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Deterministic Prediction of the Number of Target Cells of HIV Infection of $CD4^+$ T-Cells: Database Management and Health Policy Implication

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Abstract

The challenge of a deterministic modelling of the number of target cells of HIV infection of $CD4^+$ T-cells especially for the purpose of database management and its health policy remains to be an open scientific problem that can be successfully tackled using the expertise of a mathematical scientist and a medical scientist. In this study, we have utilized a powerful numerical simulation based on a sophisticated Matlab programming simplification to predict a weekly number of target cells of HIV infection. We report here that the endemic HIV infection unanimously tends to destroy these target cells with a few instances when the target cells indicate some tendency of surviving. The results which we have obtained have not been seen elsewhere, they are presented here and discussed.

1.0 Introduction

In the academic literature of the application of numerical mathematics in medicine, it is a common practice to formulate and analyse a mathematical model of HIV infection of $CD4^+$ T-Cells [1]. However, it is a rare contribution to design a data base system which has strong elements of capacity building strategy that is replicable and computationally efficient. This insight will play vital role in the HIV/AIDS health policy and planning in a developing country like Nigeria where computer literacy in a health research is in high demand. It is against this urgent need that we have developed a sustainable data base system which has the capability to predict the number of target cells of HIV infection of $CD4^+$ T-Cells over weekly data collection assumption. Other related mathematical models [2, 3, 4, 5] which describe the dynamics of HIV/AIDS without random noise implications and without weekly predicted assumptions only provide adequate theoretical bases with rare policy formulations.

2.0 Mathematical Formulation

Following [2], we consider the system of time dependent non-linear first order ordinary differential equations

$$\frac{dT}{dt} = s - dT + aT(1 - \frac{T}{T_{\max}}) - \beta TV \quad (1)$$

$$\frac{dI}{dt} = \beta_1 TV - \delta I \quad (2)$$

$$\frac{dV}{dt} = \rho I - cV \quad (3)$$

with the initial conditions $T(0) \geq 0, I(0) \geq 0, V(0) \geq 0$ .The notation T is called the number of target cells while the notations I and V are called the number of infected cells and the viral load of the virions at time t in the unit of days. The notation s stands for the rate at which new T cells are created from sources within the body such as the thymus whereas the

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notation a is called the maximum proliferation rate of target cells. The notation $T_{\max}$ stands for the T population density at which proliferation shuts off whereas the notation d stands for the death rate of the T cells. The notation β_1 is represented by the exponential equation $\beta_1 = \beta e^{-m\tau}$ where β is the infection rate constant whereas the term $e^{-m\tau}$ accounts for cells that are infected τ time units later. The notation δ stands for the death rate of infective cells whereas the notation ρ is the reproductivity rate of the infected cell. The notation c represents the clearance rate constant of virions.The precise parameter values are

$$d = 0.01, \delta = 0.5, c = 10, a = 6.8, T_{\max} = 1300, s = 5, \beta = 0.0002, \rho = 1000$$

This model formulation did not look at the data base implementation with its health policy. This omitted idea is a vital issue for the purpose of mitigating against this endeamic infection. It against this background that we have proposed the present method of tackling this problem.

3.0 Method of Solution

In this pioneering study which we have not seen elsewhere, we have used the mathematical technique of a numerical simulation on a non-linear system of first order ordinary differential equations that model the HIV infection of $CD4^+$ T-Cells to predict the number of target cells over every week and over every two weeks simplifying assumptions. We would expect this new insight to provide useful planning information for HIV/AIDS systematic early intervention. For the purpose of this study, the maximum proliferation rate of target cells is modified between 10 percent and 50 percent as well as between 110 percent and 150 percent under a fixed initial data or initial condition in which the number of target cells at time t is 1000, the number of infected cells at time t is 1 and the viral load of the virions at time t is 1. The experimental time was taken to be 180 days.

4.0 Results and Discussion

The results of our present analysis are displayed below and discussed. Here, $T_{wp}(t)$ stands for the weekly predicted number of target cells of HIV infection with T (1) representing the number of target cells on the first day of week 1 while T (8) and T (176) represent the number of target cells on the eighth day of week 2 and on the 176^{th} day of week 26. Due to the huge data values, we will here only comment on the incidence of the number of target cells for the second and third weeks as these relate to the controlled number of target cells when the maximum proliferation rate of target cells remains fixed for the value of 6.8. We have similarly compared these data for the twenty-fifth and twenty-sixth weeks. For the second week data values denoted by T (8) which starts at day 8, the original number of target cells is 176. When the maximum proliferation rate of target cells is varied by 10 percent (that is when a = 0.68), the number of target cells that remains is 47 implying that about 73.3 percent of the number of target cells of the HIV infection has been destroyed. When the maximum proliferation rate of target cells is varied by 20 percent and 30 percent (that is when a = 1.36 and a = 2.04),the numbers of target cells that remain are 68 and 105 implying that about 61.4 percent and 40.3 percent of the number of target cells of the HIV infection have been destroyed. In contrast, a 40 percent variation of the maximum proliferation rate of target cells does not change the original number of the target cells of the HIV infection while a 50 percent variation of the maximum proliferation rate of target cells tends to increase the original number of the target cells by 73.3 percent approximately. This observation shows an evidence that the number of target cells can survive in the midst of HIV infection due to a 50 percent variation of the maximum proliferation rate of target cells.

For the third week data values denoted by T (15) which starts at day 15, the original number of target cells is 420. The numbers of target cells due to 10 percent, 20 percent, 30 percent and 50 percent variations of the maximum proliferation rate of target cells are 394, 129, 218 and 225. These empirical results indicate that 6.2 percent, 69.3 percent, 48.1 percent and 46.4 percent of the number of target cells of the HIV infection have been destroyed. In this scenario, a 40 percent variation of the maximum proliferation rate of target cells has indicated a 9.5 percent increase in the original number of target cells showing the possibility of the survival of the target cells. In contrast, when the maximum proliferation rate of target cells ranges between a = 7.48 and a = 10.20, the number of target cells of the HIV infection generally tends to be destroyed. Irrespective of the ranges of the variation of the maximum proliferation rate of target cells, it is more likely for the target cells to be destroyed than their tendency to survive. In the symbol 'count (bn: an:n) as shown in Table 1', **'n'** specifies the number of target cells when a = 6.8, **'an'** specifies the number of target cells that is bigger than the number of target cells when a = 6.8, **'bn'** specifies the number of target cells that is smaller than the number of target cells when a = 6.8.

Table 1: Predicting the number of target cells of HIV infection when the maximum proliferation rate of target cells ranges between a = 0.68 and a = 3.40

Example	**T_{wp} (t)**	**T_w (t) a = 6.8**	**a = 0.68**	**a = 1.36**	**a = 2.04**	**a = 2.72**	**a = 3.40**	**count (bn: an:n)**
1	T (1)	1000	1000	1000	1000	1000	1000	0:0:5
2	T (8)	176	47	68	105	176	305	3:1:1
3	T (15)	420	394	129	218	446	255	4:1:0
4	T (22)	456	265	188	300	229	178	5:0:0
5	T (29)	428	212	228	309	193	330	5:0:0
6	T (36)	391	257	247	277	270	202	5:0:0
7	T (43)	359	260	255	251	270	280	5:0:0
8	T (50)	333	245	256	241	235	237	5:0:0
9	T (57)	312	249	255	242	247	252	5:0:0
10	T (64)	295	252	253	247	258	253	5:0:0
11	T (71)	282	250	252	250	249	245	5:0:0
12	T (78)	272	249	251	251	247	255	5:0:0
13	T (85)	263	250	250	251	251	246	5:0:0
14	T (92)	257	250	250	250	251	252	5:0:0
15	T (99)	251	250	250	250	249	249	5:0:0
16	T (106)	247	250	250	250	250	250	0:5:0
17	T (113)	244	250	250	250	250	250	0:5:0
18	T (120)	242	250	250	250	250	250	0:5:0
19	T (127)	240	250	250	250	250	250	0:5:0
20	T (134)	238	250	250	250	250	250	0:5:0
21	T (141)	237	250	250	250	250	250	0:5:0
22	T (148)	237	250	250	250	250	250	0:5:0
23	T (155)	237	250	250	250	250	250	0:5:0
24	T (162)	237	250	250	250	250	250	0:5:0
25	T (169)	237	250	250	250	250	250	0:5:0
26	T (176)	237	250	250	250	250	250	0:5:0

Looking at example 2, the predicted number of target cells is 176 for a =6.8 while the predicted number of target cells is 147 for a = 0.68. That is, the number of target cells which remains due to the new value of the proliferation rate is 147. The percentage proportion that remains has been calculated using the formula $Pd(\%) = (1 - \frac{147}{176})100$. Therefore, the percentage proportion of the target cells destroyed is 16.5. Using the same idea, for a = 1.36, the percentage proportion of the target cells destroyed is $Pd(\%) = (1 - \frac{68}{176})100$ = 61.4. When the values of the maximum proliferation rate are 2.04 and 2.72, the predicted percentage proportions of the target cells destroyed are 40.3 and zero respectively.In contrast, for a = 3.40, the number of target cells predicted has increased from 176 to 305.The same explanation is true for example 3 and other examples as displayed in Table 1 and Table 2.

From Table 1, we have observed that out of 130 predicted simulations, six (6) data points indicate no change from the original number of target cells while sixty seven (67) data points indicate smaller values from the original number of target cells and fifty seven (57) data points indicate bigger values from the original number of target cells. Therefore, our simulation study shows that there is more chance of target cells being destroyed weekly than their chance of survival.

Table 2: Predicting the number of target cells of HIV infection when the maximum proliferation rate of target cells ranges between a = 7.48 and a = 10.20.

Example	T_{Wp} (t)	T_W (t) a = 6.8	a = 7.48	a = 8.16	a = 8.84	a = 9.52	a = 10.20	count (bn: an:n)
1	T (1)	1000	1000	1000	1000	1000	1000	0:0:5
2	T (8)	176	90	57	45	43	47	5:0:0
3	T (15)	420	161	81	80	127	258	5:0:0
4	T (22)	456	168	97	176	461	467	3:2:0
5	T (29)	428	159	136	413	318	86	5:0:0
6	T (36)	391	150	219	421	99	203	4:1:0
7	T (43)	359	146	347	169	171	482	4:1:0
8	T (50)	333	149	420	117	480	97	4:1:0
9	T (57)	312	159	329	196	210	207	4:1:0
10	T (64)	295	176	204	401	109	465	3:2:0
11	T (71)	282	200	151	347	271	98	4:1:0
12	T (78)	272	231	158	154	438	220	4:1:0
13	T (85)	263	266	217	138	133	442	4:1:0
14	T (92)	257	297	316	258	145	97	2:3:0
15	T (99)	251	319	372	421	418	238	1:4:0
16	T (106)	247	324	301	253	269	413	0:5:0
17	T (113)	244	311	204	137	111	96	4:1:0
18	T (120)	242	286	165	175	235	262	3:2:0
19	T (127)	240	258	182	344	457	380	1:4:0
20	T (134)	238	233	248	373	150	95	3:2:0
21	T (141)	237	216	332	179	136	285	3:2:0
22	T (148)	237	207	341	141	386	347	2:3:0
23	T (155)	237	207	258	241	302	95	2:3:0
24	T (162)	237	214	189	407	113	312	3:2:0
25	T (169)	237	227	175	269	215	313	3:2:0
26	T (176)	237	244	212	143	464	96	3:2:0

In this scenario, we have found that out of 130 data points, five (5) data points reflect no change from the original value of the target cells whereas seventy nine (79) data points reflect values smaller than the original value of the target cells and fourty six (46) data points reflect values bigger than the original value of the target cells. Therefore, for the maximum proliferation rate interval [7.48, 10.20], the chance of the target cells being destroyed is higher compared to other options.

5.0 Conclusion

In this challenging study, we have successfully used the mathematical technique of a deterministic numerical simulation to predict the number of target cells of the HIV infection due to a variation of the maximum proliferation rate of target cells of HIV infection of $CD4^+$ T-Cells. The database computer system designed to generate these data sets if well managed is capable to provide sound HIV infection monitoring strategy which can form a major component of Nigerian HIV/AIDS health policy. The implication of the target cells which get destroyed due to a variation of the maximum proliferation rate of target cells of HIV infection of $CD4^+$ T-Cells would require a further collaboration in order to find a realistic mitigation measure against this observation. In a further analysis, this study can be extended to determine the impact of varying the maximum proliferation rate of target cells of HIV infection of $CD4^+$ T-Cells on the number of infected cells and the viral load of the virions which we did not consider in this present study because of the coupled system of interaction.

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