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Fracture analysis of some engineering materials using Boundary Element Method (BEM)

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Abstract

Boundary element method (BEM) has been used in the study of engineering materials such as alumina, iron steel, mild steel, low carbon steel, stainless steel, concrete, silica glass and PVC. Special crack-tip element method was used to evaluate stress intensity factor for centre, single-edge and double-edge crack for various values of specimen size (geometry), microstructural parameter and volume fraction. Materials behaviour increases exponentially with increase in specimen size in the range of

0.1< a/w <0.8. The response is high for alumina, low for low carbon steel, mild steel, iron and stainless steel showing high and low resistance to crack growth. The asymptotic behaviour of concrete, silica glass and PVC gives rise to crack growth at 0.105 $MPam^{\frac{1}{2}}$ and 0.2 specimen geometry. The ratio of shear modulus at (-0.2-1.1) x10^{-5},

(0 to -2)x10^{-6}, -1.46 x10^{-4} to -2 x10^{-6}, respectively for concrete, silica glass and PVC exhibited brittle fracture (failure) while at 1.03 to 1.39, 1.04 to 1.39, 1.07 to 2.23 for low carbon steel, iron steel and mild steel, stainless steel and alumina was characterized with ductile fracture.

Keywords: Engineering materials, specimen geometry, stress intensity factor, crack-tip etc.

1.0 Introduction

Engineering materials are very important in the development of any society as they are used in the construction of structures such as buildings, communication masts, bridges, flyovers, automobile, power plants, refineries, etc. It is observed that most structures are collapsing on daily basis. This has attracted the attention of the different tiers of government, companies, non-governmental organizations (NGO), Nigeria Society of Engineers (NSE), Council for Regulation of Engineering in Nigeria (COREN) and scientist as a whole. Some people have blamed the failure on engineers who design and supervise such projects. Others are of the opinion that the engineering materials used for the construction are substandard while some other researchers have attributed the problem to crack failure in engineering materials. In a bid to address the problem, COREN has made it mandatory for all Nigerian engineers to belong to the professional body. However, no matter the ingenuity exhibited by engineers and scientist in the design and construction, failure of machine parts and structures must occur as each machine member has its life span.

The study of fracture mechanics began in earnest during World War 1 by English aeronautical engineer, Griffith [1] who used his theory to explain the failure of brittle materials. However, Griffith's approach was too primitive for engineering applications and is only good for brittle materials. For ductile materials, the milestone that was set by Griffith [1] did not come true. Because of this setback Griffith work was largely ignored by the engineering community.

Griffith's theory provides excellent agreements with experimental data for brittle materials such as glass. For ductile materials such as steel, his experiments showed that the product of the square root of the flaw length (a) and the stress at fracture (σ_f) was nearly constant, which is expressed by equation

$$\sigma_f \sqrt{a} \approx C \qquad (1)$$

where C is a constant

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He later found an expression for the constant C in terms of the surface energy of the crack by solving the elasticity problem of a finite crack in an elastic plate.
Griffith found that

$$C = \sqrt{\frac{2E\gamma}{\pi a}} \tag{2}$$

where (E) is the Young's modulus of the material and (γ) is the surface energy density of the material. The surface energy (γ) predicted by Griffith's theory is usually unrealistically high for ductile materials and even in materials that appear to be brittle.
Onuu and co-woeker[2 - 4] investigated methods for predicting growth rate of cracks in solids. Various older procedures with different validity criteria are used to study and characterize crack growth resistance in solids [5 - 7]

1.1 Boundary Element Method

The boundary element attempts to use the given boundary conditions to fit boundary values into the integral equation, rather than values throughout the space defined by a partial differential equation. Once this is done, in the post-processing stage, the integral equation can then be used again to calculate numerically the solution directly at any desired point in the interior of the solution domain.
BEM is applicable to problems for which Green's function can be calculated. These usually involve fields in linear homogenous media. This places considerable restrictions on the range and generality of problems to which boundary element can usefully be applied. Nonlinearities can be included in the formulation, although they will generally introduce volume integrals which then require the volume to be discretised, before solution can be attempted, removing one of the most often cited advantages of BEM. BEM has emerged as a powerful alternative to the finite element method. The most important features of BEM is that it reduces the dimensionality of the problem by one, resulting in a smaller system of equations and a considerable reduction in the data required for the analysis.

2.0 Similar Methods and Formulation

2.1 J-integral method

The use of J-Integral method to investigate growth rate of cracks in solids reveals that specimen size and geometry $\left(a/W\right)$ has effect on the critical values of fracture toughness (K_{Ic}) where W is the width of the crack. Onuu [8] showed that K_{Ic} decreases with decrease in $\left(a/W\right)$. According to Onuu [8] for a given geometry, K_{Ic} depends on both specimen width and initial crack length and that J-Integral values increases as the applied load increases for all specimen geometry except for $\left(a/W\right)$ = 0.21, where the unbroken ligament was so small that a very small force is needed to bring about fracture. Linear elastic fracture mechanics (LEFM) and secant intercept procedure have been used by Onuu and Ajepong [3] to determine the critical value of fracture toughness (K_{Ic}), critical elastic-energy release rate per crack-tip extension, G_{1c}, and the plastic-zone radius (for mode I loading) for the ST 60 Mn steel in their estimation of plane strain fracture mechanics parameters for this material.
The investigation on plane stress fracture mechanics parameter of locally produced steel using crack opening displacement (COD) approach by Onuu and Adjepong [4] was to predict full-scale structural behaviour.
For the steel specimen examined, critical values of the COD (COD_{crit}, δ_c) at the tip of the crack for various crack-lengths to net-width ratios of the test piece were determined. This ranged from 0.09 to 0.85mm corresponding to crack-length to net-with ratio that varied from 0.92 to 0.67, respectively. This investigation has shown that for a fixed net-width, COD_{crit} decreases with crack-length or increases with unbroken ligament. The critical value of COD could be a measure of the resistance of a material to fracture initiation and propagation.

Table 1 Some materials constants, Ashby and Jones [9]

Materials	Young's modulus E/GNm^{-2}	Poisson Ratio V	Expansion coefficient $K^{-1}\times 10^{-6}$	Fracture toughness $MNm^{\frac{1}{2}}$
Alumina	390	0.25	7.0	3-5
Iron	190	0.30	13.0	150
Low alloy steel	200-210	0.30	15.0	50
Stainless steel	190-200	0.30	11.0	30
Mild steel	196	0.30	15.0	140
Silica glass	94	0.16	0.50	0.0008-0.0048
Aluminium & alloys	69-79	0.35	2.20	20-50
Concrete (reinforced)	45-50	0.3	10.0	10-15
PVC	0.003-0.01	0.41	70.0	2.0-4.7

2.2 Crack-Tip element method (CTEM)

The ultimate task in fracture mechanics analysis is the calculation of the stress intensity factor which is a local parameter. The most common methods of evaluation are the J-integral method and the near-tip displacement method as mentioned above. The later is much preferred computationally since the calculation is straight forward. However, to obtain accurate results, the singularity nature of the crack displacement has to be modelled correctly.

The required singularity can be achieved by placing special elements at crack-tips [10 - 11]. Discontinuous quarter-point crack-tip element was used in the present formulation. The stress intensity factors are calculated as

$$K_n = \frac{G}{k+1}\sqrt{\frac{2\pi}{r}\Delta u_{n,t}(r)} \tag{3}$$

where $k = 3 - 4v$ for plane strain problems, r is the distance from crack-tip to the nearest node on the upper crack-face. $\Delta u_n(r)$ and $\Delta u_t(r)$ denote the relative normal and tangential displacement at r. The results show that the inclusion of the special crack-tip elements led to improved accuracy and efficiency in the stress intensity factor calculation.

The fracture in a cracked body can be in any of these modes, or a combination of two or three modes. Consider a crack problem in an infinite domain, the stress components can be expressed as [12 - 13]

$$\sigma_{11} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) \tag{4}$$

$$\sigma_{22} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \frac{K_{II}}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{5}$$

$$\sigma_{12} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} + \frac{K_{II}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{6}$$

where K_I and K_{II} are the stress intensity factors corresponding to the opening mode and the in-plane shear mode, respectively, and the size of r is much smaller than the crack length. Integrating equations (4), (5) and (6) using the strain displacement and stress-strain relations, the displacement components in the vicinity of the crack-tip are

$$u_1 = \frac{1}{4G}\sqrt{\frac{r}{2\pi}}\left\{K_I\left[(2K-1)\cos\frac{\theta}{2}-\cos\frac{3\theta}{2}\right]+K_{II}\left[(2k+3)\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right]\right\} \tag{7}$$

$$u_2 = \frac{1}{4G}\sqrt{\frac{r}{2\pi}}\left\{K_I\left[(2K-1)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2}\right] - K_{II}\left[(2k-3)\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\right]\right\} \quad (8)$$

$$u_3 = 4\frac{K_{III}}{G}\sqrt{\frac{r}{2\pi}}\sin\left(\frac{\theta}{2}\right) \quad (9)$$

where K_{III} is the stress intensity factors corresponding to the out-of-plane shear mode.
We avoided using the interpolating polynomial function on the crack-tip element in this analysis because of the $o\left(\sqrt{r}\right)$ behaviour of the near tip displacement field. we let the displacement be represented by

$$U_i \approx B_I\sqrt{r}\ . \quad (10)$$

Besides the unknown displacement of the collocation nodes, an unknown constant B_i needs to be obtained. Let r and θ polar coordinate system with origin at the crack-tip, such that $\theta = \pm\pi$ defines the crack faces.
The relative displacement near a traction free crack-tip from equations 7, 8, and 9 can be written as

$$\Delta u_x = u_x(\theta=\pi) - u_x(\theta=-\pi) = \frac{k+1}{G}K_{II}\sqrt{\frac{r}{2\pi}} \quad (11)$$

$$\Delta u_y = u_y(\theta=\pi) - u_y(\theta=-\pi) = \frac{k+1}{G}K_I\sqrt{\frac{r}{2\pi}} \quad (12)$$

$$\Delta u_z = u_z(\theta=\pi) - u_z(\theta=-\pi) = \frac{k+1}{G}K_{III}\sqrt{\frac{r}{2\pi}} \quad (13)$$

As $r \rightarrow 0$ the leading terms in the displacement approaches infinity, while other terms remain finite or approaches zero. Calculation of the Mode III Stress Intensity Factor Near tip displacement extrapolation is used to evaluate the numerical values of the stress intensity factor. The relative displacements of the crack surfaces are calculated using the Double boundary element method DBEM and are used in the near crack-tip stress field equations to obtain the stress intensity factor.

Due to singular behaviour of the stress around the crack tip, it is reasonable to expect a better approximation by replacing the normal discontinuous quadratic element with a transition element possessing the same order of singularity at the crack-tip. The stress intensity factors are given by

$$K_I = B_x\frac{G}{k+1}\sqrt{2\pi} \quad (14)$$

$$K_{II} = B_y\frac{G}{k+1}\sqrt{2\pi} \quad (15)$$

$$K_{III} = B_z\frac{G}{4}\sqrt{2\pi} \quad (16)$$

where G is the Shear modulus, (v) is the Poisson's ratio,

$k = 3-4v$ for plane strain.

B_y = Compliance constant for Centre Crack (CC)

B_x= Compliance constant for Single-edge Crack (SEC)

B_z= Compliance constant for Double-edge Crack (DEC)

where $\left(a/_W\right)$ = crack size (specimen geometry)

a=crack length, w=crack width, π = 3.142

The compliance constants are;

$$B_y = \left[1 + 0.50\left(a/_W\right)^2 + 20.46\left(a/_W\right)^4 + 81.72\left(a/_W\right)^6\right]^{\frac{1}{2}} \quad (17)$$

Fracture analysis of some… *Arikpo and OnuuJ of NAMP*

$$B_x = 1.12 - 0.23(a/W) + 10.55(a/W)^2 - 21.71(a/W)^3 + 30.38(a/W)^4 \quad (18)$$

$$B_z = 1.12 + 0.41(a/W) - 4.78(a/W)^2 + 15.44(a/W)^3 \quad (19)$$

Shear moduli were calculated by solving two fundamental boundary value problems using the values obtained from Table 1 for Young's modulus (E) and Poisson's (v) ratio in the different materials as shown in equations

$$G = \frac{E}{2(1+v)} . \quad (20)$$

Eischen [14] developed BEM used to calculate the upper and lower shear modulus boundaries of the materials as shown in equations.

$$G_L^{(3)} = \left[\left\langle \frac{1}{G} \right\rangle - \frac{\phi_1 \phi_2 \left(\frac{1}{G_2} - \frac{1}{G_1} \right)^2}{\left\langle \frac{1}{\tilde{G}} \right\rangle + \langle G \rangle_\eta} \right]^{-1} \quad \text{(lower bound)} \quad (21)$$

$$G_U^{(3)} = \langle G \rangle - \frac{\phi_1 \phi_2 \left(\frac{1}{G_2} - \frac{1}{G_1} \right)^2}{\langle \tilde{G} \rangle + \langle G \rangle_\eta} \quad \text{(upper bound)} \quad (22)$$

where $G_U^{(3)}$ = upper bound shear modulus

$G_L^{(3)}$ = lower bound shear modulus

$\tilde{G} =$ average shear modulus

$G_2 =$ highest value of the shear modulus

$G_1 =$ lowest value of the shear modulus

$G =$ shear modulus of a particular material

$\phi_2 =$ material volume fraction (set values 0.1-0.85)

$\phi_1 =$ material volume fraction

$\langle G \rangle_\eta =$ shear modulus as a function of the microstructural parameter

2.3 Microstructural parameter η_2

The microstructural parameter (η_2) was calculated using the boundary element method and the three-point bound formulas for the effective shear modulus as shown in the equation

$$\eta_2 = \left(\frac{\phi_1 \phi_2 (G_2 - G_1)^2}{\langle G \rangle - G_u^{(3)}} - \langle \tilde{G} \rangle - G_1 \right) \Big/ (G_2 - G_1) \quad (23)$$

Crack configurations for centre, single-edge and double-edge crack for the different modes is shown in Figs. 1, 2 and 3, respectively, where a is the crack length, W is the crack width and σ is the applied stress.

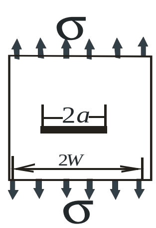

FIG.1: Centre crack for material under mode I loading

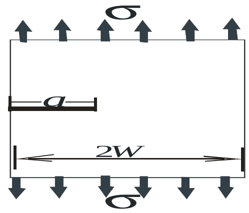

FIG.2: Single-edge Crack for material under mode II loading

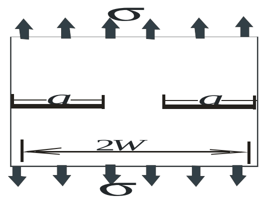

FIG.3: Double-edge Crack for material under mode III loading

3.0 Results and Discussion

Figure 4 shows that the plot of normalised stress intensity factor for alumina, low carbon steel, iron steel, mild steel and stainless steel increases exponentially with increased in specimen size (geometry). This is a strong evidence for the resistance effect of centre crack growth for these materials. Concrete, and PVC show asymptotic response to specimen size (geometry) but with crack initiation at 0.105 $MPam^{\frac{1}{2}}$ and 0.2 specimen size for silica glass.

Figures 5 and 6 show the comparative difference in the effect of stress intensity factor on materials with single and double edge crack. These materials behave in like manner. Low carbon steel, iron, mild steel and stainless steel superpose each other in behaviour.

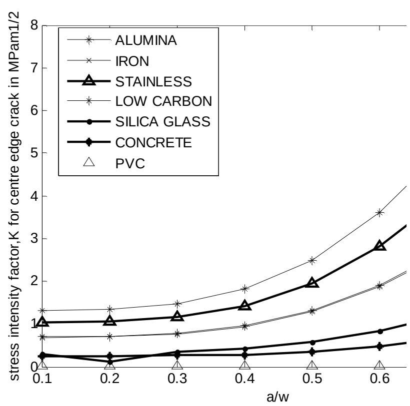

Fig.4: Normalised stress intensity factor of materials with centre-crack

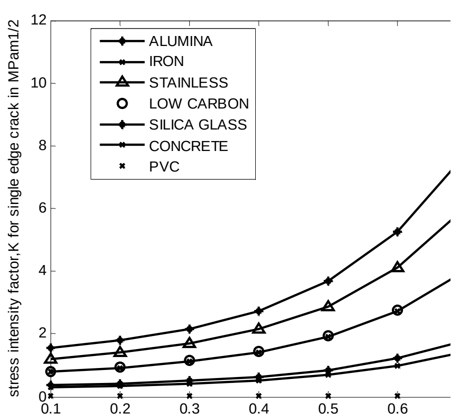

Fig.5: Normalised stress intensity factor of materials with single-edge crack

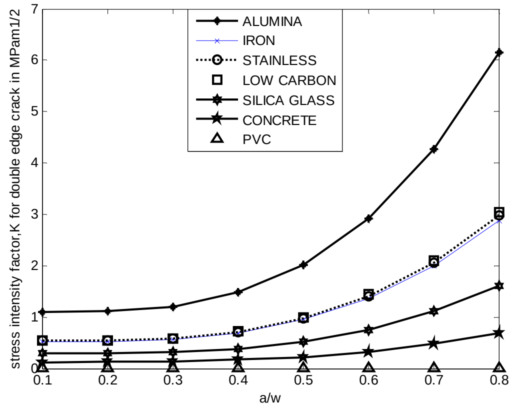

Fig.6: Normalised stress intensity factor of materials with double-edge crack

Results for the eight different materials at volume fractions in the range $0.1 \le \phi_2 \le 0.85$ are shown in Fig.7. This figure shows that microstructural parameter increases linearly with increase in volume fraction (ϕ_2) for the materials. Specifically, the increase rate is high for alumina but moderate in lowcarbon steel, mild steel, iron, and concrete. Similar results were obtained in the study of elastic behaviour of composites by boundary element method by Eischen, [14] who found that microstructural parameter rises in rigorous three-point bounds on the effective shear modulus. The agreement with this result is extremely good, thus providing confidence in the new results for microstructural parameter. Dependent of the ratio of shear modulus (lower to upper bounds) on the microstructural parameter such as grain size is shown in Fig. 8. The curve for alumina shows a high logarithmic increment to that of low

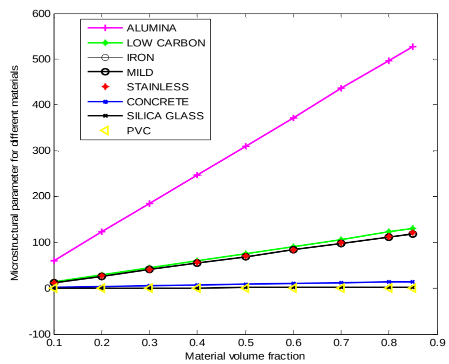

Fig.7: Effect of microstructural parameter to volume fraction for different materials.

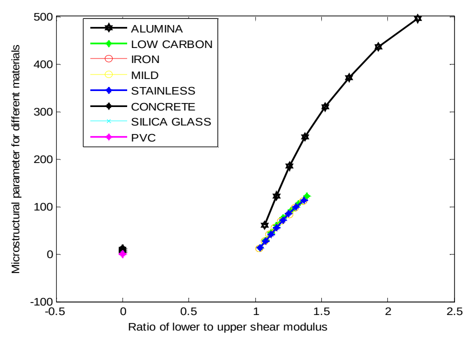

FIG.8: Effect of microstructural parameter to the bounds of shear modulus for different materials.

carbon steel, iron steel, mild steel and stainless steel while the plot for concrete, silica glass as well as PVC is different. As the grain size gets smaller, fracture becomes more brittle. This is due to the fact that in smaller grains, dislocation have less space to move before they hit a grain boundary. Thus, the material's fracture is more brittle, while low carbon steel, iron steel, mild steel and stainless steel the grain size is larger when compared to PVC and largest in alumina.

4.0 Conclusion

The following conclusions are drawn from the results presented here:

I. There is relevance of boundary element method as a tool for studying fracture characterization of some engineering materials under plane-strain condition for $k = 3 - 4v$

II. The results show that alumina, iron and stainless steel (Figure 4, 5 and 6) values were higher than other materials under study. This outcome verified that there was an effect of specimen size and geometry on modes I, II and III stress intensity factor for centre, single-edge, and double-edge crack for a fixed net-with (10mm). Stress intensity factor increases with crack-length and the ratio of lower to upper bounds shear modulus as well as material volume fractions.

III. The result of microstructural parameter to shear modulus bounds obtained from this study shows that the effect of microstructural parameter on metals was quite different from ceramics.

IV. The ratio of lower to upper bounds shear modulus for concrete at $(-0.2\text{-}1.1)\times10^{-5}$, silica glass at 0 to -2×10^{-6} and PVC at -1.46×10^{-4} to -2×10^{-6} exhibited brittle fracture (failure), while at 1.03 to 1.39, 1.04 to 1.37, 1.07 to 2.23 for low carbon steel, iron steel and mild steel, stainless steel as well as alumina exhibited ductile fracture (failure).

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