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A Mathematical Study of *Chlamydia Trachomatis* Infection in a Human Carrier

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Abstract

This paper presents a mathematical model describing the dynamics of Chlamydia Trachomatis infection in a human carrier. The model incorporated relevant feature such as recovery through drug administration. The existence and uniqueness of solutions of the model were examined by actual solution. The stability analysis of the critical points was conducted. The results show that it is globally asymptotically stable under certain conditions. The system of equations were solved analytically using parameter-expanding method and direct integration. The results are presented graphically and discussed. Our results showed that the concentration of free extracellular Chlamydia particles, number of infected, uninfected and recovered epithelial cells are significantly influenced by the parameters involved.

Keywords: Chlamydia, *Chlamydia trachomatis*, Sexually transmitted diseases (STDs),stability criteria

1.0 Introduction

Chlamydia trachomatis is a ubiquitous human pathogen that is responsible for the most prevalent bacterial sexually transmitted diseases (STDs) worldwide [1]. As an obligate intracellular bacterium, it has a distinctive biphasic developmental cycle [2]. The cycle begins when metabolically inactive elementary bodies (EBs) infect the host cell and reside in a vacuole termed an inclusion body. EBs differentiate into non-infectious, metabolically active reticulate bodies that multiply by binary fission and re-differentiate into EBs after 30–48 hours and then are released from the cell by lysis or exocytosis to initiate a new round of infection [2].

Men or women who have receptive anal intercourse can be infected with Chlamydial infection in the rectum, which can cause rectal pain, discharge, or bleeding. Chlamydia can also be found in the throats of women and men as result of having oral sex with an infected partner [3, 4]. Chlamydial infection of the cervix can spread to the rectum and the greater the number of sex partners, the greater the risk of infection [4]. The disease also affects the vagina in females and the urethra in males and can be treated with antibiotics, but if left untreated can lead to sterility. Chlamydia as the most common $S.T.D_S$ worldwide, often causes asymptomatic infections for a long duration of time without detection and hence capable of infecting a very large population through transmission and sexual contacts over time [5].

A lot of work dealing with Chlamydia has been reported by many researchers [6 - 9]. In this paper, we extend the model of Wilson [10] by incorporating recovery through drug administration. This present study investigates the criteria under which the rate of recovery of infected cells through drug administration could lead to the stability of the equilibrium point. To simulate the dynamics analytically using parameter-expanding method and direct integration, we assume that the free extracellular chlamydia particles are produced at a constant rate.

2.0 Model Formulation

We modify the model of Wilson [10] by incorporating recovery through drug administration. In addition, we assume that the free extracellular chlamydia particles are produced at a constant rate. Arising from the above, the relevant mathematical equations are:

$$\frac{dC}{dt} = P_C + K_2 \mathrm{I} - \mu_C C \quad (1)$$

$$\frac{dE}{dt} = P_E E - \mu_E E - K_1 CE + \omega R \quad (2)$$

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$$\frac{d\mathrm{I}}{dt} = -K_2\mathrm{I} - \gamma\mathrm{I} + K_1CE - \mu_E\mathrm{I} - \delta\mathrm{I} \qquad (3)$$

$$\frac{dR}{dt} = (\delta+\gamma)\mathrm{I} - \mu_E R - \omega R \qquad (4)$$

As initial condition based on our assumptions, we choose

$$C(0)=C_0>0,\quad E(0)=E_0>0,\quad \mathrm{I}(0)=\mathrm{I}_0>0,\quad R(0)=R_0=0, \qquad (5)$$

where $C(t)$ is the concentration of free extracellular Chlamydia particles, $E(t)$ is the number of uninfected mucosal epithelial cells (main host cell for Chlamydia), $\mathrm{I}(t)$ is the number of Chlamydia-infected epithelial cell at time t, $R(t)$ is the number of epithelial cells which recovered from Chlamydia-infection, K_2 is the rate at which Chlamydia particles are released from infected cells, K_1 is the rate of Epithelial cell infection which may be influenced by antibodies, P_E is the Reproduction rate of mucosal epithelial cells, μ_E is the natural death rate of epithelial cells, μ_C is the natural death rate of Concentrated Chlamydia particles, γ is the rate of clearance (recovery) of infected cells due to cell-mediated immunity, δ is the recovery rate due to drug administration, P_C is the recruitment rate of the Chlamydia particles due to external interaction and ω is the waning off Immunity.

3.0 Method of Solution

3.1 Existence and Uniqueness of Solution

Theorem 1 Let $P_E = 0,\quad \mu_E = \mu_C = \mu$. Then the equations (1) – (4) with initial conditions (5) has a unique solution for all $t \geq 0$.

Proof: Let $P_E = 0,\quad \mu_E = \mu_C = \mu$ and $\phi = C + E + I + R$,we obtain

$$\frac{d\phi}{dt} = P_C - \mu\phi,\quad \phi(0) = (C_0 + E_0 + I_0 + R_0) = \phi_0 \qquad (6)$$

Using method of integrating factor (see Boyce and Diprima[11], p.16), we obtain the solution of problem (6) as

$$\phi(t) = \frac{P_C}{\mu}\left(1 - e^{-\mu t}\right) + \phi_0 e^{-\mu t} \qquad (7)$$

Then, we obtain

$$C(t) = \left(\frac{P_C}{\mu} + \left(\phi_0 - \frac{P_C}{\mu}\right)e^{-\mu t}\right) - \left(E(t) + I(t) + R(t)\right) \qquad (8)$$

$$E(t) = \left(\frac{P_C}{\mu} + \left(\phi_0 - \frac{P_C}{\mu}\right)e^{-\mu t}\right) - \left(C(t) + I(t) + R(t)\right) \qquad (9)$$

$$I(t) = \left(\frac{P_C}{\mu} + \left(\phi_0 - \frac{P_C}{\mu}\right)e^{-\mu t}\right) - \left(C(t) + E(t) + R(t)\right) \qquad (10)$$

$$R(t) = \left(\frac{P_C}{\mu} + \left(\phi_0 - \frac{P_C}{\mu}\right)e^{-\mu t}\right) - \left(C(t) + E(t) + I(t)\right) \qquad (11)$$

Hence, there exists a unique solution of problem (1) - (4). This completes the proof.

3.2 Stability of the Critical Points

To obtain the critical points, we set

$$\frac{dC}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

Let $C = x,\quad E = y,\quad I = z,\quad R = v$

Then, the steady states of (1) – (4) satisfy the following algebraic system:

$$\left.\begin{array}{c} P_C + K_2 z - \mu_C x = 0 \\ \tau y - K_1 x y + \omega v = 0 \\ K_1 x y - \alpha z = 0 \\ \beta z - \sigma v = 0 \end{array}\right\}, \qquad (12)$$

where $\alpha = (K_2 + \gamma + \mu_E + \delta),\ \beta = \delta + \gamma,\ \sigma = \mu_E + \omega,\ \tau = (P_E - \mu_E)$

Solving (12) simultaneously, we obtain

$$P_1 = \left(\frac{P_C}{\mu_C},\ 0,\ 0,\ 0\right) \qquad (13)$$

$$P_2 = \left(\begin{array}{c} \frac{\alpha\sigma\tau}{K_1(\alpha\sigma - \beta\omega)},\ \frac{(\alpha\sigma\tau\mu_C - P_C\alpha\sigma K_1 + \beta P_C K_1\omega)}{K_1 K_2\sigma\tau},\ \frac{\alpha\sigma\tau\mu_C - P_C\alpha\sigma K_1 + \beta P_C K_1\omega}{K_2(\alpha\sigma K_1 - \beta K_1\omega)}, \\ \frac{\beta(\alpha\sigma\tau\mu_C - P_C\alpha\sigma K_1 + \beta P_C K_1\omega)}{\sigma K_2(\alpha\sigma K_1 - \beta K_1\omega)} \end{array}\right)$$

$$= (\phi_1, \phi_2, \phi_3, \phi_4) \qquad (14)$$

Theorem 2: If $\frac{P_C}{\mu_C} \neq \frac{\alpha\sigma\tau}{K_1(\alpha\sigma - \beta\omega)}$ there exist two equilibria.

Proof: The infection-free equilibrium is given by $P_1 = \left(\frac{P_C}{\mu_C},\ 0,\ 0,\ 0\right)$

If $y \neq 0,\quad z \neq 0,\quad v \neq 0,$ then $x = \frac{\alpha\sigma\tau}{K_1(\alpha\sigma - \beta\omega)}.$

Hence the other equilibrium is

$$P_2 = \left(\begin{array}{c} \frac{\alpha\sigma\tau}{K_1(\alpha\sigma - \beta\omega)},\ \frac{(\alpha\sigma\tau\mu_C - P_C\alpha\sigma K_1 + \beta P_C K_1\omega)}{K_1 K_2\sigma\tau},\ \frac{\alpha\sigma\tau\mu_C - P_C\alpha\sigma K_1 + \beta P_C K_1\omega}{K_2(\alpha\sigma K_1 - \beta K_1\omega)}, \\ \frac{\beta(\alpha\sigma\tau\mu_C - P_C\alpha\sigma K_1 + \beta P_C K_1\omega)}{\sigma K_2(\alpha\sigma K_1 - \beta K_1\omega)} \end{array}\right)$$

This completes the proof.

Now, let us denote this infected equilibrium points $(\phi_1, \phi_2, \phi_3, \phi_4)$ where each component corresponds to an earlier specified value.

We let

$x^* = x - \phi_1,\quad y^* = y - \phi_2,\quad z^* = z - \phi_3,\quad v^* = v - \phi_4$

Then

$$\frac{dx^*}{dt} = K_2 z^* - \mu_C x^* \quad (15)$$

$$\frac{dy^*}{dt} = \tau y^* - K_1\phi_2 x^* - K_1\phi_1 y^* + \omega v^* \quad (16)$$

$$\frac{dz^*}{dt} = K_1\phi_2 x^* + K_1\phi_1 y^* - \alpha z^* \qquad (17)$$

$$\frac{dv^*}{dt} = \beta z^* - \sigma v^* \qquad (18)$$

Thus

$$\begin{pmatrix} x^* \\ y^* \\ z^* \\ v^* \end{pmatrix}' = A \begin{pmatrix} x^* \\ y^* \\ z^* \\ v^* \end{pmatrix}, \tag{19}$$

where

$$A = \begin{pmatrix} -\mu_C & 0 & K_2 & 0 \\ -q & -r & 0 & \omega \\ q & s & -\alpha & 0 \\ 0 & 0 & \beta & -\sigma \end{pmatrix}$$

and $q = K_1\phi_2, \quad r = K_1\phi_1 - \tau, \quad s = K_1\phi_1$

Thus

$$|A - \lambda I| = 0$$

implies

$$\lambda_1 = -\sigma$$

and

$$P(\lambda) = \lambda^3 + (\mu_C + r + \alpha)\lambda^2 + (\mu_C r + \mu_C \alpha + r\alpha - K_2 q)\lambda + (\mu_C r\alpha + K_2 q(s - r)) = 0 \tag{20}$$

Theorem 3: Let $q < 0$. Then Equation (20) has three negative roots or one negative root and two complex roots.

Theorem 4: The infected (endemic) equilibrium is globally asymptotically stable if $q < 0.$

Proof of theorems

The proof of the theorems 3 and 4 involved using the

(i) **Descartes rule of signs:**

The number of positive zeros of a polynomial with real coefficients is either equal to the number of variations in sign of the polynomial or less than this by an even number and

(ii) **Routh-Hurwitz criteria** [12]:

All zeros of $\lambda^3 + \alpha\lambda^2 + \beta\lambda + \gamma = 0$ *have negative real parts if and only if* $\alpha\beta - \gamma > 0$.

Therefore, all zeros of (20) have negative real parts if and only if

$$(\mu_C + r + \alpha)(\mu_C r + \mu_C \alpha + r\alpha - K_2 q) - (\mu_C r\alpha + K_2 q(s - r)) > 0$$

That is

$$(\mu_C + r + \alpha)(\mu_C r + \mu_C \alpha) + r\alpha(r + \alpha) - K_2 q(\mu_C + \alpha + s) > 0 \quad if \quad q < 0.$$

Proof of theorem 3

From $P(\lambda)$ in (20), we obtain

$$P(-\lambda) = -\lambda^3 + (\mu_C + r + \alpha)\lambda^2 - (\mu_C r + \mu_C \alpha + r\alpha - K_2 q)\lambda + (\mu_C r\alpha + K_2 q(s - r)) = 0$$

So the number of change in sign is 3, if $q < 0$. Hence by Descartes rule of signs, $P(\lambda)$ have either three negative roots or one negative root and two complex roots. This completes the proof.

Proof of theorem 4

Since the inequality holds if $q < 0$. By theorem 3 and Routh-Hurwitz criteria, (20) has

(i) Either three negative roots or

(ii) One negative root and two complex roots whose real parts are equal and negative.

So in either case the equilibrium is globally asymptotically stable. This completes the proof.

3.3 Solution by Parameter-expanding Method

Suppose the solution $x(t), y(t), z(t) and\ v(t)$ in (1) - (4) can be expressed as

$$\left.\begin{aligned} x &= x_0 + K_1 x_1 + K_1^2 x_2 + h.o.t \\ y &= y_0 + K_1 y_1 + K_1^2 y_2 + h.o.t \\ z &= z_0 + K_1 z_1 + K_1^2 z_2 + h.o.t \\ v &= v_0 + K_1 v_1 + K_1^2 v_2 + h.o.t \end{aligned}\right\}, \tag{21}$$

where $h.o.t.$ read "higher order terms in K_1, and $C = x, \quad E = y, \quad I = z, \quad R = v$. In our analysis we are interested only in the first two terms.

Substituting (21) into (1) - (4), and processing, we obtain:

$$\frac{dx_0}{dt} = P_C + K_2 z_0 - \mu_C x_0, \qquad x_0(0) = x_0 \tag{22}$$

$$\frac{dy_0}{dt} = \tau y_0 + \omega v_0, \qquad y_0(0) = y_0 \tag{23}$$

$$\frac{dz_0}{dt} = -\alpha z_0, \qquad z_0(0) = z_0 \tag{24}$$

$$\frac{dv_0}{dt} = \beta z_0 - \sigma v_0, \qquad v_0(0) = v_0 \tag{25}$$

$$\frac{dx_1}{dt} = K_2 z_1 - \mu_C x_1, \qquad x_1(0) = 0 \tag{26}$$

$$\frac{dy_1}{dt} = \tau y_1 - x_0 y_0 - \omega v_1, \qquad y_1(0) = 0 \tag{27}$$

$$\frac{dz_1}{dt} = -\alpha z_1 + x_0 y_0, \qquad z_1(0) = 0 \tag{28}$$

$$\frac{dv_1}{dt} = \beta z_1 - \sigma v_1, \qquad v_1(0) = 0 \tag{29}$$

Solving equations (22) – (29) by direct integration, we obtain

$$x(t) = e^{-\mu_C t}\left(\frac{P_C}{\mu_C}e^{\mu_C t} + \frac{K_2 z_0}{\mu_C - \alpha}e^{(\mu_C - \alpha)t}\right) + \left(x_0 - \frac{P_C}{\mu_C} + \frac{K_2 z_0}{\mu_C - \alpha}\right)e^{-\mu_C t} +$$

$$K_1\left(\begin{array}{l} ste^{-\sigma t} + \frac{s_1}{\alpha}e^{-(\mu_C+\alpha)t} - \frac{s_2}{\mu_C}e^{-2\mu_C t} + \frac{s_3}{(\tau+\mu_C)}e^{\tau t} + \frac{s_4}{(\alpha-\mu_C)^2}\left(1+(\alpha-\mu_C)t\right)e^{-\alpha t} + \\ \frac{s_5}{\mu_C - 2\alpha}e^{-2\alpha t} + \frac{s_6}{(\tau+\mu_C-\alpha)}e^{(\tau-\alpha)t} + \frac{s_7}{\alpha}e^{-\alpha t} + \frac{s_8}{\tau}e^{(\tau-\mu_C)t} + \frac{s_9}{\mu_C-\alpha}e^{(\mu_C-\alpha)t} + \\ \left(\begin{array}{l} \frac{s_2}{\mu_C} - \frac{s_1}{\alpha} - \frac{s_3}{\tau+\mu_C} - \frac{s_4}{(\alpha-\mu_C)^2} - \frac{s_5}{\mu_C-2\alpha} - \frac{s_6}{(\tau+\mu_C-\alpha)} - \frac{s_7}{\alpha} \\ -\frac{s_8}{\tau} - \frac{s_9}{\mu_C-\alpha} \end{array}\right)e^{-\mu_C t} \end{array}\right) \tag{30}$$

$$y(t) = \frac{\beta\omega z_0}{(\sigma-\alpha)(\tau+\alpha)}e^{-\sigma t} - \frac{\beta\omega z_0}{(\sigma-\alpha)(\tau+\alpha)}e^{-\alpha t} - \frac{v_0}{(\tau+\sigma)}e^{-\sigma t} +$$

$$\left(y_o + \frac{\beta\omega z_0}{(\sigma-\alpha)(\tau+\alpha)} - \frac{\beta\omega z_0}{(\sigma-\alpha)(\tau+\alpha)} - + \frac{v_0}{(\tau+\sigma)}\right)e^{\tau t} +$$

$$K_1\left(\begin{array}{l} -\frac{r(1+(\sigma+\tau)t)e^{-\sigma t}}{(\tau+\sigma)^2} - \frac{r_1}{(\tau+\sigma+\alpha)}e^{-(\sigma+\alpha)t} + \frac{r_2 e^{-(\sigma+\mu_c)t}}{(\tau+\sigma+\mu_c)} + r_3 te^{\tau t} - \\ \frac{r_4\left((\alpha^2-\sigma\alpha+\alpha\tau-\sigma\tau)t+\sigma+\tau\right)e^{(\alpha+2\tau)t}}{(\alpha+\tau)^2} - \frac{-r_5 e^{-2\alpha t}}{(\tau+2\alpha)^2} - \frac{r_6}{\alpha}e^{-(\alpha-\tau)t} + \\ \frac{r_7 e^{-(\sigma-\alpha-\mu_c)t}}{(\sigma-\alpha-\mu_c-\tau)} - \frac{r_8}{\mu_C}e^{-(\mu_C-\tau)t} + \frac{r_9}{(\sigma-\alpha-\tau)}e^{(\sigma-\alpha)t} - \frac{r_{10}}{(\tau+\sigma)}e^{-\sigma t} + \\ \frac{a_1 e^{(2\tau+\sigma)t}}{(\tau+\sigma)} - \frac{a_2 e^{-(\sigma+\alpha)t}}{(\sigma+\tau+\alpha)} + \frac{a_3 e^{-(\sigma+\mu_C)t}}{(\sigma+\tau+\mu_C)} + a_4 te^{\tau t} + \frac{a_5}{(\alpha+\tau)}e^{-\alpha t} + \frac{a_6 e^{-2\alpha t}}{(\tau+2\alpha)} \\ -\frac{a_7 e^{-(\alpha-\tau)t}}{\alpha} + \frac{a_8 e^{-(\alpha+\mu_C)t}}{(\alpha+\tau+\mu_C)} \end{array}\right) \tag{31}$$

$$z(t) = z_0 e^{-\alpha t} +$$

$$K_1\left(\begin{array}{l} \dfrac{a_1}{(\alpha-\sigma)}e^{-\sigma t} - \dfrac{a_2}{\sigma}e^{-(\alpha+\sigma)t} + \dfrac{a_3}{(\alpha-\sigma-\mu_C)}e^{-(\sigma+\mu_C)t} + \dfrac{a_4}{(\tau+\alpha)}e^{\tau t} - a_5 t e^{-\alpha t} + \dfrac{a_6}{\alpha}e^{-2\alpha t} + \\ \dfrac{a_7}{\tau}e^{(\tau-\alpha)t} + \dfrac{a_8}{\mu_C}e^{-(\alpha+\mu_C)t} + \dfrac{a_9}{(\alpha+\tau-\mu_C)}e^{(\tau-\mu_C)t} + \\ \left(\begin{array}{l} \dfrac{a_2}{\sigma} - \dfrac{a_1}{(\alpha-\sigma)} - \dfrac{a_3}{(\alpha-\sigma-\mu_C)} - \\ \dfrac{a_4}{(\tau+\alpha)} - \dfrac{a_6}{\alpha} - \dfrac{a_7}{\tau} - \dfrac{a_8}{\mu_C} - \dfrac{a_9}{(\alpha+\tau-\mu_C)} \end{array}\right)e^{-\alpha t} \end{array}\right) \quad (32)$$

$$v(t) = \frac{\beta z_0}{(\sigma-\alpha)}e^{-\alpha t} + \left(v_0 - \frac{\beta z_0}{\sigma-\alpha}\right)e^{-\sigma t} +$$

$$K_1\left(\begin{array}{l} pte^{-\sigma t} + \dfrac{p_1}{\alpha}e^{-(\sigma+\alpha)t} - \dfrac{p_2}{\mu_C}e^{-(\sigma+\mu_C)t} + \dfrac{p_3}{(\tau+\sigma)}e^{\tau t} + \dfrac{p_4}{(\alpha-\sigma)^2}\left(1+(\alpha-\sigma)t\right)e^{-\alpha t} + \\ \dfrac{p_5}{\sigma-2\alpha}e^{-2\alpha t} + \dfrac{p_6}{(\tau+\sigma-\alpha)}e^{(\tau+\sigma-\alpha)t} + \dfrac{p_7}{(\sigma-\alpha-\mu_C)}e^{(\sigma-\alpha)t} + \dfrac{p_8}{(\sigma+\tau-\mu_C)}e^{(\tau-\mu_C)t} + \\ \dfrac{p_9}{\sigma-\alpha}e^{(\sigma-\alpha)t} + \left(\begin{array}{l} \dfrac{p_2}{\mu_C} - \dfrac{p_1}{\alpha} - \dfrac{p_3}{\tau+\sigma} - \dfrac{p_4}{(\alpha-\sigma)^2} - \dfrac{p_5}{\sigma-2\alpha} - \dfrac{p_6}{(\tau+\sigma-\alpha)} - \\ \dfrac{p_7}{(\sigma-\alpha-\mu_C)} - \dfrac{p_8}{(\sigma+\tau-\mu_C)} - \dfrac{p_9}{\sigma-\alpha} \end{array}\right)e^{-\sigma t} \end{array}\right), \quad (33)$$

where

$$b = \frac{P_C}{\mu_C}, \quad b_1 = \frac{K_2 z_0}{\mu_C - \alpha}, \quad b_2 = x_0 - \frac{P_C}{\mu_C} - \frac{K_2 z_0}{\mu_C - \alpha}, \quad b_3 = \frac{\beta\omega z_0}{(\sigma-\alpha)(\tau+\sigma)},$$

$$b_4 = \frac{\beta\omega z_0}{(\sigma-\alpha)(\tau+\sigma)}, \quad b_5 = \frac{v_0}{\tau+\sigma}, \quad b_6 = y_0 + \frac{\beta\omega z_0}{(\sigma-\alpha)(\tau+\sigma)} - \frac{\beta\omega z_0}{(\sigma-\alpha)(\tau+\sigma)} + \frac{v_0}{\tau+\sigma}$$

$$a_1 = \left(bb_3 - bb_5\right) = b\left(b_3 - b_5\right), \quad a_2 = b_1\left(b_3 - b\right), \quad a_3 = a_3 = b_2\left(b_3 - b_5\right),$$

$$a_4 = bb_6, \quad a_5 = bb_4, \quad a_6 = b_1 b_4, \quad a_7 = b_1 b_6, \quad a_8 = b_2 b_4, \quad a_9 = b_2 b_6$$

$$p = \frac{\beta a_1}{\alpha-\sigma}, \quad p_1 = \frac{\beta a_2}{\sigma}, \quad p_2 = \frac{\beta a_3}{(\alpha-\sigma-\mu_C)}, \quad p_3 = \frac{\beta a_4}{(\tau+\alpha)}, \quad p_4 = \beta a_5,$$

$$p_5 = \frac{\beta a_6}{\alpha}, \quad p_6 = \frac{\beta a_7}{\tau}, \quad p_7 = \frac{\beta a_8}{\mu_C}, \quad p_8 = \frac{\beta a_9}{(\alpha+\tau-\mu_C)},$$

$$p_9 = \left(\frac{a_2}{\sigma} - \frac{a_1}{(\alpha-\sigma)} - \frac{a_3}{(\alpha-\sigma-\mu_C)} - \frac{a_4}{(\tau+\alpha)} - \frac{a_6}{\alpha} - \frac{a_7}{\tau} - \frac{a_8}{\mu_C} - \frac{a_9}{(\alpha+\tau-\mu_C)}\right)$$

$$s = \frac{k_2 a_1}{\alpha-\mu_C}, \quad s_1 = \frac{k_2 a_2}{\mu_C}, \quad s_2 = \frac{k_2 a_3}{(\alpha-2\mu_C)}, \quad s_3 = \frac{k_2 a_4}{(\tau+\alpha)}, \quad s_4 = k_2 a_5,$$

$$s_5 = \frac{k_2 a_6}{\alpha}, \quad s_6 = \frac{k_2 a_7}{\tau}, \quad s_7 = \frac{k_2 a_8}{\mu_C}, \quad s_8 = \frac{k_2 a_9}{(\alpha+\tau-\mu_C)},$$

$$s_9 = \left(\frac{a_2}{\mu_C} - \frac{a_1}{(\alpha-\mu_C)} - \frac{a_3}{(\alpha-2\mu_C)} - \frac{a_4}{(\tau+\alpha)} - \frac{a_6}{\alpha} - \frac{a_7}{\tau} - \frac{a_8}{\mu_C} - \frac{a_9}{(\alpha+\tau-\mu_C)}\right)$$

$$r = p\omega, \quad r_1 = \frac{p_1\omega}{\alpha}, \quad r_2 = \frac{p_2\omega}{\mu_C}, \quad r_3 = \frac{p_3\omega}{(\tau+\sigma)}, \quad r_4 = \frac{p_4\omega}{(\alpha-\sigma)},$$

$$r_5 = \frac{p_5\omega}{\sigma-2\alpha}, \quad r_6 = \frac{p_6\omega}{(\tau+\sigma-\alpha)}, \quad r_7 = \frac{p_7\omega}{(\sigma-\alpha-\mu_C)}, \quad r_8 = \frac{p_8\omega}{(\sigma+\tau-\mu_C)}, \quad r_9 = \frac{p_9\omega}{\sigma-\alpha}$$

$$r_{10} = \left(\frac{p_2}{\mu_C} - \frac{p_1}{\alpha} - \frac{p_3}{\tau+\sigma} - \frac{p_4}{(\alpha-\sigma)^2} - \frac{p_5}{\sigma-2\alpha} - \frac{p_6}{(\tau+\sigma-\alpha)} - \frac{p_7}{(\sigma-\alpha-\mu_C)} - \frac{p_8}{(\sigma+\tau-\mu_C)} - \frac{p_9}{\sigma-\alpha} \right)$$

The computations were done using computer symbolic algebraic package MAPLE.

4.0 Results and Discussion

Analytical solutions given by (30) - (33) are computed for the values of $P_C = 0.1,\ P_E = 0.02,\ K_1 = 1,\ K_2 = 0.1,\ \mu_C = 0.02,\ \mu_E = 0.01,\ \gamma = 0.2,\ \delta = 0.1,\ \omega = 0.04$

The cells population and concentration values are depicted graphically in Figures 1 – 4.

Figure 1 depicts the graph of $y(t)$ against t for different values of P_C. It is observed that the number of uninfected mucosal epithelial cells decreases as the recruitment rate of the Chlamydia particles increases. This is as a result of interaction between uninfected mucosal epithelial cells and free extracellular Chlamydia particles.

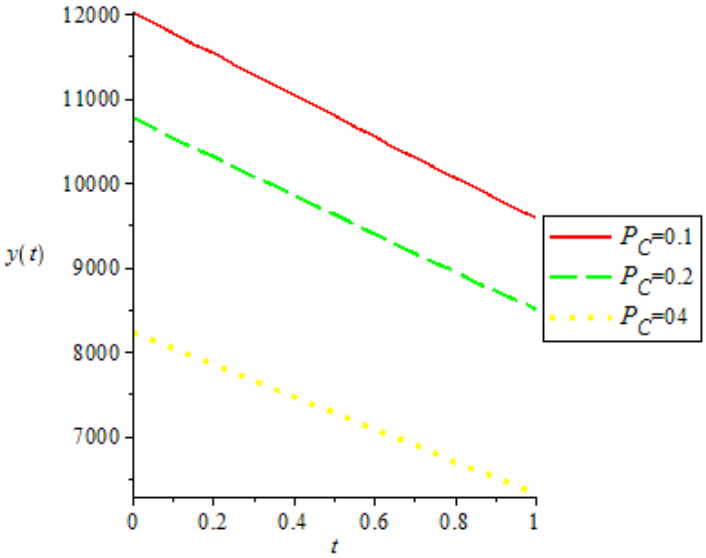

Figure 1: Plots of $y(t)$ against t for different values of P_C and $P_E = 0.02, K_1 = 1, K_2 = 0.1, \mu_C = 0.02, \mu_E = 0.01, \gamma = 0.2, \delta = 0.1, \omega = 0.04, v_0 = 0, x_0 = 50, y_0 = 30, z_0 = 10$

Figure 2 depicts the graph of $z(t)$ against t for different values of γ. It is observed that the number of infected mucosal epithelial cells decreases as the clearance rate (recovery) of the infected cells due to cell-mediated immunity increases.

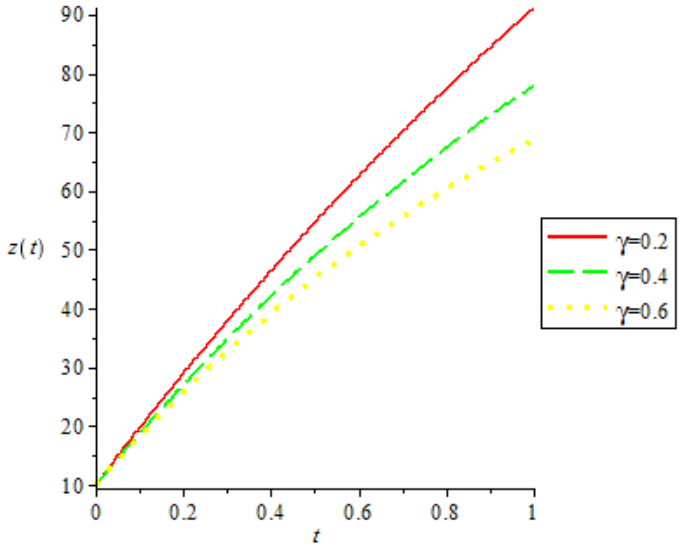

Figure 2: Plots of $z(t)$ against t for different values of γ and $P_C = 0.1$, $P_E = 0.02$, $K_1 = 1$, $K_2 = 0.1$, $\mu_C = 0.02$, $\mu_E = 0.01$, $\delta = 0.1$, $\omega = 0.04$, $v_0 = 0$, $x_0 = 50$, $y_0 = 30$, $z_0 = 10$

Figure 3 depicts the graph of $z(t)$ against t for different values of δ. It is observed that the number of infected mucosal epithelial cells decreases as the recovery rate due to drug administration increases.

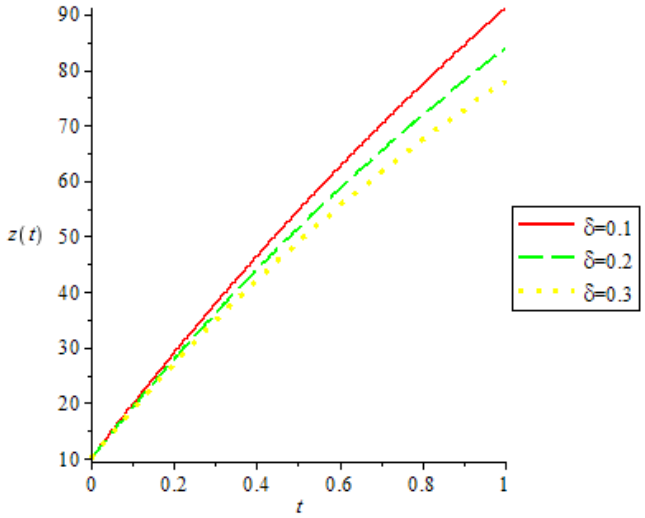

Figure 3: Plots of $z(t)$ against t for different values of δ and $P_C = 0.1$, $P_E = 0.02$, $K_1 = 1$, $K_2 = 0.1$, $\mu_C = 0.02$, $\mu_E = 0.01$, $\gamma = 0.2$, $\omega = 0.04$, $v_0 = 0$, $x_0 = 50$, $y_0 = 30$, $z_0 = 10$

Figure 4 depicts the graph of $x(t)$ against t for different values of μ_C. It is observed that the concentration of free extracellular Chlamydia particles decreases as natural death rate of Chlamydia particles increases.

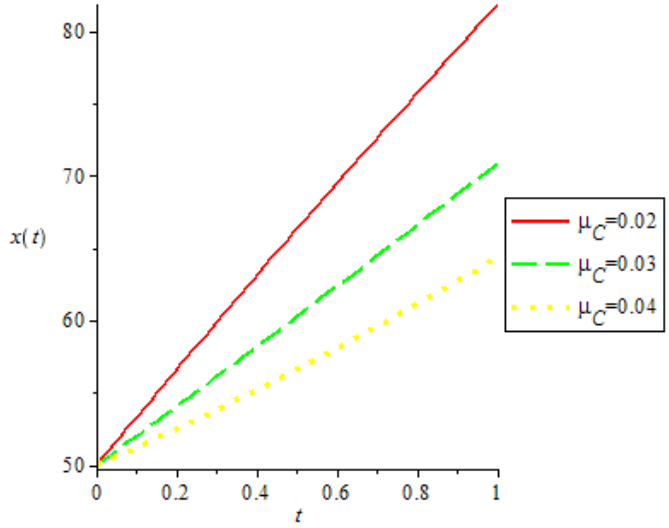

Figure 4: Plots of $x(t)$ against t for different values of μ_C and $P_C = 0.1$, $P_E = 0.02$, $K_1 = 1$, $\delta = 0.1$, $K_2 = 0.1$, $\mu_E = 0.01$, $\gamma = 0.2$, $\omega = 0.04$, $v_0 = 0$, $x_0 = 50$, $y_0 = 30$, $z_0 = 10$

It is worth pointing out that the effect observed in Figures 1 - 4, is an indication that if there is no interaction between uninfected mucosal epithelial cells and free extracellular Chlamydia particles the diseases can be eradicated or minimized.

5.0 Conclusion

The system of equations formulated to describe the dynamics of *Chlamydia Trachomatis* infection in a human carrier is solved analytically using parameter expanding method and direct integration technique. The governing parameters of the problem are the rate at which Chlamydia particles are released from infected cells (K_2), rate of Epithelial cell infectionwhich may be influenced by antibodies (K_1), reproduction rate of mucosal epithelial cells (P_E), natural death rate of epithelial cells (μ_E), natural death rate of Concentrated Chlamydia particles (μ_C), rate of clearance (recovery) of infected cells due to cell-mediated immunity (γ), recovery rate due to drug administration (δ), recruitment rate of the Chlamydia particles due to external interaction (P_C) and waning off Immunity (ω). We provide criteria under which drug administration and waning off immunity could provide a stable infected equilibrium. It is discovered that the concentration of free extracellular Chlamydia particles, number of infected, uninfected and recovered epithelial cells are significantly influenced by the parameters involved.

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