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# A Simplified Numerical Solution Method for Taitel and Dukler's Model of Stratified Gas-Liquid Flow in Horizontal and Slightly Inclined Pipelines.

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## *Abstract*

***Stratified gas-liquid flow is a flow regime commonly encountered in horizontal and slightly inclined pipelines. It is generally considered to be a prerequisite for severe slugging in pipeline-riser systems. The motivation for the work discussed in this paper stem from the widespread presentation of multiphase flow regime maps in terms of superficial velocities. In this study, a simplified numerical solution method for the Taitel and Dukler's model is presented for the special case of low flow rates characteristic of severe slugging regime. The solution method was implemented using MATLAB® and comparison of the results with those of Taitel and Dukler and other investigators were satisfactory.***

**Keywords:** Gas-liquid flow, Stratified flow, Flow regime map, Superficial velocities, Taitel and Dukler model

**Nomenclature:**

| | |
|---|---|
| $A_L$ | Cross sectional area of the liquid phase, $m^2$ |
| $A_G$ | Cross sectional area of the gas phase, $m^2$ |
| $D$ | Diameter of the pipe, $m$ |
| $F$ | Froude number |
| $f_L$ | Friction factor |
| $f_{LS}$ | Friction factor when only liquid flows in the pipe |
| $h_L$ | Equilibrium liquid level, $m$ |
| $g$ | Acceleration due to gravity, m/s$^2$ |
| $M_G$ | Molecular weight of gas, *kg/kmol* |
| $P$ | Pressure, *Pa* |
| $R$ | Universal gas constant, *J/mol-K* |
| $S_L$ | Wetting periphery of the liquid phase, $m^2$ |
| $T$ | Temperature, *K* |
| $u_G$ | Superficial gas velocity, *m/s* |
| $u_{GS}$ | Maximum superficial gas velocity, *m/s* |
| $u_L$ | Superficial liquid velocity, *m/s* |
| $v_L$ | Liquid phase kinematic viscosity, $m^2/s$ |
| $\alpha_G$ | Average gas holdup in the pipeline |
| $\beta$ | Angle of inclination, ° |
| $\rho_L$ | Liquid phase density, $kg/m^3$ |
| $\rho_G$ | Liquid phase density, $kg/m^3$ |
| $\tau_L$ | Shear stress, $N/m^2$ |

## 1.0 Introduction

Stratified gas-liquid flow is one of the common flow regimes encountered in horizontal and slightly inclined pipelines. It has been reported to be a precursor to severe slugging in pipeline-riser systems [1,2]. Stratified flow, either smooth or wavy occurs in a horizontal or slightly inclined pipeline because of gravitational separation of the phases, with the liquid phase flowing at the bottom of the pipe and the gas phase at the top [3-7]. Taitel and Dukler [3] were the first set of researchers to develop a comprehensive analytical model for stratified gas-liquid flow in horizontal and near horizontal pipelines. They also

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proposed a transition criterion from stratified flow to non-stratified flow based on Kelvin-Helmholtz instability [8]. The boundary between stratified flow and non-stratified flow is usually given by a plot of superficial gas velocity against superficial liquid velocity [3,9-11].

A simplified numerical solution method for the Taitel and Dukler's stratified flow model for gas-liquid flow in horizontal and near horizontal pipelines is proposed in this study for the special case of low flow rates which characterises the severe slugging regime. The simplified numerical solution method was implemented in MATLAB® and the results (in terms of superficial velocities) were compared with those of Taitel and Dukler [3] and other researchers.

## 2.0 Background

Taitel and Dukler [3] suggested that the criterion at which transition from stratified flow occurs is given by:

$$F^2\left[\frac{1}{C_2^{\,2}}\frac{\tilde{u}_G^{\,2}}{\tilde{A}_G}\frac{d\tilde{A}_L}{d\tilde{h}_L}\right] \geq 1 \tag{1}$$

where $F$ is a Froude number modified by the density ratio and it is defined as:

$$F = \sqrt{\frac{\rho_G}{\rho_L - \rho_G}}\frac{u_{GS}}{\sqrt{Dg\cos\beta}} \tag{2}$$

The dimensionless variables are defined as follows:

$$\tilde{u}_G = \frac{u_G}{u_{GS}} = \frac{\tilde{A}}{\tilde{A}_G} = \frac{A/D^2}{\tilde{A}_G} = \frac{\pi/4}{\tilde{A}_G} \tag{3}$$

$$\tilde{A}_G = 0.25\left[\cos^{-1}(2\tilde{h}_L - 1) - (2\tilde{h}_L - 1)\sqrt{1-(2\tilde{h}_L - 1)^2}\right] \tag{4}$$

$$C_2 = 1 - \tilde{h}_L \tag{5}$$

$$\frac{d\tilde{A}_L}{d\tilde{h}_L} = \sqrt{1-(2\tilde{h}_L - 1)^3} \tag{6}$$

The equilibrium level is defined as:

$$\tilde{h}_L = \frac{h_L}{D} \tag{7}$$

Taitel and Dukler [3] presented a generalised flow regime map for horizontal and slightly inclined two-phase flow and they used $F$ versus $X$ as coordinates to evaluate the transition from stratified flow to non-stratified flow. The variable $X$ is the square root of the ratio of superficial liquid pressure drop to superficial gas pressure drop and is defined as follows:

$$X = \left[\frac{\left|(dP/dx)_{LS}\right|}{\left|(dP/dx)_{GS}\right|}\right]^{\frac{1}{2}} \tag{8}$$

where: $\left|(dP/dx)_{LS}\right|$ = pressure drop if the liquid flows alone in the pipe

$\left|(dP/dx)_{GS}\right|$ = pressure drop if the gas flows alone in the pipe

Taitel and Dukler [3] recalculated the flow regime transition boundaries in terms of superficial liquid velocity and superficial gas velocity in order to make a comparison with the flow regime maps of other investigators. However, the details of the procedure used for the recalculation was not provided. Similarly, Barnea [4], presented a generalised flow regime map for two-phase flow in pipes for the whole range of pipe inclinations making use of $F$ versus $\tilde{h}_L$ as the coordinates for the transition from stratified to non-stratified flow. Furthermore, it was suggested that the transition line could also be plotted in terms of superficial liquid velocity and superficial gas velocity.

As a follow up, Taitel [12] further developed the approach presented in Taitel and Dukler [3] for calculating the equilibrium liquid level in stratified flow and applied it to the special case where the gas velocity is small (which is typical of severe slugging flow regime) and arrived at a momentum balance of shear stress and gravity on the liquid phase as follows:

$$\tau_L S_L = \rho_L g A_L \sin\beta \tag{9}$$

The sheer stress ($\tau_L$) was defined as follows:

$$\tau_L = f_L \frac{\rho_L u_L^{\ 2}}{2} \qquad (10)$$

The friction factor ($f_L$) can be calculated from the Moody chart making use of the appropriate hydraulic diameter. However, for smooth pipes, the friction factor may be calculated by making use of the following equation [12]:

$$f_L = C_L \left( \frac{4 A_L u_L}{S_L v_L} \right)^{-m} \qquad (11)$$

where: $C_L$= 0.046 and m = 0.2 (for turbulent flow)
$C_L$= 16 and m = 1 (for laminar flow)
$A_L$= the cross-sectional area of the liquid phase
$S_L$= the wetting periphery of the liquid phase
$u_L$= the actual velocity of the liquid phase
$v_L$= kinematic viscosity of the liquid phase

The cross-sectional area of the liquid phase ($A_L$) and the wetting periphery of the liquid phase ($S_L$), in terms of the equilibrium liquid level ( $\tilde{h}_L$ ), are given as follows [12]:

$$A_L = 0.25 D^2 \left[ \pi - \cos^{-1}(2\tilde{h}_L - 1) + (2\tilde{h}_L - 1)\sqrt{1 - (2\tilde{h}_L - 1)^2} \right] \qquad (12)$$

$$S_L = D\left[ \pi - \cos^{-1}(2\tilde{h}_L - 1) \right] \qquad (13)$$

The average gas fraction (gas holdup) in the pipeline is given by:

$$\alpha_G = 1 - \frac{A_L}{A} \qquad (14)$$

Taitel [12] suggested a trial and error solution of Equation (9) as a means of evaluating the gas fraction in the pipeline. The general solution that was obtained is presented in the form of a dimensionless function as follows:

$$\alpha_G = f\left[ \frac{(\rho_L - \rho_G)\, g \sin\beta}{(dP/dx)_{LS}} \right] \qquad (15)$$

The pressure drop when the liquid flows alone in the pipe *$(dP/dx)_{LS}$* is given as:

$$\left( \frac{dP}{dx} \right)_{LS} = f_{LS} \frac{\rho_L u_{LS}^{\ 2}}{2} \qquad (16)$$

$f_{LS}$ is the value of the friction factor when the liquid flows alone in the pipe. Under conditions of turbulent flow, $f_{LS}$ is approximately equal to $f_L$ [12].

## 3.0 Proposed Simplified Numerical Solution Method

Taking the transition criterion presented in Equation (1) into consideration, the maximum superficial velocity for stratified flow can be obtained by rearranging Equation (2) as follows:

$$u_{GS} \leq F\sqrt{Dg\cos\beta}\sqrt{\frac{\rho_L - \rho_G}{\rho_G}} \qquad (17)$$

Since gas densities are strongly dependent on temperature and pressure conditions unlike liquid densities which are fairly constant over a reasonable range of temperature and pressure conditions, the gas densities may be calculated by using a transformation of the ideal gas equation as follows:

$$\rho_G = \frac{P}{RT} M_G \qquad (18)$$

where:$\rho_G$= gas density at the operation pressure and temperature, kg/m$^3$
$P$= operating pressure, *Pa*
$R$= gas constant = 8.314 *J/mol-K*
$T$= operating temperature, *K*
$M_G$= molecular weight of gas, kg/kmol

Equation (17) shows that the maximum superficial gas velocity ($u_{GS}$) is a function of pipe diameter, pipe inclination, fluid densities and equilibrium liquid level. Thus, when the pipe diameter, pipe inclination and fluid densities are known, the maximum superficial gas velocities for various equilibrium liquid level values can easily be calculated. This may be seen as an indirect way of calculating the superficial gas velocity at the transition boundary between stratified flow and non-stratified flow. A similar procedure was adopted for the calculation of the corresponding superficial liquid velocity at the transition boundary. This procedure which may be considered novel is outlined as follows:
The first step was to combine Equations (10) and (11) with a view to eliminating $f_L$. This resulted in:

$$\tau_L = C_L \left( \frac{4A_L u_L}{S_L v_L} \right)^{-m} \frac{\rho_L u_L^{\ 2}}{2} \qquad (19)$$

Similarly, combining Equations (9) and (19) with a view to eliminating $\tau_L$ and subsequently rearranging the resulting equation resulted in the following:

$$u_L = \left[ \left( \frac{2gA_L \sin\beta}{C_L S_L} \right) \left( \frac{4A_L u_L}{S_L v_L} \right)^m \right]^{\frac{1}{2}} \qquad (20)$$

It can be observed that Equation (20) is an implicit function of the equilibrium liquid level ($\tilde{h}_L$) and the liquid velocity ($u_L$). This observation follows from the fact that for any given angle of pipe inclination (or declination), and assuming constant values for $g$, $v_L$and $C_L$, the variables $A_L$and $S_L$ are dependent on the equilibrium liquid level as given by Equations (12) and (13). Thus, when the value of the equilibrium liquid level is specified, the superficial liquid velocity ($u_L$) can be determined by combining Equations (12) and (13) with (20) and solving numerically. Following from that, $u_{LS}$ can readily be calculated as follows:

$$u_{LS} = \alpha_L u_L = (1-\alpha_G) u_L \qquad (21)$$

Alternatively, Equations (20) and (21) can be combined to eliminate $u_L$ to give a new equation in terms of $u_{LS}$ as follows:

$$u_{LS} = \frac{1}{(1-\alpha_G)} \left[ \left( \frac{2gA_L \sin\beta}{C_L S_L} \right) \left( \frac{4A_L u_{LS}}{S_L v_L (1-\alpha_G)} \right)^m \right]^{\frac{1}{2}} \qquad (22)$$

With the value of the equilibrium liquid level and the constant parameters ($\beta$, g, $v_L$ and $C_L$) known, the superficial liquid velocity can be calculated by combining Equation (22), with Equations (12), (13) and (14).

## 4.0 Application of the Simplified Numerical Solution Method

The proposed simplified numerical solution method was tested to determine its validity by using it to simulate three case scenarios. The simulation results obtained were compared with results (i.e. flow regime maps) obtained by other researchers.

### 4.1 Case 1

The proposed solution method was used to simulate the flow of an air–water system at 1 atm and 25 $^oC$ through a 2.5 cm diameter horizontal pipeline. The solution scheme was implemented in MATLAB. It should be pointed out that since the pipe is horizontal, the angle of inclination $\beta$ should be zero. However, in order for the computation in MATLAB not to be aborted, a $\beta$ value of 0.05° was used in place zero. Figure 1 shows the stratified/non-stratified boundary predicted by the proposed simplified numerical solution method for the flow of an air-water system through a 2.5 cm pipeline at a pressure of 1 atm and a temperature of 25 $^oC$.

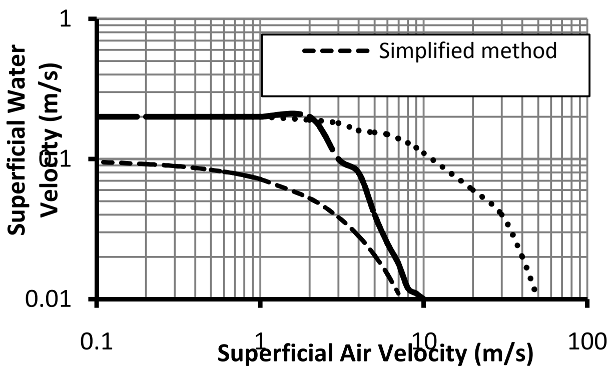


**Figure 1:** Boundary prediction for the flow of Air-Water through a 2.5 cm diameter pipe at 1 atm and 25 $^oC$ ($\beta$= $0^o$)

The Figure also compares the results obtained in this study with those of Taitel and Dukler [3] and Mandhane et al. [10] for the same system configuration. It was observed that the prediction of the proposed numerical solution method displayed a trend similar to that of the other researchers that the results were compared with. However, the proposed method predicted lower boundary values when compared with those earlier reported by Taitel and Dukler [3] and Mandhane et al. [10]. It should be noted that the lower boundary values predicted by the simplified numerical solution method do not in any way invalidate the method. This is because the solution method was based on the momentum balance approximation suggested by Taitel [12] for severe slugging flow regime; hence it only covered the portion of stratified flow regime that can give rise to severe slugging. The lower boundary values were actually expected since severe slugging is normally associated with low liquid and gas flow rates.

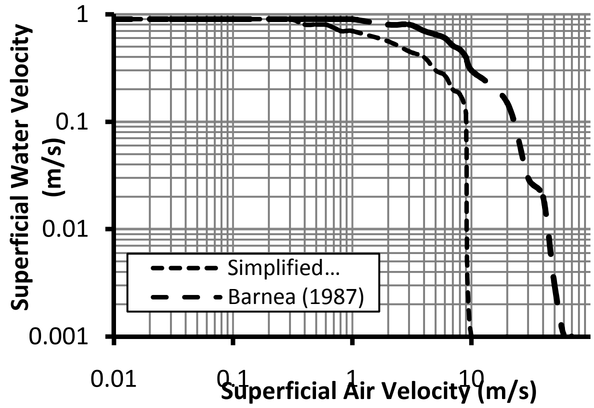


**Figure 2:** Boundary prediction for the flow of Air-Water through a 5.1 cm diameter pipe at 1 bar and 25 $^oC$ ($\beta$= -1$^o$)

## 4.2: Case 2

The proposed solution method was also used to simulate the flow of an air–water system at 1 bar and 25 $^oC$ through a 5.1 cm diameter pipeline with a declination angle of 1°. The solution method proposed was also implemented in MATLAB and the simulation results were compared with those reported in literature. Figure 2 shows the stratified/non-stratified boundaries predicted by the proposed simplified numerical solution method for the flow of an air-water system through a 5.1 cm pipeline at a pressure of 1 bar and a temperature of 25 $^oC$. The Figure also includes the boundaries obtained by Barnea [4] for the same system configuration. In terms of trend, it was observed that the simulation results obtained in this work were similar to those reported by Barnea [4]. Nevertheless, the proposed solution method predicted lower boundary values. As stated earlier, the lower boundary values predicted were expected.

## 4.3: Case 3

The last case simulated by the proposed solution method was the flow of an air–water system at 1 bar and 25 $^oC$ through a 5.1 cm diameter pipeline with a declination angle of 30°. Similarly, the solution was implemented in MATLAB and the simulation results were compared with those reported in literature. Just like cases 1 and 2, a trend similar to that reported in the literature was obtained. The solution method also predicted lower boundary values.

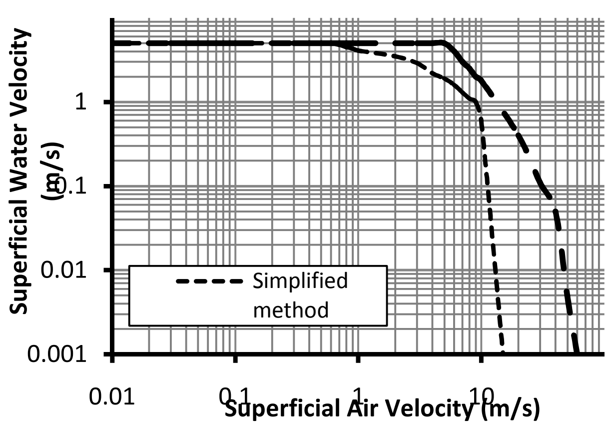


Figure 3: Boundary prediction for the flow of Air-Water through a 5.1 cm diameter pipe at 1 bar and 25 $^oC$ ($\beta$= -30$^o$)

In comparing the predicted boundaries shown in Figures 2 and 3, it can be observed that there was an upward boundary shift when the angle of declination (negative inclination) was increased from 1° in Figure 2 to 30° in Figure 3. This observation was actually expected following results previous reported in literature. The upward shift may be regarded as means of validation of the proposed solution method and this goes to show the generic and robust nature of the proposed simplified numerical solution method.

## 5.0 Conclusion

A simplified numerical solution method for the Taitel and Dukler's stratified flow model for gas-liquid flow in horizontal and near horizontal pipeline has been proposed for the special case of low flow rates that characterises the severe slugging regime. The simplified numerical solution method has been implemented in MATLAB® and comparisons of the results (in terms of superficial velocities) with those of other investigators were satisfactory.

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