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Mathematical Modeling of Effluent Distribution in Ikpoba River,Benin City,Nigeria

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Abstract

A mathematical model for Ikpoba River Pollution was developed in this paper using ordinary differential equations and the solution by laplace transform method.The model was used to determine the level of pollution concentration along the downstream of the river.The concentration distribution decrease with distance and varies with time. The results of the first and second order ordinary differential equations using laplace transform modelling of Ikpoba river effluent discharge showed a range of value close to 429mg/L with a 10m distance apart. The model also take into consideration the water quality parameters such as Biochemical oxygen demand BOD, Chemical oxygen demand COD, Dissolve oxygen DO, and pH.This decrease in biochemical oxygen demand BOD effluent concentration decreases the dissolve oxygen DO Concentration which is safe for Aquatic life (Fish and Aquatic animal).

Keywords: Effluent, Water Quality, Water Pollution, Model, Ikpoba River

1.0 Introduction

Brewery and bottling companies in Nigeria have improved on quality of their products but much is yet to be done in the area of waste management in line with the Federal Environmental Protection Agency (FEPA) Regulation of effluent discharge into rivers. In Nigeria today for instance, almost all brewery and bottling companies discharge their effluent with treatment into rivers [1].

In Benin City, University of Benin, University of Benin Teaching Hospital, Guinness Nigeria PLC, and Bendel Brewery Ltd., discharge their effluents into Ikpoba River. The Breweries are located along Ikpoba Hill Road in Ikpoba Okha Local Government Area, while University of Benin and University of Benin Teaching Hospital are located along Ugbowo Road in Benin City. From the description given, it is evident that this river is used by, the inhabitants around them for domestic and agricultural purposes.

Therefore discharge of untreated brewery effluent into Ikpoba River may likely cause epidemic. The result of this study would be helpful in finding long time solution to the river pollution problem.

To determine the concentration of industrial effluent being discharged by the companies into Ikpoba river and its ecological effect on aquatic life. Assessing the effect of effluent loading in the river on human health arising from the usage of river water for domestic purpose.

Pimpunchatet al. [2] presented a simple mathematical model for river pollution

and investigate the effect of aeration on the degradation of pollutant, their model consists of a pair of coupled reaction-diffusion-advection equations for the pollutant and dissolved oxygen concentrations. Advection-diffusion equation: is the partial differential equation that governs the motion of a conserved scalar field as it is advected by a known velocity vector field.

The discharge of wastewater and effluent into surface water bodies and the resultant deterring change in water ecology have been reported by several researchers [3 – 14]

Moreover, Swayne *et al*.[15] observed that poorly organized and unregulated disposal of industrial and domestic wastes are regarded as major causes of deterioration of aquatic environment. Odiete. [16] states that changes brought about by pollution in water bodies may create hazards bothto human and animal health and may render water unfit for domestic, industrial and agricultural activities and otherwise.

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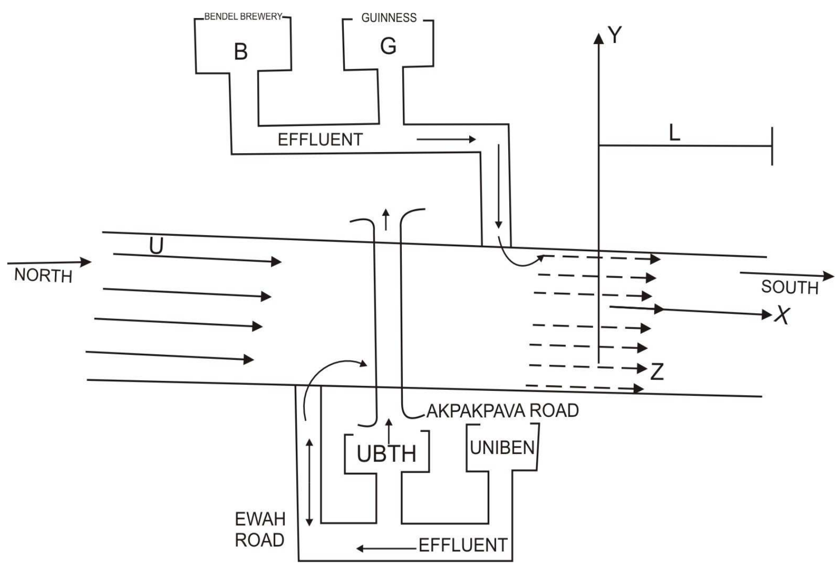
Salem et al.[17] also stated that water pollution from human activities,either industrial or domestic is a major problem in many countries as a result approximately 25 million persons die every year.He assessed the water quality in a river and considered many factors: the level of dissolved oxygen,Biochemical oxygen demand,chemical oxygen demand,chloride,nitrates,phosphates,suspended solids and presence of bacteria.
The study represents the attempt for the reseachers to study the problem of pollution in Ikpoba river and we think that this mathematical analysis would provide a better planning for water quality control.

2.0 Methodology

The samples of the effluent concentration were taken downstream the river.The various points of discharge into the river are upstream of the sampling location.The water quality parameters (BOD,COD,DO,and PH) were analysed based on the samples obtained within a 10 metre distance apart. First order differential equation by integrating factor was used to model the concentration of the river downstream with respect to sampling distance apart,chemical decay,discharge rate of the sources of discharge,speed of the river,cross sectional area, width,length and depth of the river which are all regarded as attribute of the river under investigation. Laplace transform was carried out on the first and second order differential equations presented by the integrating factor technique the concentration was modelled with respect to discharge rate,speed of the river,cross sectional area and the chemical decay for the first order laplace while the entire attribute of the river under investigation were considered for the second order laplace transform.
The data obtained were analysed using the following mathematical techniques stated above.



MODEL OF EFFLUENT DISCHARGE TO IKPOBA RIVER

Figure 1: Model of effluent discharge to Ikpoba River

2.1 Model Development

(I) With partial differential equation (PDE)
Y is measured along the width of the river
X is measured along the length L of the river
Z measured the depth of Q the river
Q = discharge rate (g/sec)
C = concentration in mg/l or ppm
U = river speed in meter/sec
A = Cross sectional area of river (m^2)
C = C(x) only, i.e. varies along the downstream only but not across river bank or down the river bed, its twice differentiable in X
Conducting a mass balance, we notice that.

$$[\text{Effluent concentration input into river}] = [\text{transport of pollutant by diffusion}] +$$
$$[\text{transport of pollutant by convention with moving stream}] + [\text{transport of pollutant by chemical decay}]$$

$$\frac{Q(x)}{A} KC'' - UC' - \beta C$$

$$KC'' - UC' - \beta C = \frac{-Q(x)}{A} (-\infty < x < \infty) \qquad (1)$$

$$KC'' = K\nabla^2 C$$

$$-\beta C = \nabla C, -\beta = reaction \quad rate \quad cons \quad \tan \quad t$$

This we see that the diffusion of chemical pollutant in the river is governed by the partial differential equation (PDE) of mathematics physics.

If we ignore the diffusion component of equation (1) the general solution is

$$UC' + \beta C = \frac{Q(x)}{A} \qquad (2)$$

Which is a first order differential equation

If the diffusion component is not ignore, the general equation is of the form.

$$KC'' + UC' + \beta C = \frac{Q(x)}{A} \qquad (3)$$

Considering these equation

$$UC' + \beta C = \frac{Q(x)}{A} \qquad (4)$$

Using integrate factors method

The method equations is the form

$$\frac{dy}{dx} + Py = Q, \qquad (5)$$

where P and Q are functions of X, using integrating factors method

let IF denote the integrating factor

then

$$IF = \ell^{\int Pdx} = \frac{dy}{dx} + Py\ell^{\int Pdx} = Q\ell^{\int Pdx} \qquad (6)$$

L.H.S is the differential coefficient of

$$y\ell^{\int Pdx}$$

$$\frac{d}{dx}\left[y\ell^{\int Pdx} \right] = Q\ell^{\int Pdx} \qquad (7)$$

integrating both sides with respect to X

$$y\ell^{\int Pdx} = \int Q\ell^{\int Pdx}\, dx \qquad (8)$$

Indicating the integrating factors by IF, this result becomes

$$Y.IF = \int Q.IF\, dx \qquad (9)$$

$$\frac{dy}{dx} + Py = Q\,(where\, P\, and\, Q\, are\, function\, of\, x) \qquad (10)$$

it can be shown that C(x) is generated by the differential equation

$$KC'' - UC' - \beta C = \frac{Q(x)}{A} \qquad (11)$$

Where K = diffusion constant in m^2/s

β = chemical decay constant in g/s/g

Co = concentration of chemical oxygen demand (Mg/L)
A = Cross sectional area of river (m^2)
C_1 = f (x) ie varies along downstream but not across riverbanks
Q = discharge rate (g/sec)
The total mass balance is

$$KC'' + UC' + \beta C = \frac{Q(x)}{A} \tag{12}$$

Boundary conditions

(i) C ($-\infty$) =0, i.e. the river is clear upstream it is unpolluted before the discharge point. Let us suppose that L = $-\infty$ and k is sufficiently small so that k tend to zero.

Then $UC' + \beta C = \frac{Q(x)}{A}$ (13)

(ii) Condition for the solution of the differential equation is to solve for C(x) and the graph of C(x) against distance.

3.0 Analysis of Results

3.1 Determination of Effluent Concentration

Effluent concentration of Ikpoba River can be determine when the diffusion component of (1) is ignore as follows:

$$UC' + \beta C = \frac{Q(x)}{A} \tag{14}$$

Where C = concentration of effluent
β = chemical decay constant (g/s/g)
U = Velocity of the river

A = cross sectional area (m^2)
Assuming that U, Q, A, β are constants

$$C(x) = U\frac{dc}{dx} + \beta C = \frac{Q(x)}{A} \tag{15}$$

Let$\mathcal{L}$ denote the Laplace transform

f(x) = f(s)

$\mathcal{L}(C) = \bar{C}$

$\mathcal{L}(C') = S\bar{C} - C_o$

$$\mathcal{L}\left(\frac{Q(x)}{A}\right) = \frac{Q}{A}.\frac{1}{5^2} = \frac{Q}{AS^2}$$

ie $\frac{d}{dt} = S$

$$UC(s)S + \beta C(s) = \frac{Q(s)}{A}$$

$$C(s)(US + \beta) = \frac{Q(s)}{A} \tag{16}$$

$$C(s) = \frac{Q(s)}{A[US + \beta]}$$

Substituting parameters:

U= 0.8m/s, A = 35.6m^2, β = 0.0447g/sec/g, Q = 32637g/sec

C (s) = $\dfrac{\dfrac{916.7697}{0.8}}{\dfrac{S+0.0447}{0.8}}$

C (s) = $\dfrac{1145.9621}{S+0.055875}$

Taking the inverse laplace transform $\mathcal{L}$

$\mathcal{L}[\dfrac{1145.9621}{S+0.055875}]$

= 1145.9621 $\ell^{-0.055875x}$

C(x) = 1145.9621 $\ell^{-0.055875x}$ (17)

This is an exponentially damped or special case of Fourier Series.

3.2 Determination of Effluent Concentration Equation when the Diffusion Component is not Ignored

However, if this term is not ignored, then the general equation for the determination of Effluent concentration of Ikpoba River is given in equation (18)

$$KC'' + UC' + \beta C = \frac{Q(x)}{A}$$

$$K\frac{d^2C}{dx^2} + \frac{dc}{dx} + \beta C = \frac{Q(x)}{A} \quad (18)$$

Let $\mathcal{L}$denote Laplace transform

$\mathcal{L}$(C) = $\overset{=}{C}$

$\mathcal{L}$(C') = S $\overset{=}{C}$ – C_o

$\mathcal{L}$(C'') = $S^2 \overset{=}{C}$ - $SC_o - C_1$

i.e. $\dfrac{d}{dt} = S,\ \dfrac{d^2}{dt^2} = S^2$

$C'' = S^2 C(s) - SC(o) - C'(o)$

Initial conditions are zero

$$K\frac{d^2C}{dx^2} + U\frac{dc}{dx} + \beta C = \frac{Q(x)}{A} \quad (19)$$

where K = diffuse constant in m^2/s

β = chemical decay constant, g/s/g

C_o = concentration of chemical oxygen demand (mg/L)

A = Cross sectional Area of river (m^2)

Q = discharge rate, g/sec

$KC(s)\,S^2 + UC(s)\,S + \beta\, C(s) = \dfrac{Q(s)}{A}$ (20)

$C(s)\,[KS + US + \beta] = \dfrac{Q(s)}{A}$

$$C(s) = \frac{Q(s)}{A\left[KS^2 + US + \beta\right]} \quad (21)$$

Substituting parameters:

U = 0.8m/s, β = 0.0447g/sec/g, A = 35.6m^2 Q= 32637g/sec, k = 3.58m^2/s

$$C(s)=\frac{32637}{35.6\left[3.58S^2+0.85+0.0447\right]}$$

$$C(s)=\frac{916.7697}{\left[3.58S^2+0.85+0.0447\right]}$$

$$=\frac{32637}{\left[3.58S^2+0.4S+0.4S+0.0447\right]}$$

3.585 [S + 0.11173] + 0.4 [S + 0.11175]

$$C(s)=\frac{916.7697}{\left[S+0.11173\right]\left[3.585+0.4\right]}$$

$$=\frac{916.7697}{\left[S+0.11173\right]\left[S+0.11173\right]}$$

$$=\frac{A}{S+0.11173}+\frac{B}{(S+0.11173)^2}$$

$$=\frac{A(S+0.11173)+B}{(S+0.11173)^2}$$

916.7696 ≡ A (S + 0.11173) + B

put S = -0.11173

but = 916.7697

comparing the highest power of S

A= 0

$$\frac{A}{S+0.11173}+\frac{B}{(S+0.11173)^2}=\frac{916.7697}{(S+0.11173)^2}$$

taking the inverse laplace transform

$$\mathcal{L}\left[\frac{916.7697}{(S+0.11173)^2}\right]$$

C(x) = 916.7697 $x\ell^{-0.11173x}$ (22)

Equation (22) is the general solution. This also an exponentially damped or special case of Fourier series.

4.0 Results

Data obtained from the preliminary investigation is presented in Table 1

Table 1: Effluent Concentration of Ikpoba River

DAY	1M	2M	3M	4M	5M	6M	7M	8M	9M	10M
1	3818	3642	3474	3314	3163	3018	2883	2753	2630	2514
2	3920	3740	3567	3505	3246	3098	2957	2824	2690	2577
3	3851	3669	3499	3338	3185	3040	2903	2772	2648	2531
4	3686	3517	3355	3188	3043	2906	2776	2652	2535	2423
5	3941	3758	3583	3418	3261	3112	2970	2836	2709	2588
6	3572	3409	3254	3106	2966	2833	2707	2611	2473	2365
7	3289	3141	3000	2866	2739	2618	2504	2395	2391	2193
8	3525	3365	3212	3066	3039	2797	2672	2555	2443	2336
9	3383	3230	3085	2946	2815	2690	2572	2459	2352	2251
10	3762	3588	3423	2366	2117	2976	2842	2715	2594	2479

First Oder Differential Equation Using Laplace Transform

Using the laplace transform obtained in equation the following effluent concentrations were obtained at various points as shown in Table 2

Table 2: Distribution of Effluent Concentration By First Order Differential Equation

Distance	Velocity(U)	Area(A)	Chemical Decay Constant	Flow rate(Q)	Concentration
1	0.8	35.6	0.0447	32637	820
2	0.8	35.6	0.0447	32637	733
3	0.8	35.6	0.0447	32637	656
4	0.8	35.6	0.0447	32637	586
5	0.8	35.6	0.0447	32637	524
6	0.8	35.6	0.0447	32637	470
7	0.8	35.6	0.0447	32637	419
8	0.8	35.6	0.0447	32637	375
9	0.8	35.6	0.0447	32637	335
10	0.8	35.6	0.0447	32637	300

Second Order Differential Equation Using Laplace Trasform

Using the laplace transform obtained in equation the following effluent concentrations were obtained at various points as presented in Table 3

Table 3: Distribution of Effluent Concentration By Second Order Differntial Equation

Distance (m)	Velocity(U) m/s	Area(A) m^2	Chemical Decay Constant g/s/g	Flow rate(Q) g/s	Concentration mg/L
1	0.8	35.6	0.0447	32637	1084
2	0.8	35.6	0.0447	32637	1025
3	0.8	35.6	0.0447	32637	969
4	0.8	35.6	0.0447	32637	916
5	0.8	35.6	0.0447	32637	867
6	0.8	35.6	0.0447	32637	820
7	0.8	35.6	0.0447	32637	775
8	0.8	35.6	0.0447	32637	733
9	0.8	35.6	0.0447	32637	693
10	0.8	35.6	0.0447	32637	655

5.0 Discussion

In his study conducted on brewery effluent on Ikpoba River, Eguaje [10] observed as follows:

i. The natural quality of Ikpoba River has been considerably affected by the effluent discharge into it by operating alcoholic beverage companies in the vicinity of the river.

ii. The pollutive effluent is highly oxygen demanding; has a high level of suspended matter, highly coloured, choking in odour and discharged in high quantities. The study further noted that the total microbal density and aquatic life have been adversely affected. Oguzie and Okhagbuzo [11], Ekhaise and Anyasi [9] carried out studies of brewery effluent discharged in the same Ikpoba river and observe that aquatic life in that system is threatened because the level of pollution is alarmingly high. Whereas the former employed statistics to analyze empirical data collected, the latter used mere data presentation to examine the problem.

The results of the first and second order ordinary differential equations using laplace transform modelling of ikpoba river effluent discharge show a range of value close to 429mg/L with a 10m distance apart. Previous Igboanugo[12] study of this nature using ordinary differential equation shown a range of value of 922mg/L within the same distance apart. This first and second order differential equations of effluent concentration distribution can be represented by a signal function,hence the signal i.e. concentration dies off (tail off) downstream as $x \rightarrow \infty$

6.0 Conclusion

The results of the first and second order ordinary differential equations using laplace transform modelling of ikpoba river effluent discharge show a range of value close to 429mg/L with a 10m distance apart. Previous igboanugo [12] study of this nature using ordinary differential equation shown a range of value of 922mg/L within the same distance apart.This decrease in biochemical oxygen demand BOD effluent concentration decreases the dissolve oxygen DO Concentration which is safe for Aquatic life (Fish and Aquatic animal)

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