Journal of the Nigerian Association of Mathematical Physics
Volume28, No. 1, (November, 2014), pp 245 – 248
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On the Effect of Boundary Conditions on Microwaves as Cancer Thermotherapy

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Abstract

We investigate the use of microwaves as a technique for cancer treatment, to heat tumor cells within the body. It is shown that the elevation of temperature of tumor cells depends on the boundary conditions. We also establish that using Dirichlet conditions is appropriate as appreciable heating occurs. On the other hand, Neumann and Robin conditions may not produce the desired treatment.

1.0 Introduction

Cancer remains a leading cause of death in the world. It is characterized by uncontrolled growth and spread of abnormal cells and tumor [1, 2]. There is no permanent cure yet. Hence, there is need for continuous research into the treatment of cancer.

Hyperthermia (or thermotherapy) is a cancer treatment that involves heating tumor cells within the body. Elevation of the temperature of tumor cells results in cell membrane damage, which, in turn, leads to the destruction of the cancer cells.

Hyperthermia treatment of cancer requires directing a carefully controlled dose of heat to the concerned tumor and surrounding tissue. This high heat must be used wisely, using little heat could mean that the cancer cells will not be killed. However, if too much heat misses the tumor target, the skin or other body tissues could be burnt.

However, microwave energy is very effective in heating cancerous tumors because tumors typically have high – water content, such tissue heat very rapidly when exposed to high – power microwaves. Microwaves can be delivered by special purpose antennas that are located adjacent to the body [3].

Our interest in this paper is to discuss the effective way of placing the antennas. Mathematically, this would mean discussing the boundary conditions of the mathematical model. Our model is taken from the work of [4, 5, 6].

2.0 Mathematical Formulation

Following [5, 7, 2], we consider the steady equation

$$k\frac{d^2T}{dx^2}+\rho_b w_b(T_b-T)+Q(T-T_\circ)^m=0 \quad (1)$$

where $T = temperature$

$T_b = temperature\ of\ \ arterial\ blood$

$T_0 = referencetemperature\ (T_b > T_0)$

$\rho_b w_b = blood\ \ perfusion$

$k = thermal\ \ conductivity$

$Q = electromagnetic\ energy\ \ per\, unit\ temperature$

$m = real\ \ number$

We may non – dimensionalize the equation by writing

$$\theta=\frac{T-T_\circ}{T_b-T_\circ},\quad \eta=\frac{x}{L} \quad (2)$$

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The non – dimensional temperature equation becomes

$$\frac{d^2\theta}{d\eta^2} - \alpha(\theta - 1) + \beta\theta^m = 0, \qquad (3)$$

Where

$$\alpha = \frac{\rho_b w_b}{k}, \ \beta = \frac{Q}{k} \qquad (4)$$

The relevant question now is how do we place the antennas to achieve what we desire? In other words what should be the boundary conditions? We have three boundary conditions:

(i). Dirichlet boundary conditions

$$\theta(0) = \theta_1, \qquad \theta(1) = \theta_2 \qquad (5)$$

(ii). Neumann boundary conditions

$$\frac{d\theta}{d\eta}(0) = \theta_3, \quad \frac{d\theta}{d\eta}(1) = \theta_4 \qquad (6)$$

(iii).Robin boundary conditions

$$\frac{d\theta}{d\eta} + h\theta = f_i, \text{ i=1,2} \qquad (7)$$

at 0 and 1 respectively.

Problem (3) subject to (5), (6) or (7) could be solved analytically or numerically depending on the value of m.

If m = 1, an analytic solution exists and it is easily seen that when the boundaries are insulated (Neumann problem) the problem has a particular solution. Thus, the cancerous tissue cannot be heated. We have a similar solution for the Robin problem. The most judicious choice is the Dirichlet problem i.e. fixed the temperatures at the boundaries.

Typically we assume $\theta_1 = \theta_2 = 0$ and $\alpha = 0.01$. The problem could be solved by shooting method and finite difference for $m > 1$. However, the shooting method gives one solution as could be seen in figure 1 for m=2, m=3 and m = 4. Using the finite difference scheme, we obtained two solutions. One coincides with the shooting method solution, while the second is due to error of discretization [2].

3.0 Conclusion

It is clear from this paper that the mode of placing the antennas is important to ensure effective heating of cancerous tumors. Moreover, the conditions on the boundaries are crucial so that the heat will not miss the tumor target and consequently burn uninfected part of the patient skin.

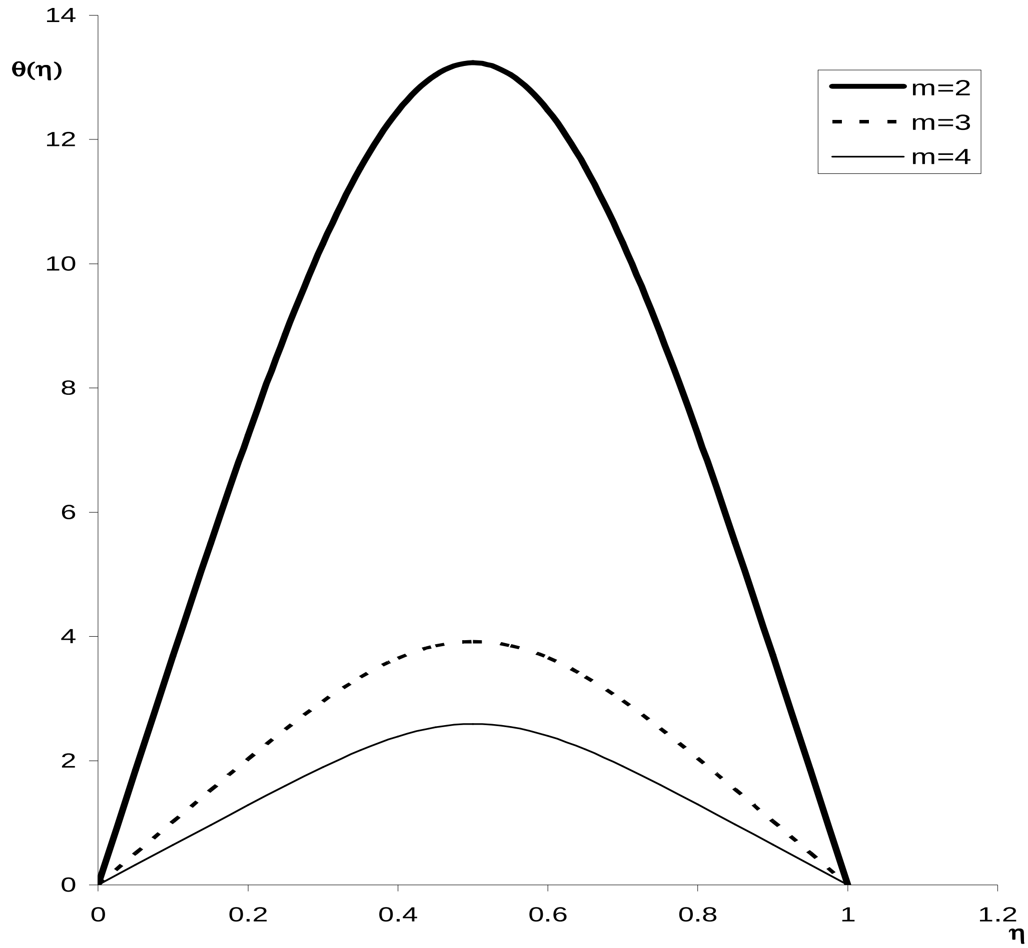

Figure 1:The graph of $\theta(\eta)$ against η for m>1 and fixed values of β=0.878, α=0.01

4.0 Acknowledgements:

The authors acknowledge useful suggestions of late Professor Reuben Olafenwa Ayeni, immediate past President of the Nigerian Association of Mathematical Physics.

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