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Hyperchaos Stabilization and Synchronization in a Flux-Controlled Memristor System

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Abstract

In this paper, the problem of stabilization and synchronization of a new two terminal memristive electronic circuit is considered. The system which was recently introduced as the fourth basic circuit element is characterized by a charge and flux-linkage relationship and it exhibit hyperchaotic dynamics for appropriate choice of parameters. Based on backstepping technique, control inputs are proposed for achieving the global stabilization and synchronization of hyperchaotic dynamics in the system. Results of numerical simulation are given to show the effectiveness of the proposed technique.

1.0 Introduction

The phenomenon of chaos in nonlinear dynamical system was first discovered by Poincare [1]. The concept did not however receive adequate attention until Lorenz encountered chaotic behaviourwhile studying his now famous model of the atmospheric convection – the Lorenz system [2]. Since then, several other systems in the physical, chemical and life sciences that exhibit chaotic dynamics have been found. In the physical sciences, here we make mention of the Henon-Heiles system [3,4], mechanical systems, like the periodically forced pendulum [1,5], the nonlinear Bloch equations [6], Bose- Einstein Condensates [7], plasma oscillations[8], gyroscopic motion [9] as well as several electronics devices [10-12], including the more recently found memristors [13-21].

Recently, the dynamical propertiesof memristive systems, including chaotic behaviour have attracted considerable attentionsince 2008; when the Hewlett Packed (HP) researchers validated the original work of Chua [14-16]. The memristor which was proposed by Leon Chua in 1971 as the fourth class of basic electrical circuit element is a device that has the hysteresis property of ferromagnetic core memory as well as the dissipation that characterizes a resistor [13]; such that by controlling the magnetic flux or flow of the electrical charge, the nonlinear resistance can be indefinitely memorized. Thus, as the name implies, it may be regarded as a form of nonlinear resistance with 'memory' ability. Due to this interesting property, memristor-based memory device appear to be the most obvious application of memristors. Infact, where capacitors are replaced with memristors, it is possible for one memristor to store a single bit of data in a DRAM-like architecture. Besides other applications in cellular and recurrent neural networks, ultra wide band receivers, programmable threshold comparators [19], to name but a few, memristors can be used within analog processing domain, for instance in chaos secure communication circuits [20]; and biomimetric and neuromorphic circuits for replicating observable biological behaviours [21].

Applications of memristors in nonlinear circuit are now active research topics and varieties of memristive chaotic circuits have been proposed. For instance, Ito [15], introduced a fourth-order memristor-based Chua's oscillator by replacing Chua's diode with an active two terminal circuit consisting of a conductance and a flux-controlled memristor. A similar circuit was also proposed in [22,23] by adapting a modified Chua's circuit.In another work, Bao et al. [16] introduced a variant of the memristor-Chua's circuit by replacing the monotone-increasing and piecewise-linear nonlinearity in [15, 22, 23] with a smooth continuous cubic monotone-increasing nonlinearity. These memristive circuits exhibit chaos when appropriate parameters are chosen.

In this paper, we consider the suppression and synchronization of chaotic behavior in a model of memristor circuit by using a systematic approach based on backstepping technique. Chaos control and synchronization for more than two decades have been considered asmain domain in which chaos theory find important applications [24-34]. The concepts were independently

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proposed by Ott,Grebogi and Yorke [24];and Pecora and Caroll [25], respectively in 1990. It is worth mentioning that different methods have been employed to stabilize unstable orbits associated with chaotic behavior as well as to synchronize chaotic systems. Increasing research has also revealed varieties of synchronization phenomena that may arise depending on the approach employed and definition of synchrony. Our choice of backstepping is based on its flexibility and effectiveness in the construction of control law and its ability to control chaos and to achieve chaos synchronization for identical and generalized cases.However, hyperchaotic systems are less investigated in this regard, partly because they are of higher dimension; so that obtaining the control laws becomes more challenging. Here, we undertake this task for strongly nonlinear system modelled by a memristor. The rest of the paper is organized a follows: In section 2 the memristor model is introduced. In sections 3 and 4, we treat chaos control and synchronization, respectively. Numerical simulations results are presented in section 5. The paper is summarized and concluded in section 6.

2.0 The Model –Memristor

In 2010, Bao et al. [16] proposed a flux-controlled nonlinear circuit, with a memristor which replaces the Chua's diode as shown in Fig. 1.

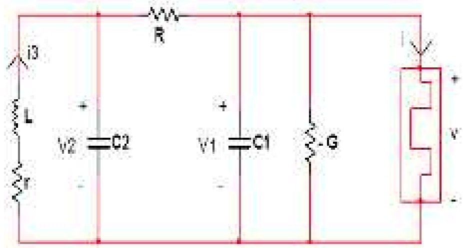


Figure 1: Chua's circuit with active memrsitor

By applying Kirchoff's laws of electrical circuit to Fig.1, the following differential equations defining the relationship between all the circuit variables $v_1, v_2, i_3,$ and ¤ are obtained:

$$\frac{dv_1}{dt} = \frac{1}{RC_1}\left[v_2 - v_1 + GRv_1 - R\ W(¤)v_1\right],$$
$$\frac{dv_2}{dt} = \frac{1}{RC_2}\left[v_1 - v_2 + Ri_3\right], \qquad (1)$$
$$\frac{di_3}{dt} = -\frac{1}{L}v_2 - \frac{r}{L}i_3,$$
$$\frac{d¤}{dt} = v_1,$$

where the nonlinear $¨ - q$ characteristic curve of the flux controlled memristor is defined by $q(¨) = a¨ + b¨^3$ and $W(¨) = dq(¨)/dt$. By redefining the state variables as¢ = *G*, *C*₂ = 1, *R*= 1, $x = v_1,\ y = v_2,\ z = i_3,\ w = ¤,\ \sim = 1/C_1,$ $= 1/L,\ ¥ = r/L$, and the nonlinear functions $q(w)$ and memductance $W(w)$ as

$$q(w) = aw + bw^3 \qquad (2)$$
$$W(w) = dq(w)/dw = a + 3bw^2 \qquad (3)$$

The dimensionless state equations with a time scale factor, k is obtained as follows [16]:

$$\dot{x} = k\sim(y - x - ¢\,x - W(w)x),$$
$$\dot{y} = k(x - y + z),$$
$$\dot{z} = -k(\ \ y + ¥\,z), \qquad (4)$$
$$\dot{w} = kx,$$

where *x, y, z* and *w* represents the rescaled voltage across C_1, voltage across C_2, current through the inductor *L*, and flux in the memristor, respectively.

For the purpose of our analysis using backstepping and to ensure that the memristor is operated in a stable chaotic state, we set the system parameters $a = 1/7$, ¢ $= 9/7$, $k = 1$ and ¥ $= 0$, and express system (4) as

$$\dot{x} = \sim[y + (a - dw^2)x]$$
$$\dot{y} = x - y + z \qquad (5)$$
$$\dot{z} = - \quad y$$
$$\dot{w} = x$$

where the parameters $d = 6/7$, $\alpha = 9.8, \beta = \frac{100}{7}$ and initial conditions are $(x, y, z, w) = (0, 10^{-10}, 0, 0)$. In Fig. 2, we show varieties of typical phase space plots of the chaotic attractor of the memristor for the given parameters set; while Fig. 3 shows the corresponding time series.

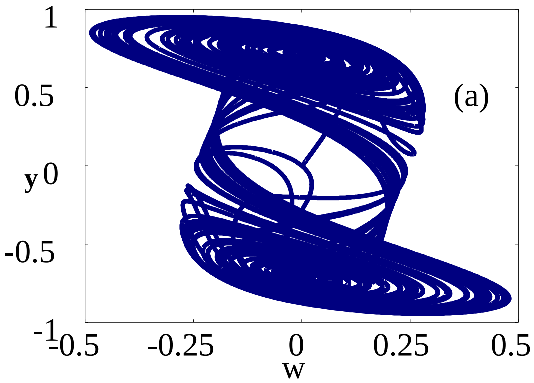


Figure 2a:The chaotic attractor of the memristor system in phase space on y-axis against w-axis.

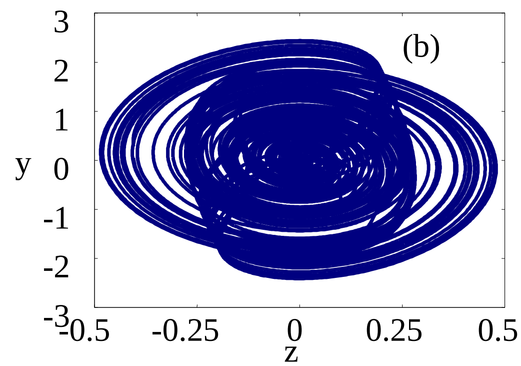


Figure 2b:The chaotic attractor of the memristor system in phase space on y-axis against w-axis

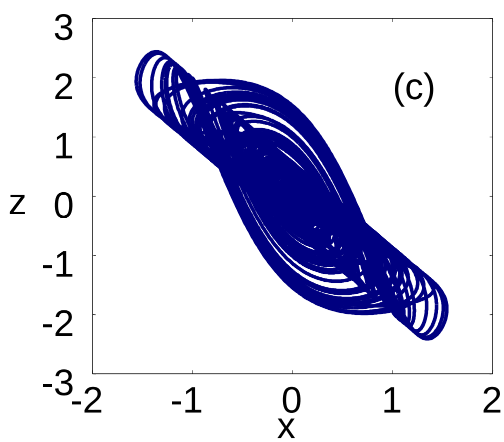


Figure 2c: The chaotic attractor of the memristor in phase space on z-axis against x-axis.

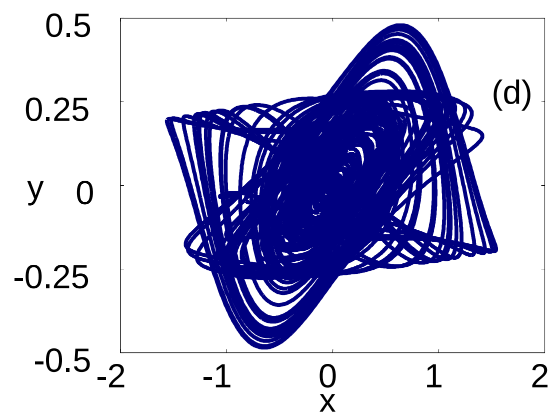
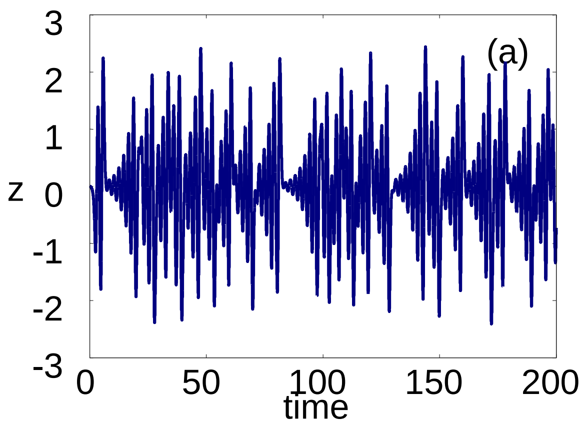


Figure 2d: The chaotic attractor of the memristor in phase space on y-axis against x-axis.



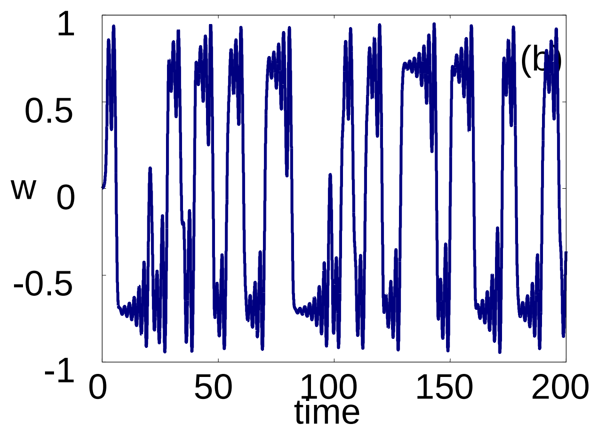


Figure 3a: The time response of the state variable z **Figure b: The time response of the state variable w.**

3.0 Chaos Control in the Memristor

Here, we focus on the design of controllers using backstepping method to eliminate the undesirable chaotic dynamicsshown in Fig.1 by stabilizing the system asymptotically. To achieve the stability, we introduce controls $u_i (i = 1, 2, 3, 4)$ to the system in Eq. (5) to obtain a controlled memristive system as follows:

$$\begin{cases} \dot{z} = -\ \ y + u_1 \\ \dot{y} = x - y + z + u_2 \\ \dot{x} = \sim[y + (a - dw^2)x] + u_3 \\ \dot{w} = x + u_4 \end{cases} \quad (6)$$

The backstepping technique employs a recursive procedure that is based on a chosen virtual control. At the *i*th step of the procedure, the *i*th-order subsystem is stabilized with respect to appropriate Lyapunov function V_i and a control input u_i. Suppose $\sim_1(z)$ is the virtual control required to stabilize the*z*-subsystem and a Lyapunov function $V_1(z)$ is assumed:

$$V_1(z) = \frac{1}{2} z \quad (7)$$

and its time-derivative $\dot{V}_1(z) = -\ \ yz + zu_1$. By choosing the control law as $u_1 = -z$ and assuming that the state y is a virtual control, that is $y \equiv \sim_1(z) = z$, then the time-derivative of $V(z)$ becomes

$$\dot{V}_1(z) = -(\ \ +1)z^2 \le 0. \quad (8)$$

Thus, $\dot{V}_1$ is negative definite.According to the Lyapunov stability theorem, the equilibrium point is globally and asymptotically stable. Notice that if we choose $u_1 = 0$, we can obtain the simplest control input, so that $\dot{V}_1(z) = -\ \ z^2 \le 0$; accordingly, the *z*-subsystem is globally stabilized.

Let the error signals between $\sim_1(z)$ and y be given by $\bar{y}$; such that $\bar{y} = y - z$. Using equation (6), we get the $(z, \bar{y})$ subsystem given by

$$\begin{aligned} \dot{z} &= -\ \ y - z \\ \dot{\bar{y}} &= x + (\ \ -1)\bar{y} + (\ \ +1)z + u_2 \end{aligned} \quad (9)$$

Suppose $\sim_2(z,\bar{y})$ is a virtual control required to stabilize the $(z,\bar{y})$ subsystem and assume a Lyapunov function of the form:

$$V_2(z,\bar{y}) = v_1(z) + \frac{1}{2}\bar{y}^2 \qquad (10)$$

By choosing a control law $u_2 = -\ \ (y+z)$ and assuming that x is a virtual control, then $x = \sim_2(z,\bar{y}) = -\bar{y}$. The time derivative of (10) gives

$$\dot{V}_2(z,\bar{y}) = -(\ \ +1)z^2 - 2\bar{y}^2 + \bar{y}[\ \ \bar{y} + \ \ z + z + u_2] = -(\ \ +1)z^2 - 2\bar{y}^2 \le 0\,, \qquad (11)$$

Equation (11) is negative definite; and according to the Lyapunov stability theorem, the equilibrium point of subsystem (9) is globally and asymptotically stable.

Suppose the error signal between $\sim_2 = (z,\bar{y})$ and x is given by $\bar{x}$,then $\dot{\bar{x}} = \dot{x} + \dot{\bar{y}}$. Using the definition for $\dot{x}$ and $\dot{\bar{y}}$ from equation (6) and (9), we get the following $(x,\bar{y},\bar{z})$ sub-system:

$$\dot{z} = -\ \ y - z$$
$$\dot{\bar{y}} = (\ \ -2)\bar{y} \qquad (12)$$
$$\dot{\bar{x}} = dz + [\sim -2 - \sim(a - dw^2)]\bar{y} + [1 + \sim(a - dw^2)]\bar{x} + u_3$$

Let $\sim_3(z,\bar{y},\bar{x})$ be a virtual control required to stabilize the $(x,\bar{y},\bar{z})$ sub-system and a Lyapunov function of the form

$$V_3(z,\bar{y},\bar{x}) = V_2(z,\bar{y}) + \frac{1}{2}\bar{x}^2 \qquad (13)$$

is assumed. Choosing the control law $u_3 = -\sim(y-z) - [2 + \sim(a - dw^2)]$ and assume that z is the virtual control, then $z \equiv \sim_3(z,\bar{y},\bar{x})$. The time-derivative of eq. (13) yields

$$\dot{V}_3(z,\bar{y},\bar{x}) = -(\ \ +1)z^2 - 2\bar{y}^2 - \sim\bar{x}^2 \le 0\ . \qquad (14)$$

This implies that $\dot{V}_3(z,\bar{y},\bar{x}) \le 0$ is negative definite. Accordingly, the equilibrium point is globally and asymptotically stable.

Now, considering the full system $(x,\bar{y},\bar{z},w)$, and by assuming a Lyapunov function of the form

$$V_4 = V_3 + \frac{1}{2}w^2 \qquad (15)$$

we can globally stabilize the full system $(x,\bar{y},\bar{z},w)$. Therefore, substituting equation (14) and (6) into the time-derivative of (15) gives

$$\dot{V}_4(z,\bar{y},\bar{x},w) = -(\ \ +1)z^2 - 2\bar{y}^2 - \sim\bar{x}^2 + w(x + u_4) \qquad (16)$$

Suppose, the control law is chosen as $u_4 = -w - x$, then,

$$\dot{V}_4(z,\bar{y},\bar{x},w) = -(\ \ +1)z^2 - 2\bar{y}^2 - \sim\bar{x}^2 - w^2 \le 0 \qquad (17)$$

is negative definite . Thus, we can conclude that the orbit will attract a stable periodic orbit and fixed point, and since the full system is asymptotically stable, all the solution of system (6) converges and the control goal is achievable.

4.0 Chaos Synchronization in the Memristor Systems

Here, we consider a drive-response configuration wherethe driver system is designatedwith the subscript 1 and the response system, having identical equations is designated with the subscript 2.Thus, for the system (1),the slave (or response) systems and master (or drive) systems are defined below respectively,

$$\dot{z}_2 = -\ \ y_2 + u_1$$
$$\dot{y}_2 = x_2 - y_2 + z_2 + u_2 \qquad (19)$$
$$\dot{x}_2 = \sim[y_2 + (y_2 + (a - dw_2^2)x_2] + u_3$$
$$\dot{w}_2 = x_2 + u_4$$

$$\dot{z}_1 = - \quad y_1$$
$$\dot{y}_1 = x_1 - y_1 + z_1 \qquad (20)$$
$$\dot{x}_1 = \sim [y_1 + (a - dw_1^2)x_1]$$
$$\dot{w}_1 = x_1$$

The error between the state variable are defined by

$$e_1 = z_2 - z_1,\ e_2 = y_2 - y_1,\ e_3 = x_2 - x_1,\ e_4 = w_2 - w_1 \qquad (21)$$

and time-derivative of the error states are written as

$$\dot{e}_1 = \dot{z}_2 - \dot{z}_1,\ \dot{e}_2 = \dot{y}_2 - \dot{y}_1,\ \dot{e}_3 = \dot{x}_2 - \dot{x}_1,\ \dot{e}_4 = \dot{w}_2 - \dot{w}_1 \qquad (22)$$

So that using equations (19) an (20), the error dynamics system is written as

$$\dot{e}_1 = - \quad e_2 + u_1$$
$$\dot{e}_2 = e_3 - e_2 + e_1 + u_2$$
$$\dot{e}_3 = \sim e_2 + \sim a e_3 - h + u_3 \qquad (23)$$
$$\dot{e}_4 = e_3 + u_4$$

where $h = \sim d(w_2^2 x_2 - w_1^2 x_1)$. Suppose $\sim_1(e_1)$ is a virtual control,and consider the $\dot{e}_1$ subsystem. Assume a Lyapunov function of the form

$$V_1(e_1) = \frac{1}{2}e_1^2 \qquad (24)$$

The derivative of (24) is

$$\dot{V}(e_1) = - \quad e_1 e_2 + e_1 u_1 \qquad (25)$$

If the control law is chosen as $u_1 = 0$ and assume that $e_2 \equiv \sim_1(e_1) = e_1$, Then,

$$\dot{V}_1(e_1) = - \quad e_1^2 \le 0 \qquad (26)$$

Let the error between $\sim_1(e_1)$ and e_2 be given by $\bar{e}_2$ such that $\bar{e}_2 = e_2 - e_1$. Derivative of $\bar{e}_2$ becomes:

$$\dot{\bar{e}}_2 = \dot{e}_2 - \dot{e}_1 \qquad (27)$$

Substitute $\dot{e}_2$ and $\dot{e}_1$ from equation (23) into (27), we get

$$\dot{\bar{e}}_2 = e_3 + (\quad -1)\bar{e}_2 + \quad e_1 + u_2 \qquad (28)$$

Let $\sim_2(e_1, \bar{e}_2)$ be a virtual control to stabilize the e_2 subsystem and assume a Lyapunov function of the form:

$$V_2(e_1, \bar{e}_2) = V_1(e_1) + \frac{1}{2}\bar{e}_2^2 \qquad (29)$$

Derivative of (29) is:

$$\dot{V}_2 = - \quad e_1^2 + \bar{e}_2[e_3 + (\quad -1)\bar{e}_2 + \quad e_1 + u_2] \qquad (30)$$

If the control law is chosen as $u_2 = (1 - \quad)e_2 - e_1$ and assuming that e_3 is the virtual control, then $e_3 \equiv \sim_2(e_1, \bar{e}_2) = -\bar{e}_2$ and

$$\dot{V}_2 = - \quad e_1^2 + \bar{e}_2[\sim_2(e_1, \bar{e}_2) + (\quad -1)\bar{e}_2 + \quad e_1 + u_2] \qquad (31)$$

$$\dot{V}_2(e_1, \bar{e}_2) = - \quad e_1^2 - \bar{e}_2^2 \le 0 \qquad (32)$$

Let the error between $\sim_2(e_1, \bar{e}_2)$ and e_3 be given by $\bar{e}_3$, such that $\bar{e}_3 = e_3 + \bar{e}_2$ and $\dot{\bar{e}}_3 = \dot{e}_3 + \dot{\bar{e}}_2$. Substitute $\dot{e}_3$ and $\dot{\bar{e}}_2$ from equation (23) and (28) into (33), we get:

$$\dot{\bar{e}}_3 = (\sim a + 1)\bar{e}_3 + (\sim - \sim a - 1)\bar{e}_2 + \sim e_1 - h + u_3 \qquad (33)$$

Let $\sim_3(e_1,\bar{e}_2,\bar{e}_3)$ be a virtual control to stabilize the e_3 subsystem and assume a Lyapunov function of the form:

$$V_3(e_1,\bar{e}_2,\bar{e}_3)=V_2(e_1,\bar{e}_2)+\frac{1}{2}\bar{e}_3^2 \quad (34)$$

The time-derivative of (35) is:

$$\dot{V}_3=- \ e_1^2-\bar{e}_2^2+\bar{e}_3[(\sim a+1)\bar{e}_3-(\sim a+1-\sim)\bar{e}_2+\sim e_1-h+u_3] \quad (35)$$

If the control law is chosen as $u_3=-\sim e_2-(\sim a+1)e_3+\sim d(w_2^2x_2-w_1^2x_1)$; and assume that e_1 is the virtual control, then $e_1=\sim_3(e_1,\bar{e}_2,\bar{e}_3)=-\bar{e}_3$

$$\dot{V}_3=- \ e_1^2-\bar{e}_2^2-\sim\bar{e}_3^2+\bar{e}_3[(\sim a+1)\bar{e}_3-(\sim a+1-\sim)\bar{e}_2 \ -h+u_3] \quad (36)$$

$$\dot{V}_3=- \ e_1^2-\bar{e}_2^2-\sim\bar{e}_3^2\le 0 \quad (37)$$

In order to achieve the stability of the e_4 subsystem, consider a Lyapunov function of the form:

$$V_4(e_1,\bar{e}_2,\bar{e}_3,e_4)=V_3(e_1,\bar{e}_2,\bar{e}_3)+\frac{1}{2}e_4^2 \quad (38)$$

The derivative of (38) is $\dot{V}_4=\dot{V}_3(e_1,\bar{e}_2,\bar{e}_3)+e_4\dot{e}_4=\dot{V}_3(e_1,\bar{e}_2,\bar{e}_3)+e_4(e_3+u_4)$ and if the control law is chosen as:

$$u_4=-e_3-e_4 \quad (39)$$

then

$$\dot{V}_4=- \ e_1^2-\bar{e}_2^2-\sim\bar{e}_3^2-e_4^2\le 0 \quad (40)$$

It is obvious that $\dot{V}_4$ is negative definite and hence systems (19) and (20) are globally synchronized; implying that the goal of synchronization has been achieved with the proposed control functions.

5.0 Numerical Results

Now, we present some numerical simulations to verify the effectiveness of the designed controllers by choosing the parameters as $\sim\ = 9.8, \quad = 100/7$; using a fourth-order Runge-Kutta algorithm with fixed integration time stepof 0.005 and initial conditions $(x,y,z,w)=(0,\ 10^{-10},\ \ 0,\ 0)$. Depicted in Figure 2 are typicallyperchaotic orbits of the memristor in different phase space. The corresponding time history is also shown in Figure 3 illustratingirregular oscillations. To show the effectiveness of \the control obtained in section 3 to stabilize the chaotic orbits of the memristor to the origin, the controls were activated at $t\ge 0$. In Figure 4, the controlled chaotic time series shows that as $t\ge 0$, the state variables $(x,y,z,w)\to(0,0,0,0)$ with asymptotically convergence of the *w* variable.

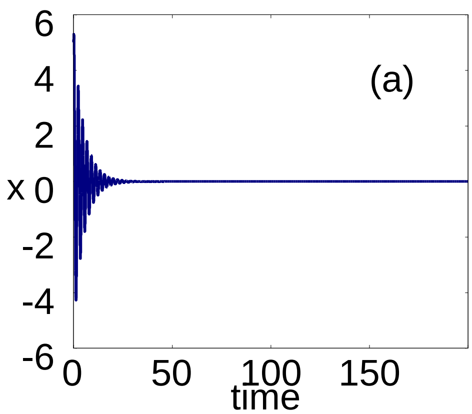


Figure 4a: Asymptotic convergence of the state variable x in the controlled state.

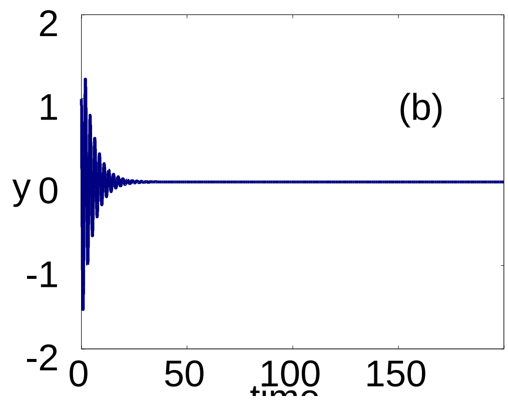


Figure 4b: Asymptotic convergence of the state variable y in the controlled state.

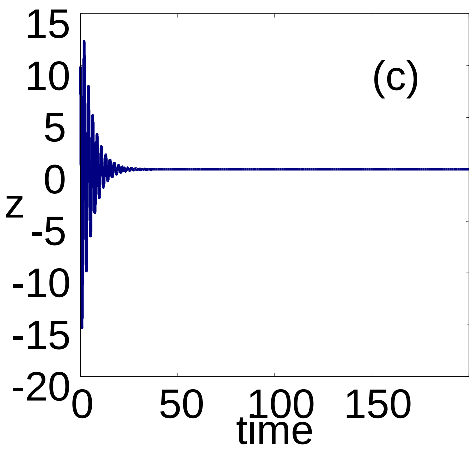


Figure 4c: Asymptotic convergence of the state variable z in the controlled state.

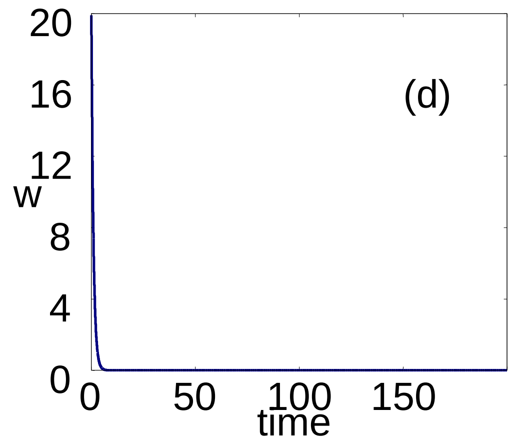


Figure 4d: Asymptotic convergence of the state variable w in the controlled state.

Similarly, numerical simulation results are presented for synchronization of the master-slave system (19) and (20) using the proposed control inputs. The parameters were set as before, while the initial conditions for the master and slave systems where chosen as $(x_1, y_1, z_1, w_1) = (0.005, 0.0001, 0.001, 0.002)$ and $(x_2, y_2, z_2, w_2) = (0.015, 0.0011, 0.011, 0.012)$, respectively. The results are shown in Figure 5. Evidently, the error dynamics $(e_1, e_2, e_3, e_4) \to (0,0,0,0)$ asymptotically showing that complete synchronization between the master and slave has been achieved.

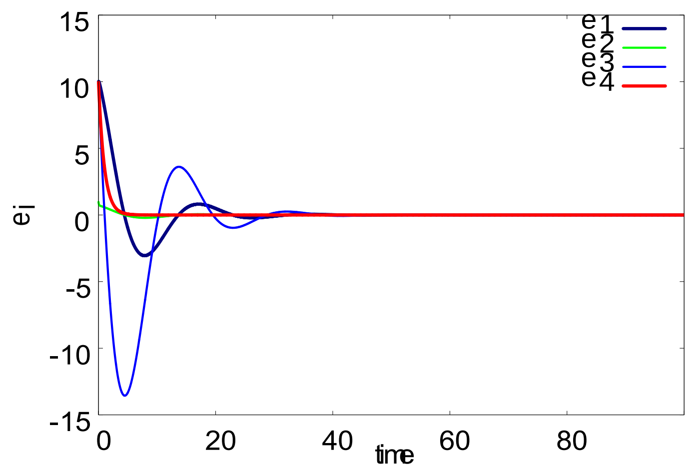


Figure 5: Asymptotic convergence of the error states to the synchronization manifold showing complete synchronization.

6.0 Concluding Remarks

In this paper,backstepping technique has been employed to achieve the global asymptotic stabilization of a hyperchaotic memristor circuit and synchronization ofmaster-slave chaotic memristor circuits, respectively. With suitable choice of controllers obtained via backstepping, the theoretical analysis and numerical simulations haveconfirmed that the system can be fully stabilized. In a master-slave configuration of two memristive circuits, appropriate control inputs would also drive the two systems into complete synchronization state. The control laws can be conveniently optimizedin order to realize minimal control inputs, thereby reducing the complexity involved in most nonlinear control approaches. Finally, we remark that the hyperchaotic orbits of the system maybe driven to a specified bounded point or projected to track a desired trajectory.

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