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Dynamic Debt Optimization Strategy in an Investment Portfolio

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Abstract

This paper study a dynamic debt optimization problem in an investment portfolio over a finite time horizon. Some group of individuals lend money from an investor which is to be paid back with interest at a stipulated time. We suppose that some debtors may failed to pay back part or the whole amount of money borrowed at the agreed time. We referred to such amount as bad debt. It is also assume that part of the bad debt will be recovered (recovered bad debt). The aim of this paper are to: provide basic background on dynamic programming in relation to debt managements; determine the possible debt thatwill accrual to a borrower at the stipulated time; and minimize the incident of bad debt on the part of a borrowee (debtor). The following were the findings: the incident of bad debt is capable of discouraging investment; when the unrecovered bad debt increases monotonically, the expected return from the investment portfolio will be monotone decreasing; when the interest rate is strictly monotone increasing, the expected return from the investment will be strictly monotone increasing; as the time period of lending increases, the possible chances of bad debt increases; and the expected returns in the investment portfolio follow exponential growth.

Keywords: Dynamic debt, optimization strategy, investment portfolio, expected returns, exponential growth, bad debt, borrowee, debtor.

1.0 Introduction

The tremendous increase in computation power over the last few decades has substantially enhanced ability to solve complex problems with their performance evaluations. In diverse area of science and engineering with the recent development in the field of optimization, these methods are now becoming lucrative inenhancing decision making processes. Dynamic programming (DP) is one of the algorithms designed to obtain solution one after another and is a powerful tool which yields classic algorithm for a variety of combinational optimization problems. In this paper, we intend to extend the algorithm to debt management. Debt managementis a multiphase concept that involved debt strategy, debt contraction, recording monitory and servicing. It is importance to note that debt management is dynamic and important for advanced and developing countries, such as Nigeria. The policies of debt management include debt management objectives, borrowing process, borrowing limit, used of funds, risk management, debt database management, critical risk management, risk of the government debt portfolio and strategy. The debt managers are responsible for the financing strategy which involves complex task of choosing a strategy that minimizes the cost of the debt portfolio within certain risk limit.

The issue of a dynamic programming approach in relation to debt management has not been study in the literature to the best of my knowledge. Hence, the need for this research work.

The work is gear towards achieving the following objectives: to(i) provide basic background to dynamic programming in relation to debt managements;(ii) determine the possible debt thatwill accrual to a borrower at time t and(iii) minimize the incident of bad debt in the part of a borrowee .

This paper will play sigmificant role in financial institutions and practitioners alike to manage debts and thereby reduce the incident of bad debt in an investment portfolios. It will also help the borrower to know the value of the debt that will accrued to him/her over time.

There have been extensive literatures that exist in the area of dynamic programming and debt management. Creativity is necessary before one can distinguish that a particular problem can be casted effectively as a dynamic program. Even clever insights to restructure the information often are essential in useful solution,[1, 2]. The idea of reasoning sub problems is the main advantage of the dynamic programming paradigm over recursion. The simplicity is what makes dynamic programming more appealing in both a full problem solving method and a subroutine solver. In more complicated algorithm solution [3, 4].

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Dynamic programming is so powerful tools that encourage tremendous growth in researches for solving sequential decision problems and research related to dynamic programming has lead to fundamental advance in theory, numerical methods, economics and now debt management. Thus, dynamic programming can be sighted as a useful "first approximation" to human decision making. But, it will undoubtedly in near future be old-fashioned by more descriptively accurate psychological models [5].

Ayadi [6] designed and implemented a dynamic program for valuing corporate debt portfolios and computing its term structure of default probabilities. They provided several theoretical properties of the debt and equity value functions. Lars and Semmler[7] considered the pricing and control of firm's debt as a major issue since Merton's (1974) seminal paper. They considered using debt finance firms to pay a premium for an idiosyncratic default risk which may face debt constraints. They further solved for asset value of the firm with debt finance by using numerical dynamic programming. Nkeki [9] considered the use of DP techniques for resource allocation problems in economics, finance and pension fund management.

In this paper, we consider dynamic debt optimization problem for an investment portfolio using dynamic programming techniques.

The remainder of this paper is organized as follows. Section 2 presents the optimization model and the objective function. The dynamic programming formulation in relation to debt management is presented in section 3. In section 4, we presents the numerical computation. Finally, section 5 concludes the paper.

2.0 The Model

In this section, we consider the model under consideration. The model dealt with the management of debt using DP techniques. We now define the notations.

2.1 Notations

$D-$ State space.

$\alpha-$ Discount factor, $0<\alpha\leq 1$.

D_0- Aggregate initial capital to be borrowed out by the investor.

D_t^i- Amount accrued to the investor's portfolio from debtor i at time t.

$P_{it}-$ Probability of bad debt arising from debtor i at time t.

$\lambda-$ Proportion of recovered bad debt.

$T-$ The final time to pay up the debt.

$r-$ Interest rate.

$n-$ Number of debtors.

ϕ_t^i- The capital function allocated to debtor i at time t.

2.2 The Objective Function of the Investment Portfolio

In this subsection, we presents the dynamic programming for resource allocation problem. We consider an investor that has D amount of naira to allocate to a series of n borrowers with interest in order to maximize return. The aim of the investor is to maximize her total return of her portfolio $G(t)$ at time t.

$$G(t)=E_t\left[\sum_{t=0}^{T}\sum_{i=1}^{n}\alpha^t\phi_t^t(D_t^i)\right], \qquad (1)$$

subject to the following constraints:

$$D_t^i = D_0^i(1+r)^t-(1-\lambda)P_{it}D_t^i,\ i=1,2,...,n;\ t=0,1,2,...,T, \qquad (2)$$

$$D_0^i\leq D_t^i, \qquad (3)$$

$$D_0\leq D_t. \qquad (4)$$

3.0 Dynamic Programming Formulation in Relation to Debt Management.

Consider any admissible policy $\pi=\{\pi_0,\pi_1,...,\pi_n\}$ any positive integer, T and any function $F:D\rightarrow\Re$ .Suppose that we accumulate the returns of the first T stages, and to them we add the terminal return

$$\alpha^T F(D_T)=\sum_{i=1}^{n}\alpha^T\phi_T^i(D_T),$$

we have a total expected return: $E\left[\alpha^T F(D_T)+\sum_{t=0}^{T}\sum_{i=1}^{n}\alpha^t \phi_T^i(D_t^i)\right]$.

The maximum of this return over π can be calculated by starting with $\alpha^T F(D_T)$ and by carrying out T iterations of the corresponding DP algorithm

$$F_{T-t}(D_t)=\max_{D_t\in D} E\left[\sum_{i=1}^{n}\alpha^{T-t}\phi_t^i(D_t^i)+F_{T-t+1}(D_{t-1})\right], t\in[0,T], \qquad (5)$$

within the initial condition

$$F_T(D_0)=\alpha^T F(D_0). \qquad (6)$$

We consider for all t, and D, the function F_t given by $F_t(D_t)=\frac{F_{T-t}(D_t)}{\alpha^{T-t}}$.

Then, $F_T(D)$ is the optimal $T-$ stage return $F_0(D)$, while the dynamic programming recursion Eq.(5) can be equivalently be written in terms of the function F_t as

$$F_t(D_t)=\max_{D_t\in D} E\left[\sum_{i=1}^{n}\phi_t^i(D_t^i)+\alpha F_{t-1}(D_{t-1})\right], t\in[0,T], \qquad (7)$$

with the initial condition $F_0(D_0)=F(D_0)$. Note that Eq.(1) can be shown to be equivalent to Eq.(7).For details see Nkeki [8], [9] and [10].

4.0 Numerical Computation

In this section, we consider the numerical implication of our model. Table 1 shows the amounts borrowed out to people which are to be paid back with interest to the investor (creditor) after an agreed period of time. We assume that the bond follows a compounded interest rate. We also assume that there will be bad debt from the debtors (or borrowees). Certain percentage of the accumulating returns of the investor are set aside as bad debt. We further assume that some proportions of the bad debt may be recovered, which we referred to as recovered bad debt and the unrecovered ones are referred to as unrecovered bad debt.

Table 1: Capital Borrowed by the Debtors from the Investment Portfolio and Probability of Bad Debt

Debtor	D^1 $\times 1000$	D^2 $\times 1000$	D^3 $\times 1000$	D^4 $\times 1000$	D^5 $\times 1000$	D^6 $\times 1000$	D^7 $\times 1000$	D^8 $\times 1000$
Amount	100	120	80	200	110	190	300	150
Prob. of Bad Debt (P_{it})	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{2}{35}$	$\frac{2}{15}$	$\frac{1}{11}$	$\frac{3}{20}$	$\frac{1}{16}$	$\frac{2}{13}$

D^i stand for Debtor i.

From Table 1, we have that the amount of bad debt for the ith individual debtor is given by $P_{it}D_t^i$, $t=0,1,2,...,8,$ where P_{it} is the probability of bad debt for the ith individual debtor at time period t. Note that $P_{i0}=0,$ since there is no bad debt at the initial time period. Hence, the total bad debt is given by $\sum_{i=1}^{8}P_{it}D_t^i$, $t=0,1,2,...,T$. Let λ be the proportion of recovered bad debt, so that $\lambda\sum_{i=1}^{8}P_{it}D_t^i$, $t=0,1,2,...,8,$ will be amount recovered from the bad debt at time t. It therefore implies that $(1-\lambda)\sum_{i=1}^{8}P_{it}D_t^i$, $t=0,1,2,...,T$, is the amount of unrecovered bad debt.

Let $D_0=\sum_{i=1}^{8}D_0^i$ be the total capital borrowed out to debtors at the beginning of the planning horizon. Let $D_t^i=D_0^i(1+r)^t-\beta P_{it}D_t^i$, $i=1,2,...,n;$ $t=0,1,2,...,8$ be the return from debtor i at time t, where $\beta=1-\lambda.$ Then,

$$F_T(D_t^i)=D_0^i+F_{T-1}(D_t^i), \text{ where } F_0=0. \qquad (8)$$

MatLab Program was used to solve Eq.(8) and the results are presented in the following tables and figures.

Table 2: Expected Return from the Investment Portfolio, for $r = 5\%$ and $\beta = 0\%$

Time (in year)	**RD 1** $\times 10^5$ **(Naira)**	**RD 2** $\times 10^5$ **(Naira)**	**RD 3** $\times 10^5$ **(Naira)**	**RD 4** $\times 10^5$ **(Naira)**	**RD 5** $\times 10^5$ **(Naira)**	**RD 6** $\times 10^5$ **(Naira)**	**RD 7** $\times 10^5$ **(Naira)**	**RD 8** $\times 10^5$ **(Naira)**
0	1.0000	1.2000	0.8000	2.0000	1.1000	1.9000	3.0000	1.5000
1	1.0500	1.2600	0.8400	2.1000	1.1550	1.9950	3.1500	1.5750
2	1.1025	1.3230	0.8820	2.2050	1.2128	2.0947	3.3075	1.6538
3	1.1576	1.3892	0.9261	2.3153	1.2734	2.1995	3.4729	1.7364
4	1.2155	1.4586	0.9724	2.4310	1.3371	2.3095	3.6465	1.8233
5	1.2763	1.5315	1.0210	2.5526	1.4039	2.4249	3.8288	1.9144
6	1.3401	1.6081	1.0721	2.6802	1.4741	2.5462	4.0203	2.0101
7	1.4071	1.6885	1.1257	2.8142	1.5478	2.6735	4.2213	2.1107
8	1.4775	1.7729	1.1820	2.9549	1.6252	2.8072	4.4324	2.2162
9	1.5513	1.8616	1.2411	3.1027	1.7065	2.9475	4.6540	2.3270
10	1.6289	1.9547	1.3031	3.2578	1.7918	3.0949	4.8867	2.4433

RD stand for Returns from Debtor.

Note that in each of the bar charts in all the Figures, RD1 is the first bar, RD2 the second bar, RD3 third bar and so on, till RD8 is the last bar.

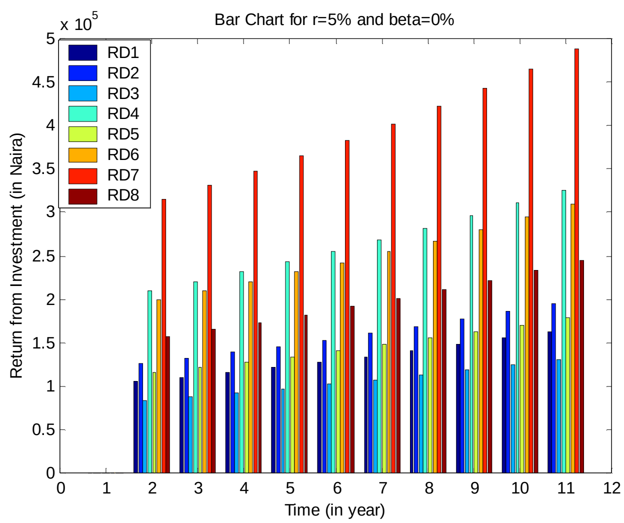

Figure 1: Bar Chart Representing the Return from the Investment for $r = 5\%$ and $\beta = 0\%$.

4.1 Discussion

Table 2 and Figure 1 show the expected return for the investor from the investment after 10years.In Table 2, we found that in the first year RD.1 amounted from 100,000 to 105,000 Naira, which shows an expected interest of 8000 Naira, in the second year, the amount rises from 100,000-110,250 Naira with additional amount of 10,250 Naira. At 10th year, the capital yielded interest of about 62,890 Naira. Similar discussions go for the RD2-RD8.

Table 3: Expected Return from the Investment Portfolio, for $r = 5\%$ and $\beta = 10\%$

Time (in year)	**RD 1** $\times 10^5$ **(Naira)**	**RD 2** $\times 10^5$ **(Naira)**	**RD 3** $\times 10^5$ **(Naira)**	**RD 4** $\times 10^5$ **(Naira)**	**RD 5** $\times 10^5$ **(Naira)**	**RD 6** $\times 10^5$ **(Naira)**	**RD 7** $\times 10^5$ **(Naira)**	**RD 8** $\times 10^5$ **(Naira)**
0	1.0000	1.2000	0.8000	2.0000	1.1000	1.9000	3.0000	1.5000
1	1.0500	1.2600	0.8400	2.1000	1.1550	1.9950	3.1500	1.5750
2	1.0920	1.3230	0.8820	2.2050	1.2128	2.0947	3.3075	1.6538
3	1.1467	1.3892	0.9261	2.3153	1.2734	2.1995	3.4729	1.7364
4	1.2040	1.4586	0.9724	2.4310	1.3371	2.3095	3.6465	1.8233
5	1.2642	1.5315	1.0210	2.5526	1.4039	2.4249	3.8288	1.9144
6	1.3275	1.6081	1.0721	2.6802	1.4741	2.5462	4.0203	2.0101
7	1.3938	1.6885	1.1257	2.8142	1.5478	2.6735	4.2213	2.1107
8	1.4635	1.7729	1.1820	2.9549	1.6252	1.6252	4.4324	2.2162
9	1.5367	1.8616	1.2411	3.1027	1.7065	2.9475	4.6540	2.3270
10	1.6135	1.9547	1.3031	3.2578	1.7918	3.0949	4.8867	2.4433

RD stand for Returns from Debtor.

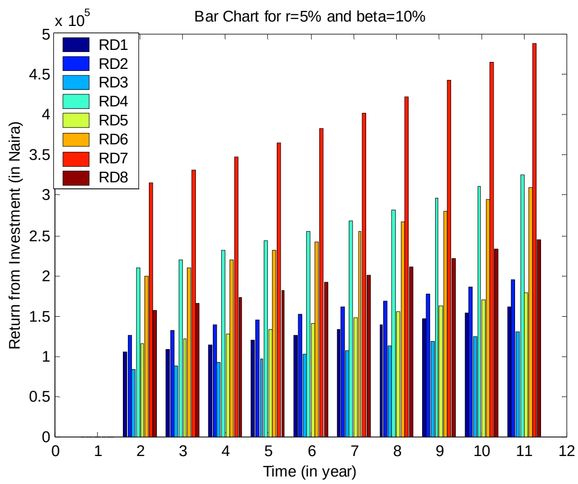

Figure 2: Bar Chart Representing the Return from Investment for $r = 5\%$ and $\beta = 10\%$

Table 3 and Figure 2 show the expected return for the investor from the investment after 10years. It was found that in the first year RD.1 amounted from 100,000 to 105,000 Naira which shows an expected interest of 5000 Naira, in the second year, the amount rises from 100,000 to 109,200 Naira with additional amount of 9,200 Naira in the third year the amount rises from 100,000-114,670 Naira, with additional amount of 14,670 Naira.At 10th year, the capital yielded an interest of about 61,350 Naira. Similar discussions go for the RD2-RD8 as well.

Table 4: Expected Return from the Investment for $r = 5\%$ **and** $\beta = 30\%$

Time (in year)	**RD 1** $\times 10^5$ **(Naira)**	**RD 2** $\times 10^5$ **(Naira)**	**RD 3** $\times 10^5$ **(Naira)**	**RD 4** $\times 10^5$ **(Naira)**	**RD 5** $\times 10^5$ **(Naira)**	**RD 6** $\times 10^5$ **(Naira)**	**RD 7** $\times 10^5$ **(Naira)**	**RD 8** $\times 10^5$ **(Naira)**
0	1.0000	1.2000	0.8000	2.0000	1.1000	1.9000	3.0000	1.5000
1	1.0500	1.2600	0.8400	2.1000	1.1550	1.9950	1.9950	1.5750
2	1.0710	1.3230	0.8820	2.2050	1.2128	2.0947	3.3075	1.6538
3	1.1255	1.3892	0.9261	2.3153	1.2734	2.1995	3.4729	1.7364
4	1.1817	1.4586	0.9724	2.4310	1.3371	2.3095	3.6465	1.8233
5	1.2408	1.5315	1.0210	2.5526	1.4039	2.4249	3.8288	1.9144
6	1.3029	1.6081	1.0721	2.6802	1.4741	2.5462	4.2213	2.0101
7	1.3680	1.6885	1.1257	2.8142	1.5478	2.6735	4.2213	2.1107
8	1.4364	1.7729	1.1820	2.9549	1.6252	2.8072	4.4324	2.2162
9	1.5082	1.8616	1.2411	3.1027	1.7065	2.9475	4.6540	2.3270
10	1.5836	1.9547	1.3031	3.2578	1.7918	3.0949	4.8867	2.4433

RD stand for Returns from Debtor.

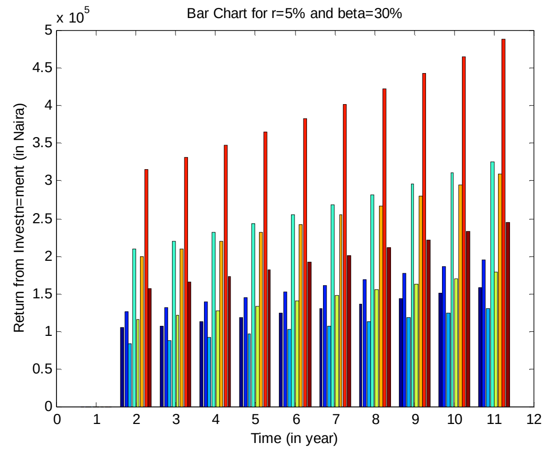

Figure 3: Bar Chart Representing the Return from Investment for $r = 5\%$ and $\beta = 30\%$

Table 4 and Figure 3 show the expected return of the investor from the investment after 10years, for the values of $r = 0.05$ and $\beta = 0.3$. In Table 4, we found that in the first year RD.1 amounted from 100,000 to 105,000 Naira which shows an expected interest of 5000 Naira.In the second year, the amount rises from 100,000 to 107,100 Naira with additional amount of 7,100 Naira. In the third year, the amount rises from 100,000-112,550 Naira with additional amount of 12,550 Naira.At 10th year, the capital yielded interest of about 58,360 Naira, similar discussions go for RD2-RD8.

Table 5: Expected Return from the Investment Portfolio, for $r = 5\%$ and $\beta = 80\%$

Time (in year)	**RD 1** $\times 10^5$ **(Naira)**	**RD 2** $\times 10^5$ **(Naira)**	**RD 3** $\times 10^5$ **(Naira)**	**RD 4** $\times 10^5$ **(Naira)**	**RD 5** $\times 10^5$ **(Naira)**	**RD 6** $\times 10^5$ **(Naira)**	**RD 7** $\times 10^5$ **(Naira)**	**RD 8** $\times 10^5$ **(Naira)**
0	1.0000	1.2000	0.8000	2.0000	1.1000	1.9000	3.0000	1.5000
1	1.0500	1.2600	0.8400	2.1000	1.1550	1.9950	3.1500	1.5750
2	1.0185	1.3230	0.8820	2.2050	1.2128	2.0947	3.3075	1.6538
3	1.0761	1.3892	0.9261	2.3153	1.2734	2.1995	3.4729	1.7364
4	1.1294	1.4586	0.9724	2.4310	1.3371	2.3095	3.6465	1.8233
5	1.1859	1.5315	1.0210	2.5526	1.4039	2.4249	3.8288	1.9144
6	1.2452	1.6081	1.0721	2.6802	1.4741	2.5462	4.0203	2.0101
7	1.3075	1.6885	1.1257	2.8142	1.5478	2.6735	4.2213	2.1107
8	1.3729	1.7729	1.1820	2.9549	1.6252	2.8072	4.4324	2.2162
9	1.4415	1.8616	1.2411	3.1027	1.7065	2.9475	4.6540	2.3270
10	1.5136	1.9547	1.3031	3.2578	1.7918	3.0949	4.8867	2.4433

RD stand for Returns from Debtor.

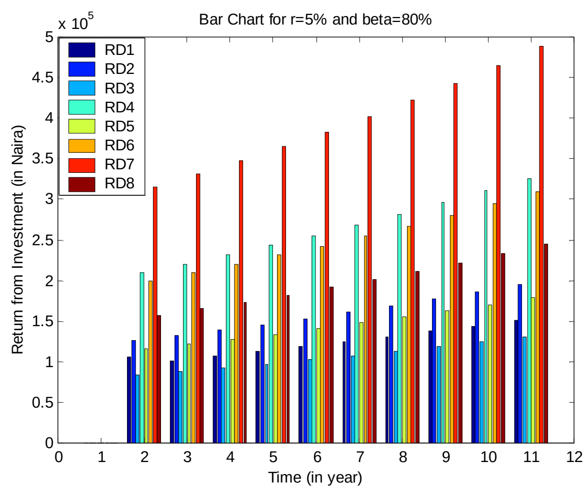

Figure 4: Bar Chart Representing the Return from Investment for $r = 5\%$ and $\beta = 80\%$

Table 5 and Figure 4 give the expected returns of the investor from the investment after 10years, for the values of $r = 0.05$ and $\beta = 0.8$. In Table 5, we found that in the first year, RD1, amounted from 100,000 to 105,000 Naira, which shows an expected interest of 5000 Naira, in the second year, the amount rises from 100,000-101,850 Naira with additional interest of 1,850 Naira, in the third, the amount rises from 100,000-107,610 Naira with additional interest of 7,610 Naira, in the forth year, the amount rises from 100,000-112,940 Naira with additional interest of about 51,360 Naira, similar discussionsalso go for the RD2-RD8.

Table 6: Expected Return from the Investment Portfolio, for $r = 5\%$ and $\beta = 100\%$

Time (in year)	**RD 1** $\times 10^5$ **(Naira)**	**RD 2** $\times 10^5$ **(Naira)**	**RD 3** $\times 10^5$ **(Naira)**	**RD 4** $\times 10^5$ **(Naira)**	**RD 5** $\times 10^5$ **(Naira)**	**RD 6** $\times 10^5$ **(Naira)**	**RD 7** $\times 10^5$ **(Naira)**	**RD 8** $\times 10^5$ **(Naira)**
0	1.0000	1.2000	0.8000	2.0000	1.1000	1.9000	3.0000	1.5000
1	1.0500	1.2600	0.8400	2.1000	1.1550	1.9950	3.1500	1.5750
2	1.0975	1.3230	0.8820	2.2050	1.2128	2.0947	3.3075	1.6538
3	1.0579	1.3892	0.9261	2.3153	1.2734	2.1995	3.4729	1.7364
4	1.1097	1.4586	0.9724	2.4310	1.3371	2.3095	3.6465	1.8233
5	1.1653	1.5315	1.0210	2.5526	1.4039	2.4249	3.8288	1.9144
6	1.2236	1.6081	1.0721	2.6802	1.4741	2.5462	4.0203	2.0101
7	1.2847	1.6885	1.1257	2.8142	1.5478	2.6735	4.2213	2.1107
8	1.3490	1.7729	1.1820	2.9549	1.6252	2.8072	4.4324	2.2162
9	1.4164	1.8616	1.2411	3.1027	1.7065	2.9475	4.6540	2.3270
10	1.4873	1.9547	1.3031	3.2578	1.7918	3.0949	4.8867	2.4433

RD stand for Returns from Debtor.

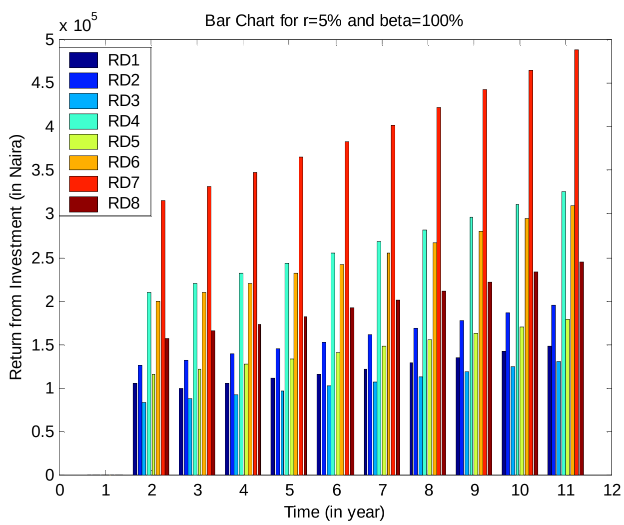

Figure 5: Bar Chart Representing Expected Return from the Investment Portfolio, for $r = 5\%$ **and** $\beta = 100\%$

Table 6 and Figure 5 give expect returns of the investor from the investment after 10years taking $r = 0.05$ and $\beta = 1.00$. It was found that in the first year RD.1 amounted from 100,000 Naira to 105,000 Naira which shows an expected interest of 5000 Naira.In the second year, the amount rises 100,000 Naira to 109750 Naira. This shows that interest of about 9750 Naira has been accrued to the investor. At 10th year, we have that interest of about 48730 Naira accrued to the investor.

We therefore conclude from Tables 2 - 6 and Figures 1 - 5 that when the unrecovered bad debt increases monotonically, the expected return from the investment will be monotone decreasing.

Table 7: Expected Return from the Investment Portfolio, for $r = 10\%$ and $\beta = 0\%$

Time (in year)	RD 1 $\times 10^5$ (Naira)	RD 2 $\times 10^5$ (Naira)	RD 3 $\times 10^5$ (Naira)	RD 4 $\times 10^5$ (Naira)	RD 5 $\times 10^5$ (Naira)	RD 6 $\times 10^5$ (Naira)	RD 7 $\times 10^5$ (Naira)	RD 8 $\times 10^5$ (Naira)
0	1.0000	1.2000	0.8000	2.0000	1.1000	1.9000	3.0000	1.5000
1	1.1000	1.3200	0.8800	2.2000	1.2100	2.0900	3.3000	1.6500
2	1.2100	1.4520	0.9680	2.4200	1.3310	2.2990	3.6300	1.8150
3	1.3310	1.5972	1.0648	2.6620	1.4641	2.5289	3.9930	1.9965
4	1.4641	1.7569	1.1713	2.9282	1.6105	2.7818	4.3923	2.1962
5	1.6105	1.9326	1.2884	3.2210	1.7716	3.0600	4.8315	2.4158
6	1.7716	2.1259	1.4172	3.5431	1.9487	3.3660	5.3147	2.6573
7	1.9487	2.3385	1.5590	3.8974	2.1436	3.7026	5.8462	2.9231
8	2.1436	2.5723	1.7149	4.2872	2.3579	4.0728	6.4308	3.2154
9	2.3579	2.8295	1.8864	4.7159	2.5937	4.4801	7.0738	3.5369
10	2.5937	3.1125	2.0750	5.1875	2.8531	4.9281	7.7812	3.8906

RD stand for Returns from Debtor.

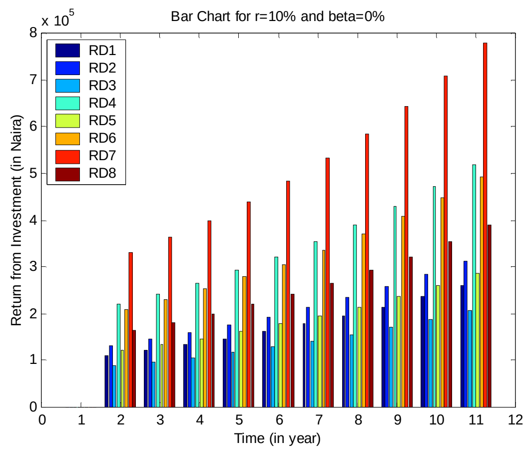

Figure 6: Bar Chart Representing the Return from Investment for $r = 10\%$ and $\beta = 0\%$

Table 7 and Figure 6 show the expected returns of the investor from the investment after 10years for taking the values of $r = 0.10$ and $\beta = 0.00$. Weobserved that in the first year, RD1 amounted from 100,000 to 110,000 Naira which shows an expected interest of 10,000 Naira, in the second year the amount rises from 100,000-121,000 Naira with additional amount of 21,000 Naira. In the third year the amount rises from 100,000-133,100 Naira with additional amount of 33,100 Naira.At10th year, the capital yielded interest of about 159,370 Naira, similar discussions go for the RD2-RD8.

Table 8: Expected Return from the Investment Portfolio, for $r = 10\%$ and $\beta = 40\%$

Time (in year)	RD 1 $\times 10^5$ (Naira)	RD 2 $\times 10^5$ (Naira)	RD 3 $\times 10^5$ (Naira)	RD 4 $\times 10^5$ (Naira)	RD 5 $\times 10^5$ (Naira)	RD 6 $\times 10^5$ (Naira)	RD 7 $\times 10^5$ (Naira)	RD 8 $\times 10^5$ (Naira)
0	1.0000	1.2000	0.8000	2.0000	1.1000	1.9000	3.0000	1.5000
1	1.1000	1.3200	0.8800	2.2000	1.2100	2.0900	3.3000	1.6500
2	1.1660	1.4520	0.9680	2.4200	1.3310	2.2990	3.6300	1.8150
3	1.2844	1.5972	1.0648	2.6620	1.4641	2.5289	3.9930	1.9965
4	1.4127	1.7569	1.1713	2.9282	1.6105	2.7818	4.3923	2.1962
5	1.5540	1.5540	1.2884	3.2210	1.7716	3.0600	4.8315	2.4158
6	1.7094	2.1259	1.4172	3.5431	1.9487	3.3660	5.3147	2.6573
7	1.8803	2.3385	1.5590	3.8974	2.1436	3.7026	5.8462	2.9231
8	2.0684	2.5723	1.7149	4.2872	2.3579	4.0728	6.4308	3.2154
9	2.2752	2.8295	1.8864	4.7159	2.5937	4.4801	7.0738	3.5369
10	2.5027	3.1125	2.0750	5.1875	2.8531	4.9281	7.7812	3.8906

RD stand for Returns from Debtor.

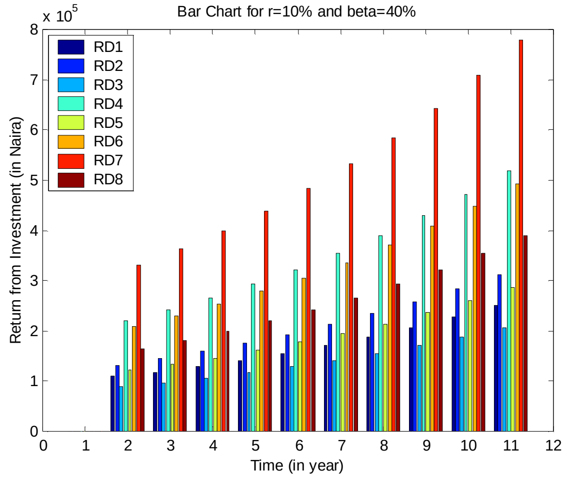

Figure 7: Bar Chart Representing Expected Return from the Investment Portfolio, for $r = 10\%$ and $\beta = 40\%$

Table 8 and Figure 7give the expected returns of the investor from the investment portfolio after 10years by taking $r = 0.10$ and $\beta = 0.40$. We found that in the first year, RD1 amounted from 100,000 Naira to 116,000 Naira which shows an expected interest of 16,000 Naira. In the second year, the amount rises from 100,000-128,440 Naira.At 10th year the capital yield interest of about 150,270 Naira, discussion for RD2-RD8 follow the same agreements.

Table 9: Return from the Investment Portfolio, for $r = 15\%$ and $\beta = 40\%$

Time (in year)	RD 1 $\times 10^5$ (Naira)	RD 2 $\times 10^5$ (Naira)	RD 3 $\times 10^5$ (Naira)	RD 4 $\times 10^5$ (Naira)	RD 5 $\times 10^5$ (Naira)	RD 6 $\times 10^5$ (Naira)	RD 7 $\times 10^5$ (Naira)	RD 8 $\times 10^5$ (Naira)
0	0.1000	1.2000	0.8000	2.0000	1.1000	1.9000	3.0000	1.5000
1	0.1150	0.1380	0.0920	0.2300	0.1265	0.2185	0.3450	0.1725
2	0.1276	0.1587	0.1058	0.2645	0.1455	0.2513	0.3967	0.1984
3	0.1470	0.1825	0.1217	0.3042	0.1673	0.2890	0.4563	0.2281
4	0.1690	0.2099	0.1399	0.3498	0.1924	0.3323	0.5247	0.2624
5	0.1944	0.2414	0.1609	0.4023	0.2212	0.3822	0.6034	0.3017
6	0.2235	0.2776	0.1850	0.4626	0.2544	0.4395	0.6939	0.3470
7	0.2571	0.3192	0.2128	0.5320	0.2926	0.5054	0.7980	0.3990
8	0.2956	0.3671	0.2447	0.6118	0.3365	0.5812	0.9177	0.4589
9	0.3400	0.4221	0.2814	0.7036	0.3870	0.6684	1.0554	0.5277
10	0.3910	0.4855	0.3236	0.8091	0.4450	0.7687	1.2137	0.6068

RD stand for Returns from Debtor.

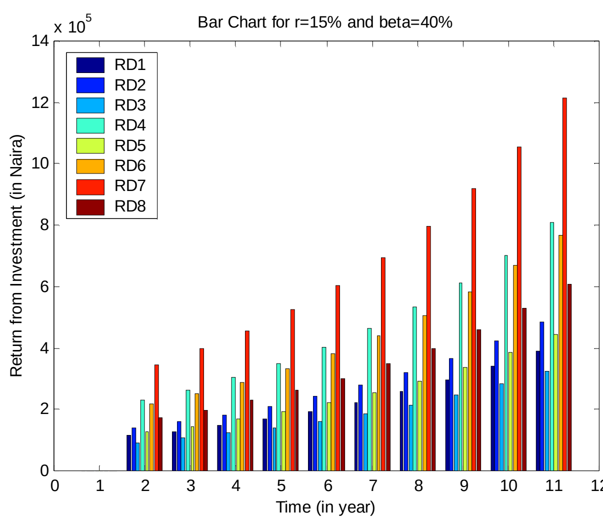

Figure 8: Bar Chart Representing Expected Return from the Investment Portfolio, for $r = 15\%$ and $\beta = 40\%$

Table 9 and Figure 8 give the expected return of the investor from the investment after 10years the value of $r = 0.15$ and $\beta = 0.40$. In this table, we found that in the first year RD1 amounted to 115,000 Naira and rises to 391,000 Naira in the 10th year.
We therefore conclude from Tables 7 - 9 that when the interest rate strictly monotone increasing, the expected return from the investment will be strictly monotone increasing as well, which is an expected result.

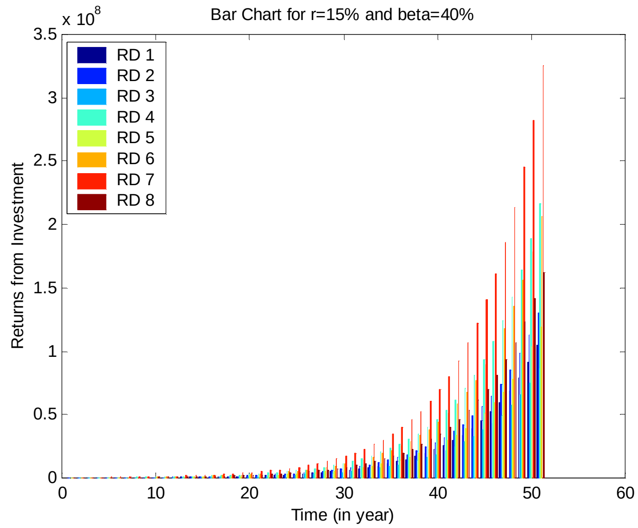

Figure 9: Bar Chart Representing Expected Return from Investment for $r = 15\%$ and $\beta = 40\%$

We observe over time from Figure 9 that the expected returns in the investment portfolio follows exponential growth.

5. 0 Conclusion

This paper studied a dynamic debt optimization problem for investor over a finite time horizon. The paper provided basic background on dynamic programming in relation to debt managements. It determined the possible debt thatwill accrual to a borrower at the stipulated time and minimize the incident of bad debt on the part of thedebtors. We found that the incident of bad debt is capable of discouraging investors. It was further found that when the unrecovered bad debt increases monotonically, the expected return from the investment portfolio will be monotone decreasing. The paper found that as the interest rate is strictly monotone increasing, the expected return from the investment portfolio will be strictly monotone increasing. Also, as the time period of lending increases, the possible chances of bad debt increases. Finally, it was found that the expected returns in the investment portfolio follow exponential growth.

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