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Optimization of the Compressive Strength Characteristics of Concrete Mixes made with Crushed Granite Chippings Using the Second-Degree Polynomial

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Abstract

This research work set out to develop a model for the compressive strength characteristics of concrete made with crushed granite chippings based on the second-degree polynomial. The crushed granite chipping was from Abagana and the river sand from Amansea, both in AnambraState. These aggregates were tested for their physical and mechanical properties based on BS 812: Part 2 & Part 3:1975. Sixty concrete cubes of dimensions 150 mm X 150mm X 150mm—three cubes for each experimental point were made, cured and tested according to BS 1881:1983. A model equation was developed and the student's t-test and the Fisher's test were used to test the adequacy of this model. The strengths predicted by the model were in complete agreement with the experimentally obtained values and the null hypothesis was satisfied.

Keywords: Optimization, Concrete, Compressive strength, Second-degree polynomial.

1.0 Introduction

1.1 Osadebe's Concrete Optimization Theory

Concrete is a four-component material of mixing water, cement, fine and coarse aggregate. These ingredients are mixed in rational proportions to achieve desired strength of the hardened concrete [1]. Let us consider an arbitrary amount, S, of a given concrete mixture and S_i, the portion of the i^{th} component of the four constituent materials of the concrete where i = 1,2,3,4, then in keeping with the principle of absolute volume or mass [2]:

$$\sum S_i = S \qquad (1)$$

Dividing through by S and substituting Z_i for S_i/S gives:

$$\sum Z_i = 1 \qquad (2)$$

Then, the compressive strength of concrete can be expressed as:

Using Taylor's theorem and the assumption that Y is continuous, equation (3) becomes:

$$Y = f(Z_i) \qquad (3)$$

$$f(Z) = f\left(Z^{(0)}\right) + \sum_{i=1}^{4} \frac{\partial f\left(Z^{(0)}\right)}{\partial Z_i}\left(Z_i - Z_i^{(0)}\right) + \frac{1}{2!}\sum_{i=1}^{3}\sum_{j=1}^{4} \frac{\partial^2 f\left(Z^{(0)}\right)}{\partial Z_i Z_j}\left(Z^{(0)}\right)\left(Z_i - Z_i^{(0)}\right)\left(Z_j - Z_j^{(0)}\right)$$

$$+ \frac{1}{2!}\sum_{i=1}^{3} \frac{\partial^2 f\left(Z^{(0)}\right)}{\partial Z_i^2}\left(Z_i - Z_i^{(0)}\right)^2 + \ldots\ldots\ldots\ \ldots\ldots \qquad (4)$$

If $b_0 = f(0)$, $b_i = \partial f(0)/\partial Z_i$, $b_{ij} = \partial f^2(0)/\partial Z_i \partial Z_j$ and $b_{ii} = \partial^2 f(0)/\partial Z^2_i$, then eqn. (4) can be written as follows:

Multiplying eqn.(2) by b_0 we have

$$f(Z) = b_0 + \sum_{i=1}^{4} b_i Z_i + \sum_{i=1}^{3}\sum_{j=1}^{4} b_{ij} Z_i Z_j + \sum_{i=1}^{4} b_{ii} Z^2{}_i + \ldots\ldots\ldots\ \ldots \qquad (5)$$

$$b_0 Z_i = b_0 \qquad (6)$$

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Also, multiplying eqn.(2) by Z_1, Z_2, Z_3 and Z_4 in succession, making Z_1^2, Z_2^2, Z_3^2 and Z_4^2 the subject of the formula, substituting into eqn. (5) and factorizing gives:

$$Y = \sum \beta_i Z_i + \sum \beta_{ij} Z_i Z_j \qquad (7)$$

where $\beta_i = b_0 + b_i + b_{ii}$ and $\beta_{ij} = b_{ij} - b_{ii} + b_{jj}$ $\quad (i,\ j = 1,2,3,4)$

1.2 The Coefficients of the Regression Equation

If the Kth response (compressive strength for the serial number k) is $y^{(k)}$, substituting the vector of the corresponding set of variables, i.e., $Z^{(K)} = [Z_1^{(K)}, Z_2^{(K)}, Z_3^{(K)}, Z_4^{(K)}]^T$ (see Table 1) into eqn.(7) generates the explicit matrix of equation (8):

$$\left[y^{(k)}\right] = [B][Z] \qquad (8)$$

Re-arranging eqn.8 yields:

$$[Z]^T [B]^T = \left[y^{(k)}\right] \qquad (9)$$

Solution of eqn. (9) gives the values of the unknown coefficients of the regression equation (eqn. 7).

1.3 The Student's T-test

The unbiased estimate of the unknown variance $\boldsymbol{S}^2$ is given in [3],

$$S_Y^2 = \frac{\sum \left(y_i - \breve{y}\right)^2}{n-1} \qquad (10)$$

If $a_i = z_i (2z_i - 1)$, $a_{ij} = 4 z_i z_j$; for $(1 \le i \le q)$ and $(1 \le i \le j \le q)$ respectively.
Then,
where ε is the error of the predicted values of the response.
The t-test statistic is given in [3]

$$\varepsilon = \sum a^2{}_i + \sum a_{ij}{}^2 \qquad (11)$$

where $\Delta y = y_0 - y_t$; y_0 = observed value, y_t = theoretical value; n = number of replicate observations at every point; ε = as defined in eqn.(11).

1.4 The Fisher's Test

The Fishers-test statistic is given by

$$t = \left(\frac{\Delta y \sqrt{n}}{S_Y}\right)\sqrt{(1+\varepsilon)} \qquad (12)$$

The values of S_1(lower value)and S_2(upper value) are calculated from equation (10).

$$(13) \quad F = \frac{S_1^{\ 2}}{S^2{}_2}$$

2.0 Materials and Method

2.1 Preparation, Curing and Testing of Cube Samples

The aggregates were sampled in accordance with the methods prescribed in BS 812: Part 1:1975 [4]. The test sieves were selected according to BS 410:1986 [5]. The water absorption, the apparent specific gravity and the bulk density of the coarse aggregates were determined following the procedures prescribed in BS 812: Part 2: 1975 [6]. The Los Angeles abrasion test was carried out in accordance with ASTM. Standard C131: 1976 [7]. The sieve analyses of the fine and coarse aggregate samples were done in accordance with BS 812: Part 1: 1975 [4] and satisfied BS 882:1992[8]. The sieving was performed by a sieve shaker. The water used in preparing the experimental samples satisfied the conditions prescribed in BS 3148:1980 [9]. The required concrete specimens were made in threes in accordance with the method specified in BS 1881: 108:1983 [10]. These specimens were cured for 28 days in accordance with BS 1881: Part 111: 1983 [11]. The testing was done in accordance with BS 1881: Part 116:1983 [12] using compressive testing machine.

Table 1:Selected Mix Ratios and Component's Fraction Based on Osadebe's Second Degree Polynomial

	MIX RATIOS				**COMPONENT'S FRACTION**			
S/NO	**S_1**	**S_2**	**S_3**	**S_4**	**Z_1**	**Z_2**	**Z_3**	**Z_4**
1	0.88	1	2.5	4	0.105	0.119	0.298	0.477
2	0.86	1	2	4	0.109	0.127	0.254	0.509
3	0.855	1	2	3.5	0.116	0.136	0.272	0.476
4	0.86	1	2	3	0.125	0.146	0.292	0.437
5	0.855	1	2.5	3.5	0.109	0.127	0.318	0.446
6	0.865	1	3	4	0.098	0.113	0.338	0.451
7	0.87	1	3	4.5	0.093	0.107	0.320	0.480
8	0.86	1	1.5	3	0.135	0.157	0.236	0.472
9	0.86	1	2.75	3.4	0.107	0.125	0.343	0.424
10	0.865	1	2	4.25	0.107	0.123	0.246	0.524
			CONTROL					
11	0.858	1	2.43	4	0.104	0.121	0.293	0.483
12	0.86	1	1.75	3	0.130	0.151	0.265	0.454
13	0.855	1	2.4	3.5	0.110	0.129	0.309	0.451
14	0.86	1	2	4.33	0.105	0.122	0.244	0.529
15	0.862	1	2.25	3.13	0.119	0.138	0.311	0.432
16	0.858	1	2	2.83	0.128	0.150	0.299	0.423
17	0.858	1	2.67	3.29	0.110	0.128	0.342	0.421
18	0.86	1	3	4.13	0.096	0.111	0.334	0.459
19	0.855	1	2	3	0.125	0.146	0.292	0.438
20	0.8595	1	2.75	4	0.100	0.116	0.319	0.465

LEGEND: S_1= water/cement ratio; S_2=Cement; S_3=Fine aggregate; S_4=Coarse aggregate, $Z_i = S_i/S$

Table 2: Z^TMatrix

Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4
0.105	0.119	0.298	0.477	0.013	0.031	0.050	0.036	0.057	0.142
0.109	0.127	0.254	0.509	0.014	0.028	0.056	0.032	0.065	0.129
0.116	0.136	0.272	0.476	0.016	0.032	0.055	0.037	0.065	0.129
0.125	0.146	0.292	0.437	0.018	0.037	0.055	0.042	0.064	0.127
0.109	0.127	0.318	0.446	0.014	0.035	0.049	0.041	0.057	0.142
0.098	0.113	0.338	0.451	0.011	0.033	0.044	0.038	0.051	0.153
0.093	0.107	0.320	0.480	0.010	0.030	0.045	0.034	0.051	0.154
0.135	0.157	0.236	0.472	0.021	0.032	0.064	0.037	0.074	0.111
0.107	0.125	0.343	0.424	0.013	0.037	0.046	0.043	0.053	0.146
0.107	0.123	0.246	0.524	0.013	0.026	0.056	0.030	0.065	0.129

Table 3 Responses of the Mix Ratios

S/NO	S_1	S_2	S_3	S_4	*RESPONSES[N/mm²]*
1	0.88	1	2.5	4	19.21
2	0.86	1	2	4	20.00
3	0.855	1	2	3.5	21.23
4	0.86	1	2	3	22.24
5	0.855	1	2.5	3.5	21.08
6	0.865	1	3	4	14.95
7	0.87	1	3	4.5	14.15
8	0.86	1	1.5	3	24.19
9	0.86	1	2.75	3.4	21.59
10	0.865	1	2	4.25	19.57

LEGEND: S_1= water/cement ratio; S_2=Cement; S_3=Fine aggregate; S_4=Coarse aggregate

2.2 Testing the Fit of the Quadratic Polynomials

The polynomial regression equation developed was tested to see if the model agreed with the actual experimental results. The null hypothesis was denoted by H_0 and the alternative by H_1.

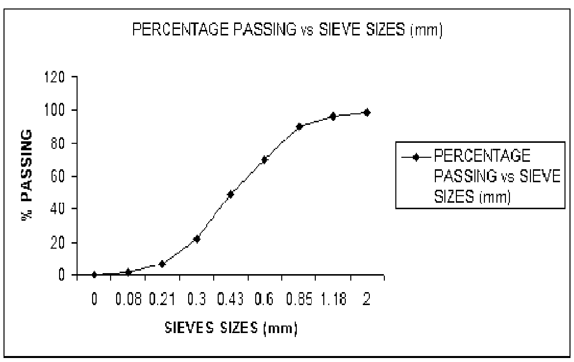

Figure 1:Grading Curve for the Fine Aggregate

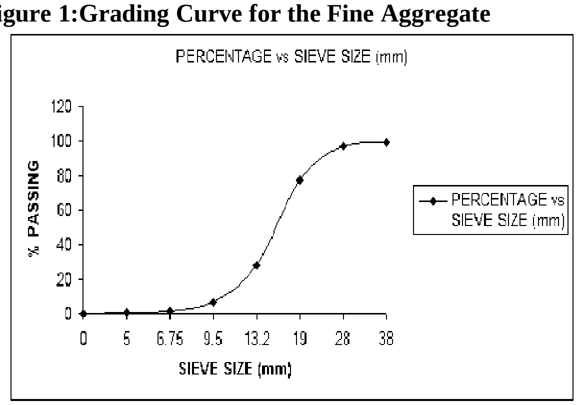

Figure 2:Grading Curve for the Crushed Granite Chippings

3. 0 Results and Discussion

3.1 Physical and Mechanical Properties of Aggregates

Sieve analyses of both the fine and coarse aggregates were performed and the grading curves shown in Figures 1 and 2. These grading curves showed the particle size distribution of the aggregates. The maximum aggregate size for the granite chipping was 20 mm and 2mm for the fine sand. The granite chippings had water absorption of 2.7%, moisture content of 44.2%, apparent specific gravity of 2.26, Los Angeles abrasion value of 22% and bulk density of 2072.4 kg/m^3.

3.2 The Regression Equation for The Compressive Strength Tests Results

Solution of Eqn.(9), given Z^T values of Table 2 and the responses (average compressive strengths) in Table 3 gave the values of the unknown coefficients of the regression equation (Eqn.7) as follows: β_1= -2878.52, β_2 = 1989.71, β_3 = -133.53, β_4 = -160.83, β_{12}= 135.92, β_{13}= -13202.90, β_{14}= 4739.91, β_{23}= 11461.35, β_{24}= -2309.10, β_{34} = 638.92. Thus, from Eqn.(7), the model equation based on Osadebe's second-degree polynomial was given by:

$$\hat{Y} = -2878.52Z_1 + 1989.71Z_2 - 133.53Z_3 - 160.83Z_4 + 135.92Z_1Z_2 - 13202.90Z_1Z_3 + 4739.91Z_1Z_4 + 11461.35Z_2Z_3 - 2309.10Z_2Z_4 + 638.92Z_3Z_4 \quad (14)$$

, where $\hat{Y}$ represented the compressive strength of the mixture in N/mm^2.

3.3 Fit of the Polynomial

Selected mix ratios and component's fraction based on Osadebe's second degree polynomial was shown in Table 1. The polynomial regression equation developed was tested to see if the model agreed with the actual experimental results. There was no significant difference between the experimental and the theoretically expected results. The null hypothesis, H_0 was satisfied.

Table 4 t –Statistic for the controlled points, granite chippings concrete compressive test, based on Osadebe's

ResponseSymbol	**i**	**j**	**a_i**	**a_{ij}**	**a_i^2**	**a_{ij}^2**	**ε**	**ÿ**	**Ŷ**	**t**
	1	2	-0.082	0.050	0.007	0.003	0.4835	23.20	23.88	-2.43
	1	3	-0.082	0.121	0.007	0.015				
	1	4	-0.082	0.120	0.007	0.0399				
	2	3	-0.092	0.142	0.01	0.020				
	2	4	-0.092	0.233	0.013	0.0543				
	3	4	-0.121	0.566	0.016	0.3202				
	4	—	-0.017	—	0.001	—				
				∑	0.052	0.4316				
	Similarly									
C_2	—	—	—	—	—	—	0.4809	22.64	22.61	0.11
C_3	—	—	—	—	—	—	0.9234	25.35	26.11	-3.10
C_4	—	—	—	—	—	—	0.4642	23.49	24.23	-2.63
C_5	—	—	—	—	—	—	0.5053	18.58	18.38	0.71
C_6	—	—	—	—	—	—	0.4966	23.77	24.33	-2.03
C_7	—	—	—	—	—	—	0.5707	19.1	18.69	1.52
C_8	—	—	—	—	—	—	0.5624	19	18.29	2.61
C_9	—	—	—	—	—	—	0.4949	23.44	24.01	-2.04
C_{10}	—	—	—	—	—	—	0.5236	20.46	20.72	-0.94

Second –Degree Polynomial

LEGEND: c_i =response; $a_i = z_i\,(2z_i - 1)$; $a_{ij} = 4\,z_i\,z_j$; $\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij}$; ÿ = experimentally-observed value; Ŷ= theoretical value; t = t-test statistic

3.4 t -Value From Table

The student's t-test had a significance level, α = 0.05 and $t_{\alpha/l(ve)} = t_{0.005(9)}$=3.69 from the standard table [13]. This was greater than any of the t values calculated in Table 4. Therefore, the regression equation for the unwashed gravel concrete was adequate.

3.5 F-Statistic Analysis

The sample variances S_1^2 and S_2^2 for the two sets of data were not significantly different (Table 5). It implied that the error(s) from experimental procedure were similar and that the sample variances being tested are estimates of the same population variance. Based on eqn.(10), we had that S_K^2 = 51.88/9 = 5.76, S_E^2 = 74.5/9 = 8.28 & F = 5.76/8.28 = 0.696. From Fisher's table [13], $F_{0.95(9,9)}$ = 3.3, hence the regression equation for the compressive strength of the unwashed gravel concrete was adequate.

Table 5 F –Statistic For The Controlled Points, Granite Concrete Compressive Strength, Based On Osadebe's Second –Degree Polynomial

Response symbol	Y_K	Y_E	Y_K- $\bar{Y}_K$	Y_E-$\bar{Y}_E$	$(Y_K$- $\bar{Y}_K)^2$	$(Y_E$-$\bar{Y}_E)^2$
C_1	23.2	23.88	1.30	1.75	1.68	3.07
C_2	22.64	22.61	0.74	0.49	0.54	0.24
C_3	25.35	26.11	3.45	3.98	11.88	15.87
C_4	23.49	24.23	1.59	2.10	2.52	4.42
C_5	18.58	18.38	-3.32	-3.74	11.04	13.99
C_6	23.77	24.33	1.87	2.21	3.49	4.88
C_7	19.1	18.69	-2.80	-3.44	7.86	11.82
C_8	19	18.29	-2.90	-3.83	8.43	14.69
C_9	23.44	24.01	1.54	1.88	2.36	3.54
C_{10}	20.46	20.72	-1.44	-1.41	2.08	1.98
$\sum$	219.03	221.25			51.88	74.5

Legend: $\bar{Y}=\Sigma Y/n$ where y is the response and n, the number of observed data (responses)
Y_k is the experimental value (response)
Y_E is the expected or theoretically calculated value(response)

3.6 Conclusion

The strengths (responses) of concrete were a function of the proportions of its ingredients: water, cement, fine aggregate and coarse aggregates. Since the predicted strengths by the model were in total agreement with the corresponding experimentally -observed values, the null hypothesis was satisfied. This meant that the model equation was valid.

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