- eration,wesubstitute(32)into(14)toobtain

$$\frac{2}{L}\sum_{i=1}^{\infty}\gamma\ (t)\int_0^L S\left[\frac{i}{L}x\right]S\left[\frac{i}{L}x\right]d$$
$$-M\frac{2}{L}\sqrt{\frac{2}{L}}\int_0^L S\ \frac{i}{L}x\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d$$
$$-2M\frac{V}{\varepsilon}\sum_{i=1}^{\infty}\gamma\ (t)\int_0^L S\ \frac{i}{L}x\sin\frac{i}{L}x\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d$$
$$-4M\ \frac{i}{\varepsilon L^4}\sum_{i=1}^{\infty}\gamma_i'(t)\int_0^L S\ \frac{i}{L}x\cos\frac{i}{L}x\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d$$
$$-2MV^2\frac{i^4\pi^4}{\varepsilon L^2}\sum_{i=1}^{\infty}\gamma_i\ (t)\int_0^L S\ \frac{i}{L}x\sin\frac{i}{L}x\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d \quad (33)$$

Evaluatingtheaboveintegrals, wehave

$$Q_1=\int_0^L S\ \frac{j}{L}x\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d\ =2S\left[\frac{j}{L}\xi\right]S\left[\frac{i}{2L}\varepsilon\right] \quad (34)$$
$$Q_2=\int_0^L S\ \frac{i}{L}x\,S\ \frac{j}{L}x\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d$$
$$=\frac{1}{i-j}C\ \frac{j}{L}\xi(i-j)S\ \frac{i}{2L}\varepsilon(i-j)-\frac{1}{i+j}C\ \frac{\pi}{L}\xi(i+j)S\ \frac{\pi}{2L}\varepsilon(i+j) \quad (35)$$
$$Q_3=\int_0^L S\ \frac{i}{L}x\,C\ \frac{j}{L}x\left\{H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right\}d$$

$$= \frac{1}{i+j} s \frac{\pi}{L} \xi(i+j) s \frac{\pi}{2L} \varepsilon(i+j) - s \frac{\pi}{L} \xi(i-j) s \frac{\pi}{2L} \varepsilon(i-j) \quad (36)$$

$$Q_4 = \int_L^L s \frac{i\pi}{L} x\, s \frac{j}{L} x \left\{ H\left(x - \xi + \frac{\varepsilon}{2}\right) - H\left(x - \xi - \frac{\varepsilon}{2}\right) \right\} d$$

$$= \frac{1}{i-j} c \frac{\pi}{L} \xi(i-j) s \frac{\pi}{2L} \varepsilon(i-j) - \frac{1}{i+j} c \frac{\pi}{L} \xi(i+j) s \frac{\pi}{2L} \varepsilon(i+j) \quad (37)$$

$$Q_5 = \int_L^L s \left[\frac{i\pi}{L} x\right] s \left[\frac{j}{2L} x\right] d = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad (38)$$

Bysubstitutingequations(34)−(38)into(33),wehave

$$\gamma \ (t) = -\frac{M}{\varepsilon} \sqrt{8L} s \left[\frac{j}{L} \xi\right] s \left[\frac{i\pi}{L} \varepsilon\right]$$

$$-2\frac{M}{\varepsilon} \sum_{i=1}^{\infty} \gamma_i''(t) \left\{ \frac{1}{i-j} c \frac{\pi}{L} \xi(i-j) s\, n \frac{\pi}{2L} \varepsilon(i-j) - \frac{1}{i+j} c \frac{\pi}{L} \xi\, (i+j) \frac{\pi}{2L} \varepsilon(i+j) \right\}$$

$$-4M \frac{i\pi}{\varepsilon L^2} \sum_{i=1}^{\infty} \gamma_i'(t) \left\{ \frac{1}{i+j} s \frac{\pi}{L} \xi(i+j) s \frac{\pi}{2L} \varepsilon(i+j) - \frac{1}{i-j} s \frac{\pi}{L} \xi\, (i-j) s \frac{\pi}{2L} \varepsilon(i-j) \right\} \quad (39)$$

$$-2M \frac{i^2\pi^2}{\varepsilon L^3} \sum_{i=1}^{\infty} \gamma_i(t) \left\{ \frac{1}{i-j} c \frac{\pi}{L} \xi(i-j) s \frac{\pi}{2L} \varepsilon(i-j) - \frac{1}{i+j} c \frac{\pi}{L} \xi\, (i+j) s \frac{\pi}{2L} \varepsilon(i+j) \right\}$$

I $\neq j, i = 1, 2, 3, \ldots$

Byreplacingtherighthandsideof(31)withtherighthandsideof(39),wefinallyobtain

$$M\, \alpha_i^2(t)\, \gamma_j + \rho\ \gamma_j''(t) + K\gamma_j = -\frac{M}{\varepsilon} \sqrt{8L} s \left[\frac{j}{L} \xi\right] s \left[\frac{i\pi}{L} \varepsilon\right]$$

$$-2\frac{M}{\varepsilon} \sum_{i=1}^{\infty} \gamma_i''(t) \left\{ \frac{1}{i-j} c \frac{\pi}{L} \xi(i-j) s \frac{\pi}{2L} \varepsilon(i-j) - \frac{1}{i+j} c \frac{\pi}{L} \xi\, (i+j) s \frac{\pi}{2L} \varepsilon(i+j) \right\}$$

$$-4M \frac{i\pi}{\varepsilon L^2} \sum_{i=1}^{\infty} \gamma_i'(t) \left\{ \frac{1}{i+j} s \frac{\pi}{L} \xi(i+j) s \frac{\pi}{2L} \varepsilon(i+j) - \frac{1}{i-j} s \frac{\pi}{L} \xi\, (i-j) s \frac{\pi}{2L} \varepsilon(i-j) \right\} \quad (40)$$

$$-2M \frac{i^2\pi^2}{\varepsilon L^3} \sum_{i=1}^{\infty} \gamma_i(t) \left\{ \frac{1}{i-j} c \frac{\pi}{L} \xi(i-j) s \frac{\pi}{2L} \varepsilon(i-j) - \frac{1}{i+j} c \frac{\pi}{L} \xi\, (i+j) s \frac{\pi}{2L} \varepsilon(i+j) \right\}$$

i $\neq j, i = 1, 2, 3, \ldots$

$$M\, \alpha_i^2(t)\, \gamma_j + \rho\ \gamma_j''(t) + K\gamma_j = -\frac{M}{\varepsilon} \sqrt{8L} s \left[\frac{j}{L} \xi\right] s \left[\frac{i\pi}{L} \varepsilon\right]$$

$$-\frac{2M}{\varepsilon} \sum_{i=1}^{\infty} \gamma_i''(t) \left\{ \frac{1}{i-j} c \frac{\pi}{L} \xi(i-j)\ s \frac{\pi}{2L} \varepsilon(i-j) \right\} + \frac{2M}{\varepsilon} \sum_{i=1}^{\infty} \gamma_i''(t) \left\{ \frac{1}{i+j} c \frac{\pi}{L} \xi(i+j) s \frac{\pi}{2L} \varepsilon(i+j) \right.$$

$$-\ 4M \frac{i\pi}{\varepsilon L^2} \sum_{i=1}^{\infty} \gamma_i'(t) \left\{ \frac{1}{i+j} s \frac{\pi}{L} \xi\, (i+j) s \frac{\pi}{2L} \varepsilon(i+j) \right\}$$

$$+4M \frac{i\pi}{\varepsilon L^2} \sum_{i=1}^{\infty} \gamma_i'(t) \left\{ \frac{1}{i-j} s \frac{\pi}{L} \xi\, (i-j) s \frac{\pi}{2L} \varepsilon(i-j) \right\}$$

$$-\ 2MV^2 \frac{i^2\pi^2}{\varepsilon L^3} \sum_{i=1}^{\infty} \gamma_i(t) \left\{ \frac{1}{i-j} c \frac{\pi}{L} \xi(i-j) s \frac{\pi}{2L} \varepsilon\, si \frac{\pi}{2L} \varepsilon(i-j) \right\}$$

$$+\ 2MV^2 \frac{i^2\pi^2}{\varepsilon L^3} \sum_{i=1}^{\infty} \gamma_i(t) \left\{ \frac{1}{i+j} c \frac{\pi}{L} \xi(i+j) s \frac{\pi}{2L} \varepsilon\, s \frac{\pi}{2L} \varepsilon(i+j) \right\}$$

i= $1, 2, 3, \ldots i \neq j$

(41)

$$M\, \alpha_i^2(t)\, \gamma_j + \rho\ \gamma_j''(t) + K\gamma_j = -M \frac{g}{\varepsilon} \sqrt{8L}\, s \left[\frac{j}{L} \xi\right] s \left[\frac{i\pi}{L} \varepsilon\right]$$

$$-2\frac{M}{\varepsilon} \sum_{i=1}^{\infty} \gamma_i''(t) \left\{ \frac{1}{i-j} cos \frac{\pi}{L} \xi(i-j) s \frac{\pi}{2L} \varepsilon(i-j) \right\}$$

$$+\frac{2M}{\varepsilon} \sum_{i=1}^{\infty} \gamma_i''(t) \left\{ \frac{1}{i+j} c \frac{\pi}{L} \xi(i+j) s \frac{\pi}{2L} \varepsilon(i+j) \right\} \quad (42)$$

where,

$$\gamma_i''(t) = (\gamma_i + 1 - 2\gamma_i + \gamma_i - 1)/h^2$$

(43)

$$\gamma_i'(t) = (\gamma_i + 1 - \gamma_i - 1)/2h$$

$$\rho \ \frac{(\gamma_i + 1 - 2\gamma_i + \gamma_i - 1)}{h^2} + M(\alpha^2(t) + k)\gamma_i = -M \frac{g}{i\pi\, \varepsilon} h^2 \sqrt{8L}\, s \left[\frac{i\pi}{L} \xi\right] s \left[\frac{j\pi}{2L} \varepsilon\right]$$

$$-\frac{2M}{\varepsilon}\left(\frac{(\gamma_t+1-2\gamma_t+\gamma_t-1)}{h^2}\right)\left\{\frac{1}{i-j}c\quad \frac{\pi}{L}\xi(i-j)\ s\quad \frac{\pi}{2L}\varepsilon(i-j)\right\}$$
$$+\frac{2M}{\varepsilon}\left(\frac{(\gamma_t+1-2\gamma_t+\gamma_t-1)}{h^2}\right)\left\{\frac{1}{i+j}c\quad \frac{\pi}{L}\xi(i+j)\ s\quad \frac{\pi}{2L}\varepsilon(i+j)\right\}$$
$$-4M\quad \frac{i\imath}{\varepsilon L^2}\left(\frac{(\gamma_t+1-\gamma_t-1)}{2h}\right)\left\{\frac{1}{i-j}s\quad \frac{\pi}{L}\xi(i-j)s\quad \frac{\pi}{2L}\varepsilon(i-j)\right\}$$
$$-2MV^2\frac{i^2\pi^2}{\varepsilon L^3}\gamma_t\left\{\frac{1}{i-j}c\quad \frac{\pi}{L}\xi(i-j)s\quad \frac{\pi}{2L}\varepsilon\ s\quad \frac{\pi}{2L}\varepsilon(i-j)\right\}$$
$$+2MV^2\frac{i^2\pi^2}{\varepsilon L^3}\gamma_t\left\{\frac{1}{i+j}c\quad \frac{\pi}{L}\xi(i+j)\varepsilon\ s\quad \frac{\pi}{2L}\varepsilon(i+j)\right\} \tag{44}$$

multiplyingthroughby$h$2wefinallyobtain,

$$\rho\quad +\frac{2M}{\varepsilon}\left[\frac{1}{i-j}c\quad \frac{\pi}{L}\xi(i-j)s\quad \frac{\pi}{2L}\varepsilon(i-j)-\frac{1}{i+j}c\quad \frac{\pi}{L}\xi(i+j)s\quad \frac{\pi}{2L}\varepsilon(i+j)\right]$$
$$+2M\quad \frac{i\imath}{\varepsilon L^2}h\left[\frac{1}{i+j}s\quad \frac{\pi}{L}\xi(i+j)s\quad \frac{\pi}{2L}\varepsilon(i+j)-\frac{1}{i-j}s\quad \frac{\pi}{L}\xi(i-j)s\quad \frac{\pi}{2L}\varepsilon(i-j)\right]\gamma_t+1$$
$$+\{Mh^2\omega_i^2h^2+Kh^2-2\rho\quad +\frac{4M}{\varepsilon}\left[\frac{1}{i+j}c\quad \frac{\pi}{L}\xi(i+j)s\quad \frac{\pi}{2L}\varepsilon(i+j)-\frac{1}{i-j}c\quad \frac{\pi}{L}\xi(i-j)s\quad \frac{\pi}{2L}\varepsilon(i-j)\right]$$
$$+2MV^2\frac{i^2\pi^2}{\varepsilon L^3}h^2\left[\frac{1}{i-j}c\quad \frac{\pi}{L}\xi(i-j)s\quad \frac{\pi}{2L}\varepsilon(i-j)-\frac{1}{i+j}c\quad \frac{\pi}{L}\xi(i+j)s\quad \frac{\pi}{2L}\varepsilon(i+j)\right]\Big\}\gamma_t+$$
$$\Big\{\rho\quad +\frac{2M}{\varepsilon}\left[\frac{1}{i-j}c\quad \frac{\pi}{L}\xi(i-j)s\quad \frac{\pi}{2L}\varepsilon(i-j)-\frac{1}{i+j}c\quad \frac{\pi}{L}\xi(i+j)s\quad \frac{\pi}{2L}\varepsilon(i+j)\right]$$
$$+2M\quad \frac{i\imath}{\varepsilon L^2}h\left[\frac{1}{i-j}s\quad \frac{\pi}{L}\xi(i-j)s\quad \frac{\pi}{2L}\varepsilon(i-j)-\frac{1}{i+j}s\quad \frac{\pi}{L}\xi(i+j)s\quad \frac{\pi}{2L}\varepsilon(i+j)\right]\Big\}\gamma_t-1$$
$$=-M\frac{g}{\varepsilon}h^2\sqrt{8L}\ s\quad \left[\frac{\imath\imath}{L}\xi\right]s\quad \left[\frac{j}{2L}\varepsilon\right] \tag{45}$$

4.0 Numerical Simulation

Computer program was developed and the following numerical data were used ρA=0.2,0.4,0.6,0.8,1.0kg/m, ξ and = 0.01,0.02 and 0.03

M=70kg,L=6m,m=7kg,v=10m/s,K=0,1,2,3and4. Hence we have the graphs of results. See Figures 1-4.

The deflection profiles of the beam are display graphically to demonstrate the effect of fixed length of the beam, fixed length of the load and the foundation constant.

5.0 Conclusion

From the response profile of the beam it was observed that irrespective of the value of the damping coefficient, there sponse amplitude of vibration increases as the fixed length of the load increases. It was also found that there sponse amplitude of vibration increases as the foundation moduli increases at fixed length of the beam.

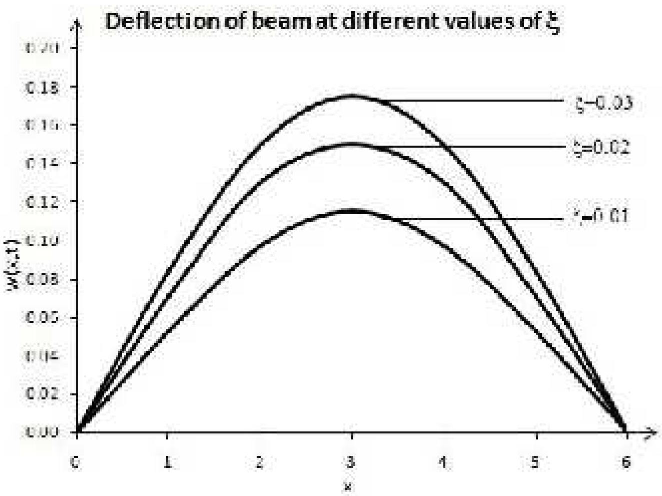

Figure1: Deflection of beam for ρA=0.02atdifferentvaluesof ξ

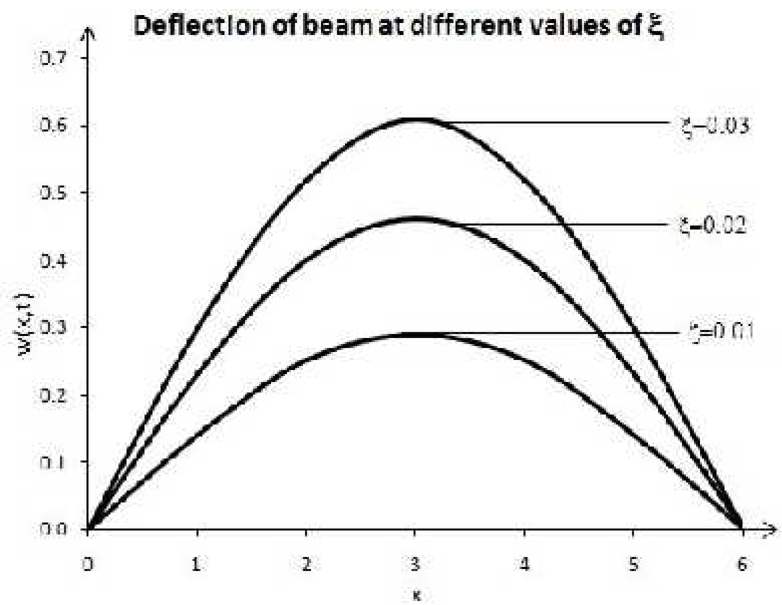

Figure2: Deflection of beam for ρA=0.04atdifferentvaluesof ξ

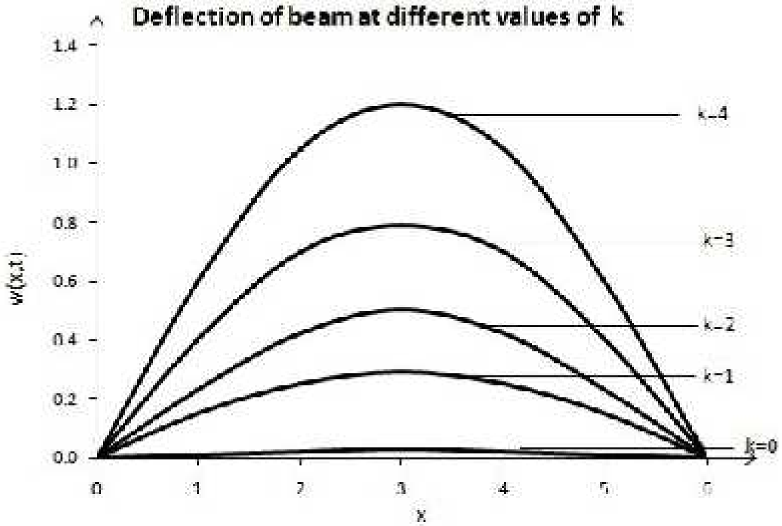

Figure 3: Deflection of beam for =0.01 at different values of k

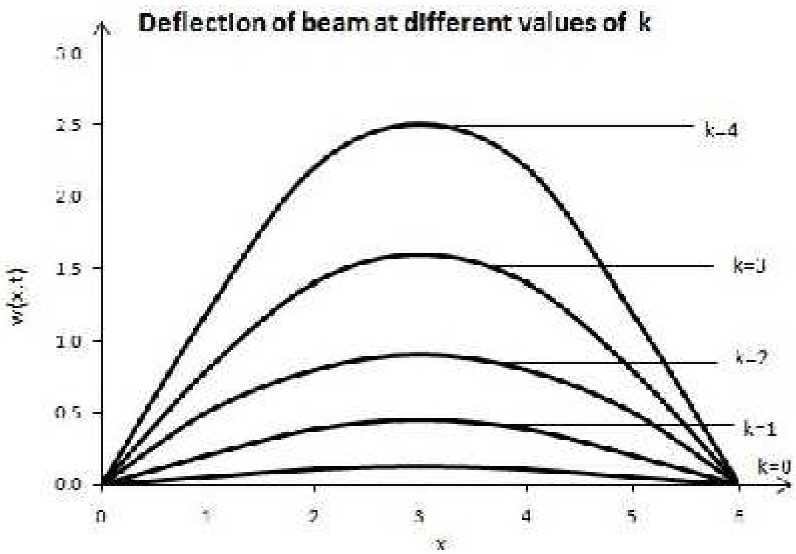

Figure 4: Deflection of beam for =0.02 at different values of k

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