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Numerical simulation study of the viral load of the virions of HIV infection of $CD4^+$ T-Cells: capacity building initiative and HIV/AIDS intervention strategy

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Abstract

One of the stresses suffered by people living with HIV/AIDS is capable to introduce some degrees of random noise into the system of modelling the viral load of the virions of HIV infection of $CD4^+$ T-cells. The quantification of this complex medical science problem requires a mathematical reasoning of a combination of deterministic and stochastic analyses in order to provide a useful insight with a strong component of capacity building initiative and HIV/AIDS intervention strategy. The predicted viral loads of the virions of HIV infection of $CD4^+$ T-cells were differentiated for two values of the maximum proliferation rate of target cells when the random-noise intensities were selected to be 0.8 and 8 respectively. These results which we have not seen elsewhere are presented in this study and discussed.

1.0 Introduction

The mathematical modelling of the HIV infection dynamics that involves the viral load of the virions of HIV infection of $CD4^+$ T-cells over an experimental time in the unit of days is not a new contribution [1]. What is also lacking in this model formulation lies in its inability to predict the viral load of the virions of HIV infection indexed by a weekly defined data points. This new dimension is rarely well researched probably due to the design of the Matlab programming language solution of tackling this problem in order to construct a sustainable capacity building initiative and HIV/AIDS intervention strategy.Other related mathematical models [2, 3, 4, 5] only describe the dynamics of HIV/AIDS without random noise analysis with a limited policy implication.Despite the stress associated with the stage of HIV infection [6], we report that the parameters which were estimated to explain the dynamics of HIV infection are beyond deterministic characteristic, hence random noise in the estimated parameters and HIV infection data [7] makes this simplifying assumption in our present study necessary.

2.0 Mathematical Formulation

Following [1], we consider the system of time dependent non-linear first order ordinary differential equations

$$\frac{dT}{dt} = s - dT + aT\left(1 - \frac{T}{T_{max}}\right) - \beta TV \qquad (1)$$

$$\frac{dI}{dt} = \beta_1 TV - \delta I \qquad (2)$$

$$\frac{dV}{dt} = \rho I - cV \qquad (3)$$

with the initial conditions $T(0) \geq 0, I(0) \geq 0, V(0) \geq 0$ .The notation T is called the number of target cells while the notations I and V are called the number of infected cells and the viral load of the virions at time t in the unit of days. The notation s stands for the rate at which new T cells are created from sources within the body such as the thymus whereas the notation a is called the maximum proliferation rate of target cells. The notation T_{max} stands for the T population density at which proliferation shuts off whereas the notation d stands for the death rate of the T cells. The

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notation β_1 is represented by the exponential equation $\beta_1 = \beta e^{-m\tau}$ where β is the infection rate constant whereas the term $e^{-m\tau}$ accounts for cells that are infected τ time units later. The notation δ stands for the death rate of infective cells whereas the notation ρ is the reproductivity rate of the infected cell. The notation c represents the clearance rate constant of virions.

3.0 Method of Solution

In this study, we have used the mathematical technique of a numerical simulation on a non-linear system of first order ordinary differential equations that model the HIV infection of $CD4^+$ T-cells to predict the viral load of the virions of HIV infection of $CD4^+$ T-cells over every week simplifying assumption. We would expect this idea to provide useful planning information for HIV/AIDS systematic early intervention. For the purpose of this study, the maximum proliferation rate of target cells is modified between 10 percent and 50 percent as well as between 110 percent and 150 percent under a fixed initial data or initial condition in which the number of target cells at time t is 1000, the number of infected cells at time t is 1 and the viral load of the virions at time t is 1. The experimental time was taken to be 180 days.

4.0 Results and Discussion

The weekly quantified viral load of the virions are presented under different choices of the initial data of the maximum proliferation rate of target cells. The range of the maximum proliferation rate of target cells between 0.68 and 3.40 shows that the viral load of the virions will be subjected to varying degrees of damage [Table 1] whereas the range of the maximum proliferation rate of target cells between 7.48 and 10.20 indicates an increase in the viral load of the virions for these weekly data scenarios [Table 2]. For the instances when the maximum proliferation rate of target cells is 7.48 with the random noise intensities of 0.8 and 8 [Table 3 and Table 4], our present simulation study predicts an increase in the viral load of the virions on a weekly basis for 50 instances.

Table 1: Predicting theweekly approximate values of viral load of the virions of HIV infection undergoing changes in the maximum proliferation rate **a** of target cells for the interval [0.68, 3.40] when other parameters are fixed

Example	**V_{wp} (t)**	**V_w (t) :a = 6.8**	**a = 0.68**	**a = 1.36**	**a = 2.04**	**a = 2.72**	**a = 3.40**
1	V (1)	1	1	1	1	1	1
2	V (8)	48441	6241	7115	8097	9384	11396
3	V (15)	40004	4992	8616	8356	13017	22663
4	V (22)	35490	3418	8735	9326	16892	16053
5	V (29)	33383	4378	8429	10587	14068	17906
6	V (36)	32399	4542	8102	11315	13374	17983
7	V (43)	31968	4174	7867	11379	14679	16983
8	V (50)	31813	4247	7725	11177	14545	18194
9	V (57)	31805	4332	7652	10987	14051	17194
10	V (64)	31898	4282	7619	10895	14249	17875
11	V (71)	32045	4269	7610	10899	14401	17499
12	V (78)	32220	4289	7608	10924	14280	17653
13	V (85)	32397	4289	7615	10948	14247	17647
14	V (92)	32569	4286	7613	10969	14310	17578
15	V (99)	32744	4282	7608	10956	14297	17669
16	V (106)	32903	4282	7620	10950	14283	17590
17	V (113)	33091	4285	7617	10954	14292	17651
18	V (120)	33246	4288	7618	10952	14297	17611
19	V (127)	33397	4284	7620	10954	14281	17633
20	V (134)	33498	4280	7625	10951	14286	17622
21	V (141)	33646	4285	7619	10954	14299	17626
22	V (148)	33755	4286	7616	10956	14288	17634
23	V (155)	33854	4285	7621	10966	14297	17623
24	V (162)	33958	4283	7616	10953	14291	17635
25	V (169)	34033	4284	7615	10956	14291	17625
26	V (176)	34094	4285	7621	10952	14297	17614

Table 2: Predicting the weekly approximate values of viral load of the virions of HIV infection undergoing changes in the maximum proliferation rate **a** of target cells for the interval [7.48, 10.20] when other parameters are fixed

Example	V_{wp} (t)	V_w (t) :a = 6.8	a = 7.48	a = 8.16	a = 8.64	a = 9.52	a = 10.20
1	V (1)	1	1	1	1	1	1
2	V (8)	48441	50903	51409	51239	50852	50537
3	V (15)	40004	47533	46328	43111	40690	40501
4	V (22)	35490	45384	41582	37438	40433	57512
5	V (29)	33383	43693	37639	38048	56590	56323
6	V (36)	32399	42070	35343	48180	51904	42792
7	V (43)	31968	40444	35876	51780	41487	54600
8	V (50)	31813	38846	39948	45063	44092	56431
9	V (57)	31805	37388	45083	39015	56491	43148
10	V (64)	31898	36115	46303	40013	47788	54881
11	V (71)	32045	35253	43606	49012	40517	55921
12	V (78)	32220	34721	39822	49705	50445	43059
13	V (85)	32397	34656	37244	42985	54092	55760
14	V (92)	32569	35123	37208	38787	43587	55207
15	V (99)	32744	36017	40378	43048	42400	42901
16	V (106)	32903	37183	44304	50408	55463	56838
17	V (113)	33091	38366	44980	46992	49322	54290
18	V (120)	33246	39273	42407	40710	40978	42783
19	V (127)	33397	39744	39256	39585	48491	57960
20	V (134)	33498	39706	37512	47066	54769	53435
21	V (141)	33646	39276	38470	49813	44517	42703
22	V (148)	33755	38584	41861	43869	41797	58916
23	V (155)	33854	37819	44479	39255	54708	52551
24	V (162)	33958	37103	43747	42460	50219	42763
25	V (169)	34033	36530	40963	49935	41302	59733
26	V (176)	34094	36231	38460	47251	47293	51667

Table 3: Predicting the weekly approximate values of viral load of the virions of HIV infection undergoing changes in the maximum proliferation rate a of target cells is 7.48when the random noise intensity is 0.8.

Example	V_{wp} (t)	V_w (t) :a = 6.8	a = 7.48	a = 7.48	a = 7.48	a = 7.48	a = 7.48
1	V (1)	1	1	1	1	1	1
2	V (8)	48441	51514	50865	51862	51570	51408
3	V (15)	40004	48418	48167	48132	47270	47141
4	V (22)	35490	44829	44068	44830	44689	43280
5	V (29)	33383	41820	40268	41611	41368	40228
6	V (36)	32399	38952	37738	37911	38257	37489
7	V (43)	31968	36013	35239	36357	36027	35221
8	V (50)	31813	35196	34594	34756	35434	35351
9	V (57)	31805	35361	35726	34983	37170	37701
10	V (64)	31898	37385	39189	38149	39193	41398
11	V (71)	32045	39756	41762	43432	42587	43080
12	V (78)	32220	41927	42615	44923	43571	43515
13	V (85)	32397	42933	41768	43500	42391	40724
14	V (92)	32569	42730	39930	40976	40232	37842
15	V (99)	32744	41395	40035	38490	39347	36096
16	V (106)	32903	39130	39323	36741	38377	34794
17	V (113)	33091	37881	40462	37030	37453	34902
18	V (120)	33246	36876	39346	38412	38753	36965
19	V (127)	33397	37989	39629	40508	40584	39290
20	V (134)	33498	39325	39371	41657	40962	42781
21	V (141)	33646	40241	38957	40071	40628	44741
22	V (148)	33755	42724	40106	38689	39436	43969
23	V (155)	33854	42363	41079	37838	39209	41424
24	V (162)	33958	39275	41418	38943	39477	39101
25	V (169)	34033	37053	39668	40506	40008	37227
26	V (176)	34094	36861	38905	41206	41891	36302

Table 4: Predicting the weekly approximate values of viral load of the virions of HIV infection undergoing changes in the maximum proliferation rate a of target cells is 7.48 when the random noise intensity is 8.

Example	V_{wp} (t)	V_w (t) :a = 6.8	a = 7.48	a = 7.48	a = 7.48	a = 7.48	a = 7.48
1	V (1)	1	1	1	1	1	1
2	V (8)	48441	52023	52170	51090	51993	51913
3	V (15)	40004	48377	47718	46850	48258	47571
4	V (22)	35490	44544	43822	43017	43718	43639
5	V (29)	33383	41160	41612	39732	40474	39744
6	V (36)	32399	38182	38071	37556	38303	36679
7	V (43)	31968	35047	35604	35488	35562	34953
8	V (50)	31813	34423	35196	35043	34415	34396
9	V (57)	31805	35194	36385	36255	34502	34265
10	V (64)	31898	38357	38421	40007	36355	37560
11	V (71)	32045	42147	41637	42593	40366	41324
12	V (78)	32220	44632	42378	43284	43635	45712
13	V (85)	32397	44511	42232	40481	43280	46705
14	V (92)	32569	43673	40466	38747	40181	44472
15	V (99)	32744	40187	38058	38289	37925	40316
16	V (106)	32903	37528	36976	38921	36135	36714
17	V (113)	33091	36109	37125	40638	37710	35028
18	V (120)	33246	36219	38300	41992	3.9862	35858
19	V (127)	33397	35998	40176	42035	4.2589	37574
20	V (134)	33498	37777	41140	41027	4.3276	40629
21	V (141)	33646	40501	40053	38282	4.2739	43101
22	V (148)	33755	42686	41949	36018	4.1087	43465
23	V (155)	33854	43779	41116	35372	3.8581	41802
24	V (162)	33958	41235	39012	36513	3.7296	38293
25	V (169)	34033	39214	37489	39556	3.7115	36461
26	V (176)	34094	37781	37668	43184	3.8340	36480

5.0 Conclusion

We have used the technique of a simulation analysis to quantify and predict the viral load of the virionsof HIV infection of $CD4^+$ T-cells for the first day of the first week that is denoted by V(1), for the first day of the second week that is denoted by V(8), for the first day of the third week that is denoted by V(15) and so forth. These results can form part of a capacity building initiative and HIV/AIDS intervention strategy.

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