

Error Calculations Between Experimental Corrosion Penetration Rate and Modelled Corrosion Penetration Rate Data in the HCl Concentration Environment: Implication for the Model Building of Sn-Al Duplex Interaction System

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Abstract

Most data obtained from experimental studies and modelled studies of growth data are not totally free of error. The impact of this type of error has the capability to bias the mathematical model that can be constructed from these data sets. This is a challenging scientific problem. We have proposed the method of p -norms and used this method to calculate and quantify the precise values of the error between the experimental corrosion penetration rate and modelled corrosion penetration rate data in the HCl concentration environment which have been previously tested to be highly statistically correlated. On the basis of this powerful numerical method, the K_1 data sets have been selected to be the best-fit time series data upon which we can construct a mathematical model in order to minimize selection bias and enhance model application. Our present contribution is a further extension of some previous statistical analyses of Sn-Al Duplex systems.

1.0 Introduction

On the basis of a statistical law, the corrosion characterizations of binary Al-Sn alloy systems have been analysed in the light of developed model equations [1]. In the work of [1], standard regression equations were utilized to measure the strength of the relationship between time series indexed corrosion penetration rate data. But for the purpose of developing a mathematical model that can be used to describe the dynamics between these data sets, it is necessary to measure the size of the error between these data sets in order to minimize selection bias when an appropriate mathematical model is constructed. According to these authors [1], the modelled corrosion penetration rate values generated using the developed model equations were in tandem with the experimental values. In order to check if these data can be used to develop a mathematical model which defines the Sn-Al Duplex interaction system, it is imperative to calculate the error between these data using the notion of mathematical P -norms [2]. Beyond the calculation of these error values, we have proposed to select the smallest error values that are consistent with best-fit selection method [3] so as to minimize the issue of selection bias. This present idea forms the basis for this study.

2.0 Method of Analysis

From the viewpoint of functional analysis, the 1-norm for the differences of two data sets is defined as the sum of the absolute values of the differences of the data while the 2-norm for the differences of two data sets is defined as the square-root of the squares of the sum of the absolute values of the differences of the data [2]. Similarly, the n th-norm for the differences of two data sets is defined as the n th-root of the n th-powers of the sum of the absolute values of the differences of the data. The 3-norm, 4-norm, 5-norm and so forth for these corrosion penetration rate data sets were calculated. The infinity-norm for the difference of two data sets is defined as the maximum value of the absolute value of the corresponding pairs of data. The HCL environment data used in this analysis were obtained from the work of Ekuma et al. [1].

3.0 Discussion of Results

The results of this analysis using the above mentioned method are presented and discussed in this section.

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Table 1: Corrosion Penetration Rate (CPR) Data for Al – 2.5% Sn in 0.5 M HCl Environment.

Time (hours)	Experimental CPR (mm/hr)	Modelled CPR (mm/hr)
12	2.08	1.763
24	1.25	1.334
36	0.90	1.083
48	0.73	0.905
60	0.62	0.767
72	0.55	0.654
84	0.51	0.559
96	0.47	0.477
108	0.44	0.404
120	0.43	0.338
132	0.42	0.28
144	0.39	0.226

Source: [1]

For the purpose of this study, we have represented the experimental CPR as *ecpr* and the modelled CPR as *mcpr*. The differences between these data sets are represented using the notation $K = ecpr - mcpr$. These data sets are displayed in Table 2.

Table 2: Calculation of the Error between Corrosion Penetration Rate (CPR) Data for Al – 2.5% Sn in 0.5 M HCl Environment.

Time (hours)	<i>ecpr</i> (mm/hr)	<i>mcpr</i> (mm/hr)	$K = ecpr - mcpr$
12	2.08	1.763	0.3170
24	1.25	1.334	-0.0840
36	0.90	1.083	-0.1830
48	0.73	0.905	-0.1750
60	0.62	0.767	-0.1470
72	0.55	0.654	-0.1040
84	0.51	0.559	-0.0490
96	0.47	0.477	-0.0070
108	0.44	0.404	0.0360
120	0.43	0.338	0.0920
132	0.42	0.28	0.1400
144	0.39	0.226	0.1640

Source: [1]

The range, variance, mean and standard deviation of *ecpr* data values have been calculated to be 2.89, 0.6806, 1.0133 and 0.825 whereas the range, variance, mean and standard deviation of *mcpr* data values have been calculated to be 2.552, 0.6027, 1.0133 and 0.7764. The calculated realistic statistical measures of spread for *K* data values such as the range, variance and standard deviation are 0.5, 0.0239 and 0.1546. The error between the *ecpr* and *mcpr* data sets has been estimated using the notion of *p*-norms. In this scenario, the estimated 1-norm, 2-norm, 3-norm, 4-norm and 5-norm for the *K* data values are 1.498, 0.5126, 0.3833, 0.3438, and 0.3287 whereas the estimated 6-norm, 7-norm, 8-norm, 9-norm, 10-norm and infinity-norm for the *K* data values are 0.3223, 0.3195, 0.3182, 0.3176, 0.3173 and 0.3170. The second data sets obtained from Ekuma et al. 2011 [3] are displayed in Table 3.

Table 3: Calculation of the Error between Corrosion Penetration Rate (CPR) Data for Al – 2.5% Sn in 1.0 M HCl Environment.

Time (hours)	<i>ecpr</i> (mm/hr)	<i>mcpr</i> (mm/hr)	$K_1 = ecpr - mcpr$
12	3.12	2.815	0.3050
24	2.08	2.201	-0.1210
36	1.66	1.843	-0.1830
48	1.46	1.588	-0.1280
60	1.25	1.391	-0.1410
72	1.14	1.229	-0.0890
84	1.07	1.093	-0.0230
96	1.01	0.975	0.0350
108	0.92	0.870	0.0500
120	0.85	0.777	0.0730
132	0.79	0.693	0.0970
144	0.74	0.616	0.1240

Source: [1]

The range, variance, mean and standard deviation of **ecpr** data values have been calculated to be 2.38, 0.4674, 1.3408 and 0.6837 whereas the range, variance, mean and standard deviation of **mcpr** data values have been calculated to be 2.199, 0.4474, 1.3409 and 0.6689. The calculated realistic statistical measures of spread for **K₁** data values such as the range, variance and standard deviation are 0.488, 0.02 and 0.1413. The error between the **ecpr** and **mcpr** data sets has also been estimated using the notion of p-norms. In this scenario, the estimated 1-norm, 2-norm, 3-norm, 4-norm and 5-norm for the **K₁** data values are 1.369, 0.4687, 0.3564, 0.3244, and 0.3131 whereas the estimated 6-norm, 7-norm, 8-norm, 9-norm, 10-norm and infinity-norm for the **K₁** data values are 0.3086, 0.3067, 0.3058, 0.3054, 0.3052 and 0.3050.

Table 4: Calculation of the Error between Corrosion Penetration Rate (CPR) Data for Al – 4.5% Sn in 0.5 M HCl Environment.

Time (hours)	ecpr (mm/hr)	mcpr (mm/hr)	K₂ = ecpr-mcpr
12	3.12	2.549	0.5710
24	1.66	1.890	-0.2300
36	1.18	1.505	-0.3250
48	0.94	1.231	-0.2910
60	0.79	1.019	-0.2290
72	0.73	0.846	-0.1160
84	0.64	0.699	-0.0590
96	0.60	0.573	0.0270
108	0.54	0.461	0.0790
120	0.50	0.360	0.1400
132	0.46	0.270	0.1900
144	0.43	0.187	0.2430

Source: [1]

The range, variance, mean and standard deviation of **ecpr** data values have been calculated to be 2.69, 0.5852, 0.9658 and 0.765 whereas the range, variance, mean and standard deviation of **mcpr** data values have been calculated to be 2.362, 0.5161, 0.9658 and 0.7184. The calculated realistic statistical measures of spread for **K₂** data values such as the range, variance and standard deviation are 0.896, 0.0691 and 0.2629. In this scenario, the estimated 1-norm, 2-norm, 3-norm, 4-norm and 5-norm for the **K₂** data values are 2.50, 0.872, 0.6661, 0.6068, and 0.5856 whereas the estimated 6-norm, 7-norm, 8-norm, 9-norm, 10-norm and infinity-norm for the **K₂** data values are 0.5773, 0.5738, 0.5723, 0.5716, 0.5713 and 0.5710.

Table 5: Calculation of the Error between Corrosion Penetration Rate (CPR) Data for Al – 4.5% Sn in 1.0 M HCl Environment.

Time (hours)	ecpr (mm/hr)	mcpr (mm/hr)	K₃ = ecpr-mcpr
12	3.33	2.724	0.6060
24	1.77	2.012	-0.2420
36	1.25	1.596	-0.3460
48	0.99	1.30	-0.3100
60	0.85	1.071	-0.2210
72	0.75	0.884	-0.1340
84	0.65	0.725	-0.0750
96	0.60	0.588	0.0120
108	0.55	0.467	0.0830
120	0.51	0.359	0.1510
132	0.47	0.261	0.2090
144	0.44	0.172	0.2680

Source: [1]

The range, variance, mean and standard deviation of **ecpr** data values have been calculated to be 2.89, 0.6806, 1.0133 and 0.825 whereas the range, variance, mean and standard deviation of **mcpr** data values have been calculated to be 2.552, 0.6027, 1.0133 and 0.7764. The calculated realistic statistical measures of spread for **K₃** data values such as the range, variance and standard deviation are 0.952, 0.0781 and 0.2795. In this scenario, the estimated 1-norm, 2-norm, 3-norm, 4-norm and 5-norm for the **K₃** data values are 2.657, 0.927, 0.7074, 0.6442, and 0.6216 whereas the estimated 6-norm, 7-norm, 8-norm, 9-norm, 10-norm and infinity-norm for the **K₃** data values are 0.6127, 0.6090, 0.6074, 0.6067, 0.6063 and 0.6060. On the basis of this analysis, the classified measures of error between corrosion penetration rate data are summarised as follows:

Table 6: Selection of best-fit data sets using the method of p-norms

Example	p-norms	K	K ₁	K ₂	K ₃
1	1-norm	1.4980	1.3690	2.5000	2.6570
2	2-norm	0.5126	0.4687	0.8720	0.9270
3	3-norm	0.3833	0.3564	0.6661	0.7074
4	4-norm	0.3438	0.3244	0.6068	0.6442
5	5-norm	0.3287	0.3131	0.5856	0.6216
6	6-norm	0.3223	0.3086	0.5773	0.6127
7	7-norm	0.3195	0.3067	0.5738	0.6090
8	8-norm	0.3182	0.3058	0.5723	0.6074
9	9-norm	0.3176	0.3054	0.5716	0.6067
10	10-norm	0.3173	0.3052	0.5713	0.6063
11	Infinity-norm	0.3170	0.3050	0.5710	0.6060

For the purpose of a further model development, the difference of the corrosion penetration rate data that is associated with the minimum size of the measures of error in terms of the values of p-norm should be selected as the best-fit corrosion penetration rate data sets. In this context, the K₁ data sets agree with the minimum values of the p-norms. Therefore, K₁ is the best-fit data sets.

4.0 Conclusion

We have utilized the method of p-norms to select the best-fit data sets which in this study agree with the K₁ time series corrosion penetration rate data sets. The statistical measures of spread for these data sets were also quantified. We would expect to develop a mathematical model of these data sets in combination with other corrosion penetration rate data sets from the NaCl concentration environment in order to minimize selection bias in our future attempt to construct a sound Al-Sn Alloy Duplex interaction model. The idea is that an approximate mathematical model can only be successfully developed from the two best-fit data sets from the HCl and NaCl concentration environments on the premise that the expected model parameters can be calculated from an experimental analysis which is highly correlated with its modelling counterpart.

References

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