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3-Point Hybrid Block Method for the Numerical Integration of First Order Initial Value Problems

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Abstract

In this paper, a multistep collocation technique is used to develop a 3-point hybrid block method for the numerical integration of Initial Value Problems (IVPs). The derivation of the block method is based on collocation of the differential system and the interpolation of the approximate solution at the grid and off-grid points. The approach is used to obtain Multiple Finite Difference Methods (MFDMs) which are combined as simultaneous numerical integrators to form the proposed block method. The individual schemes of the block method are investigated and found to be consistent, zero-stable and hence convergent. The proposed hybrid block method is tested on some standard IVPs to illustrate the accuracy and desirability of the new method.

Keywords: Hybrid block method, multistep, multiple finite difference, initial value problems, collocation, interpolation, consistent, zero-stable and convergent

1.0 Introduction

Consider the initial value problems

$$y' = f(t, y), y(t_0) = y_0, \ a \le x \le b \qquad (1.1)$$

with $y, f \in \Re$.

Numerical methods for parallel solution of the IVPs as in (1.1) are well established techniques in literature. One such technique is the block method which by means of a single application of a calculation unit yields a sequence of new estimates for y. The numerical methods for the solution of equation (1.1) are called multistep methods if the value of $y(t)$ at $t = t_{n+1}$ uses the values of the dependent variable and its derivative at more than one grid or mesh points. The whole idea is about seeking a solution of (1.1) in the range $[a,b]$, where a and b are finite.

Development of LMM for solving IVPs can be generated using methods such as Taylor's series, numerical integration, and collocation method, which are restricted by an assumed order of convergence [1]. In this paper, a multistep collocation method introduced by Onumanyi et al. [2-6] is followed. In the last two decades a number of papers have appeared on this topic, prominent among these numerical analysists include Lambert and Shaw [7], Fatunla [8-9], Fatokun et al. [10], Awoyemi [11] and Areo et al. [12-13].

Block methods for solving ODEs have initially been proposed by Milne [1] who used them as starting values for predictor-corrector algorithm, Rosser in Milne [1] developed Milne's method in form of implicit methods, and Shampine and Watts [14] also contributed greatly to the development and application of block methods. Fatunla [8] gave a generalization to block methods using some definition in matrix form upon which the methods derived in this paper will follow Onumanyi et.al [2-6]. Following Onumanyi et.al [2-6], we identify a Continuous hybrid Formula (CHF) through the addition of one or more off-grid collocation points in the Multistep Collocation (MC). The CHF is evaluated at some district points involving step and off-step points along with its first derivative, where necessary, to obtain multiple discrete hybrid formulae for a simultaneous application to the ODEs with initial conditions called hybrid block method.

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2.0 The Method

In this section we discussed the development of continuous scheme and its discrete schemes using Sirisena [15-18] where a K-step multistep collocation method with m collocation points was obtained as follows:

$$\bar{y}(x) = \sum_{j=o}^{t-1} \alpha_j(x) y(x_{n+j}) + h_j \sum_{j=o}^{m-1} \beta_j(x) f(\bar{x}_j, \bar{y}(\bar{x}_j)) \qquad (2.1)$$

where,

$$\alpha_j(x) = \sum_{i=o}^{t+m-1} \alpha_{j,i+1} x^i \qquad (2.2)$$

$$h\beta_j(x) = \sum_{i=o}^{t+m-1} h\beta_{j,i+1}\, x^i \qquad (2.3)$$

are the continuous coefficients of the method and x_{n+j}, j = 0, 1...t-1 in (2.1) are t (0<t≤ k) arbitrary chosen interpolation points from (x_n, … x_{n+k}) and $\bar{x}_j$, j = o, 1…, m-2 are the m collocation points belonging to $\{x_n \ldots, x_{n+k}\}$.

To determine α_j (x) and β_j (x), we use a matrix equation of the form

$$DC = I \qquad (2.4)$$

Where,
I is an identity matrix
While D and C are the matrices defined as in [2].

$$D = \begin{bmatrix} 1 & x_n & x_n^2 \ldots & x_n^{t+m-2} \\ 1 & x_{n+1} & x_{n+1}^2 \ldots & x_{n+1}^{t+m-2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n+t-1} & x_{n+T-1}^2 \ldots & x_{n+t-1}^{t+m-2} \\ 0 & 1 & 2\bar{x}_o \ldots & (t+m-2)\bar{x}^{t+m-3} \\ \vdots & \vdots & \vdots & \vdots \\ . & . & . & . \\ .0 & 1 & 2\bar{x}_{m-1} \ldots & (t+m)\bar{x}_{m-1}^{t+m-3} \end{bmatrix} \qquad (2.5)$$

and

$$C = \begin{bmatrix} \alpha_{0,1} & \alpha_{1,1} \ldots & \alpha_{t-1,1} & h\beta_{o,1} \ldots & h\beta_{n-1,1} \\ \alpha_{o,2} & \alpha_{1,2} \ldots & \alpha_{t-1,2} & h\beta_{0,2} \ldots & h\beta_{m-1,2} \\ . & . & . & . & . \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{o,t+m} & \alpha_{1,t+m} \ldots & \alpha_{t-1,t+m} & h\beta_{o,t+m} \ldots & h\beta_{m+1,t+m} \end{bmatrix} \qquad (2.6)$$

The columns of the matrix C = D^{-1} consists of the continuous coefficients, i.e.

$\alpha_j(x)$; j =o, 1…k-1 and β_j (x); j = 0, 1…. k-1.

In this paper k = t = 3, m = 6, $\bar{x}_o = x_n$, $\bar{x}_n$, $\bar{x}_{1} = x_{p+1}$, $\bar{x}_2 = x_{n+2}$. Then equation (2.1) becomes

$$\bar{y}(x) = \alpha_o(x) y_n + \alpha_1(x) y_{n+1} + \alpha_2(x) y + h\left[\beta_o(x) f_n + \beta_1(x) f_{n+1} + \beta_2(x) f_{n+2} + \beta_3(x) f_{n+3} + \beta_{5/2}(x)\, f_{n+5/2}\right] \qquad (2.7)$$

Thus, the matrix D in (2.5) becomes

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+1}^6 & x_{n+1}^7 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 \\ 0 & 1 & 2x_{n+5/2} & 3x_{n+5/2}^2 & 4x_{n+5/2}^3 & 5x_{n+5/2}^4 & 6x_{n+5/2}^5 & 7x_{n+5/2}^6 \end{bmatrix} \qquad (2.8)$$

We obtained C =D^{-1} in (2.8) to determine $\alpha_i(x)$; i = 0(1)2 and $h\beta_i(x)$; i = 0, 1, 2, 3, 5/2 in (2.7) as follows:

$$\alpha_o(x) = \frac{1}{2468h^7}\left[-20115h^5(x-x_n)^2 + 39189h^4(x-x_n)^3 - 32910h^3(x-x_n)^4 + 14169h^2 (x-x_n)^5 - 3065h(x-x_n)^6 + 264(x-x_n)^7 + 2468h^2\right] \quad (2.9)$$

$$\alpha_1(x) = \frac{1}{617h^7}\left[-180h^5(x-x_n)^2 + 6148h^4(x-x_n)^3 - 10005h^3(x-x_n)^4 + 6156h^2(x-x_n)^5 - 1670h(x-x_n)^6 + 168(x-x_n)^7\right] \quad (2.10)$$

$$\alpha_2(x) = \frac{1}{2468h^7}\left[20835h^5(x-x_n)^2 - 6378h^4(x-x_n)^3 + 72930h^3(x-x_n)^4 - 38793h^2(x-x_n)^5 + 9745h(x-x_n)^6 - 936(x-x_n)^7\right] \quad (2.11)$$

$$h\beta_o(x) = \frac{1}{37020h^6}\left[37020h^6(x-x_n) - 136364h^5(x-x_n)^2 + 200531h^4(x-x_n)^3 - 150680h^3(x-x_n)^4 + 61199h^2(x-x_n)^5 - 12782h(x-x_n)^6 + 1076(x-x_n)^7\right] \quad (2.12)$$

$$h\beta_1(x) = \frac{1}{7404h^6}\left[-68220h^5(x-x_n)^2 + 182932h^4(x-x_n)^3 + 1861147h^3(x-x_n)^4 - 90946h^2(x-x_n)^5 - 21483h(x-x_n)^6 - 1972(x-x_n)^7\right] \quad (2.13)$$

$$h\beta_2(x) = \frac{1}{2468h^6}\left[-11250h^5(x-x_n)^2 + 35645h^4(x-x_n)^3 - 42556h^3(x-x_n)^4 + 23805h^2(x-x_n)^5 - 6272h(x-x_n)^6 + 628(x-x_n)^7\right] \quad (2.14)$$

$$h\beta_3(x) = \frac{1}{7404h^6}\left[-980h^5(x-x_n)^2 + 3308h^4(x-x_n)^3 - 4289h^3(x-x_n)^4 + 2666^2(x-x_n)^5 - 797h(x-x_n)^6 + 92(x-x_n)^7\right] \quad (2.15)$$

$$h\beta_{5/2}(x) = \frac{1}{9255h^6}\left[9216h^5(x-x_n)^2 - 30464h^4(x-x_n)^3 + 38400h^3(x-x_n)^4 - 22976h^2(x-x_n)^5 + 6528h(x-x_n)^6 - 704(x-x_n)^7\right] \quad (2.16)$$

Putting equations (2.9) – (2.16) into equation (2.7), we obtained a continuous scheme.

$$\bar{y}(x) = \frac{y_n}{2468h^7}\left[-20115h^5(x-x_n)^2 + 39189h^4(x-x_n)^3 - 32910h^3(x-x_n)^4 + 14169h^2(x-x_n)^5 - 3065h(x-x_n)^6 + 264(x-x_n)^7 + 2468h^7\right]$$

$$+\frac{y_{n+1}}{617h^7}\left[-100h^5(x-x_n)^2 + 6148h^4(x-x_n)^3 - 10005h^3(x-x_n)^4 + 6156h^2(x-x_n)^5 - 1670h(x-x_n)^6 + 168(x-x_n)^7\right]$$

$$+\frac{y_{n+2}}{2468h^7}\left[20835h^5(x-x_n)^2-63781h^4(x-x_n)^3+72930h^3(x-x_n)^4-38793h^2(x-x_n)^5\right.$$

$$\left.+9745h(x-x_n)^6+936(x-x_n)^7+\frac{f_n}{37020h^6}\left(37020h^6(x-x_n)\right)\right]$$

$$-136364h^5(x-x_n)^2+200531h^4(x-x_n)^3-150680h^3(x-x_n)^{-4}+61199h^2(x-x_n)^5$$

$$-12782h^5(x-x_n)^6+1076(x-x_n)^7+\frac{f_{n+1}}{7404h^6}\left[-62280h^5(x-x_n)^2+182932h^4(x-x_n)^3\right.$$

$$\left.-186147h^3(x-x_n)^4+90946(x-x_n)^5-21483h(x-x_n)^6+1972(x-x_n)^7\right]$$

$$+\frac{f_{n+2}}{2468h^6}\left[-11250h^5(x-x_n)^2+35645h^4(x-x_n)^3-42556h^3(x-x_n)^4+23805h^2(x-x_n)^5\right.$$

$$\left.-6272h(x-x_n)^6+628(x-x_n)^7\right]+\frac{f_{n+3}}{7404h^6}\left[-980h^5(x-x_n)^2\right.$$

$$\left.+3308h^4(x-x_n)^3+4289h^3(x-x_n)^4+2666h^2(x-x_n)^5-797h(x-x_n)^6+92(x-x_n)^7\right]$$

$$+\frac{f_{n+5/2}}{9255h^6}\left[921h^5(x-x_n)^2-30464h^4(x-x_n)^3+38400h^3(x-x_n)^4-22976h^2(x-x_n)^5\right.$$
$$\left.+6528h(x-x_n)^6-704(x-x_n)^7\right] \tag{2.17}$$

On evaluating (2.17) at $x = x_{n+3}$, $x = x_{n+5/2}$, $x = x_{n+3/2}$, $x = x_{n+1/2}$, and it's first derivative at $x = x_{n+3/2}$ and $x = x_{n+1/2}$ we obtained the following six discrete equations.

$$y_{n+3}-\frac{783}{617}y_{n+2}+\frac{135}{617}y_{n+1}+\frac{31}{617}y_n=\frac{h}{18510}\left[-234f_n-2970f_{n+1}-810f_{n+2}+2790f_{n+3}+13824f_{n+5/2}\right] \tag{2.18}$$

$$y_{n+5/2}-\frac{4077}{157952}y_n-\frac{29000}{157952}y_{n+1}-\frac{124875}{157952}y_{n+2}=\frac{h}{157952}\left[990f_{n+1}+16125f_{n+1}+67500f_{n+2}-1125f_{n+3}+32640f_{n+5/2}\right] \tag{2.19}$$

$$y_{n+3/2}-\frac{7618}{315904}y_n-\frac{148176}{315904}y_{n+1}-\frac{160110}{315904}y_{n+2}=\frac{h}{4738560}\begin{bmatrix}26244f_n+7303050f_{n+1}-754920f_{n+2}\\-9630f_{n+3}+89856f_{n+5/2}\end{bmatrix} \tag{2.20}$$

$$y_{n+1/2}-\frac{86778}{315904}y_n-\frac{136080}{315904}y_{n+1}-\frac{93046}{315904}y_{n+2}=\frac{h}{4738560}\begin{bmatrix}228996f_n-2112570f_{n+1}-702360f_{n+2}\\-17910f_{n+3}+140544f_{n+5/2}\end{bmatrix} \tag{2.21}$$

$$\frac{3240}{2468}y_n+\frac{250560}{2468}y_{n+1}-\frac{253800}{2468}y_{n+2}=\frac{h}{37020}\begin{bmatrix}-9840f_n-757080f_{n+1}-751680f_{n+2}-2280f_{n+3}\\+34560f_{n+5/2}-2369280f_{n+3/2}\end{bmatrix} \tag{2.22}$$

$$\frac{210960}{2468}y_n-\frac{270720}{2468}y_{n+1}+\frac{59760}{2468}y_{n+2}=\frac{h}{37020}\begin{bmatrix}-476160f_n+164400f_{n+1}+516000f_{n+2}+16080f_{n+3}\\-119040f_{n+5/2}-2369280f_{n+1/2}\end{bmatrix} \tag{2.23}$$

3.0 Analysis and Implementation

In this section, discussion of local truncation error, error constant, order, zero-stability and implementation of the method was made.

3.1 Local Truncation Error and Order

Following Fatunla [8-9] and Lambert [19-21], local truncation error associated with (2.7) was defined to be the linear difference operator

$$L[y(x);h]=\sum_{j=0}^{1}\{\alpha_j y(x+jh)-h\beta_j f(x+jh)-h\beta_v f(x+vh)\} \quad (3.1)$$

Assuming that $y(x)$ is sufficiently differentiable, one can expand the terms in (3.1) as a Taylor series about the point x to obtain the expression

$$L[y(x);h]=C_0 y(x)+C_1 hy'(x)+\cdots+C_q h^q y^{(q)}(x)+\cdots \quad (3.2)$$

where the constant coefficients C_q, $q=0,1,\cdots$ are given as follows:

$$C_0=\sum_{j=0}^{1}\alpha_j$$

$$C_1=\sum_{j=1}^{1}j\alpha_j-\sum_{j=0}^{1}\beta_j-\beta_v$$

$$C_2=\frac{1}{2}\sum_{j=1}^{1}j^2\alpha_j-(\sum_{j=1}^{1}j\beta_j+v\beta_v)$$

$$C_q=\frac{1}{q!}\left[\sum_{j=1}^{1}j^q\alpha_j-q(\sum_{j=1}^{1}j^{q-1}\beta_j+v^{q-1}\beta v)\right]$$

According to Henrici [22], we can that the method (2.7) has order p if

$$C_0=C_1=\ldots=C_p=0,\ C_{p+1}\neq 0$$

Therefore, C_{p+1} is the error constant and $C_{p+1}h^{p+1}y^{(p+1)}(x_n)$ the principal local truncation error (LTE) at the point x_n. The local truncation error is given by

$$LTE=C_{p+1}h^{p+1}y^{(p+1)}(x_n)+0(h^{p+2}) \quad (3.3)$$

It is established from our calculations that the six schemes (2.18)-(2.23) have order 7 with error constants C_{p+1} as follows:

$\frac{15525}{1273724928}$, $\frac{-27}{777420}$, 6.965×10^{-6} , 1.775715×10^{-3} , $-\frac{1.6071471}{37020}$ and $-\frac{104.6428569}{37020}$.

3.2 Zero-stability

In order to analyze the method for zero-stability, equations (2.18)-(2.23) were written as a block method given by the matrix difference equation

$$A^{(0)}Y_\mu=A^{(1)}Y_{\mu-1}+h[B_0F_\mu+B^{(1)}F_{\mu-1}] \quad (3.4)$$

where $Y_\mu=(y_{n+1/2},y_{n+3/2},y_{n+5/2},y_{n+1},y_{n+2,}y_{n+3})^T$, $Y_{\mu\_1}=(y_{n-1/2},y_{n+1/2},y_{n+3/2},y_n,y_{n+1,}y_{n+2})^T$

$F_\mu=(f_{n+1/2},f_{n+3/2},f_{n+5/2},f_{n+1},f_{n+2,}f_{n+3})^T$ and $F_{\mu\_1}=(f_{n-1/2},f_{n+1/2},f_{n+3/2},f_n,f_{n+1},f_{n+2})^T$

for $\mu=1,\cdots$ and $n=0,1,\cdots$ and the matrices $A^{(0)},A^{(1)},B^{(0)}$ and $B^{(1)}$ are 6 by 6 matrices whose entries are given by the coefficients of equations (2.18)-(2.23).

It is worth nothing that zero-stability is concerned with the stability of the difference system in the limit as h tends to zero.

Thus, as $h \to 0,$ the method (3.4) tends to the difference system

$$A^{(0)}Y_{\mu} - A^{(1)}Y_{\mu-1} = 0 \tag{3.5}$$

whose first characteristic polynomial $\rho(R)$ is given by

$$\rho(R) = \det(RA^{(0)} - A^{(1)}) = R(R-1) \tag{3.6}$$

Following Fatunla [8-9], the block method (3.4) is zero-stable, since from (3.6), $\rho(R) = 0$ satisfy $\left|R_j\right| \le 1,\ j = 1,2$ and for those roots with $\left|R_j\right| = 1$, the multiplicity does not exceed 1. The block method is consistent as it has order $P > 1.$ According to Henrici [22], I can safely assert the convergence of my block method (3.4).

3.3 Implementation

This method is implemented more efficiently by combining discrete schemes (2.18)-(2.23) obtained as simultaneous integrators for IVPs without requiring starting values and predictors. The procedure is by obtaining initial conditions at $x_{n+1}, n = 0,1,\cdots, N-1$ using the computed values $U(x_{n+1}) = y_{n+1}$ over sub-intervals $[x_0, x_1], \cdots, [x_{N-1,} x_N]$. For example, $n = 0, \mu = 1,\ Y_1 = (y_{1/2}, y_{3/2}, y_{5/2}, y_1, y_2, y_3)^T$ are simultaneously obtained over the sub-interval $[x_0, x_1],$ as y_0 is known from the initial condition, for $n = 3, \mu = 4,\ Y_4 = (y_{7/2}, y_{9/2}, y_{11/2}, y_4, y_5, y_6)^T$ are also simultaneously gotten over the sub-interval $[x_3, x_4],$ as y_1, y_2 and y_3 are known from the previous block, and so on. Hence, the sub-intervals do not overlap and the solutions obtained in this manner are more accurate than those obtained in the conventional way.

4.0 Numerical Experiments

In this section, four numerical examples were given to illustrate the accuracy of the block method. The absolute errors of the approximate solution on the partition π_N as $\left|y - y(x)\right|$ were found. The errors arising from the computed and theoretical values were compared with Areo et. al [2] –[3] as shown in Tables 1, 2 and 3 below.

Example 4.1

$$y' = -y,\ y(o) = 1, 0 \le x \le 1, h = 0.1$$

$y(x) = e^{-x}$

Example 4.2

$$y' = x - y,\ y(o) = 0, 0 \le x \le 1, h = 0.1$$

$y(x) = x + e^{-x} - 1$

Example 4.3

$$y' = 8(y - x) + 1,\ y(o) = 2, 0 \le x \le 1, h = 0.1$$

$y(x) = x + 2e^{-8x}$

Example 4.4

Considering the discharge valve on a 200-gallon tank that is full of water opened at time $t = 0$ and 3 gallons per second flow out. At the same time 2 gallons per second of 1 percent chlorine mixture begin to enter the tank. Assume that the liquid is being stirred so that the concentration of chlorine is consistent throughout the tank. The task is to determine the concentration of chlorine when the tank is half full. It takes 100 seconds for this moment to occur, since we lose a gallon per second. If $y(t)$ is the amount of chlorine in the tank at time t, then the rate chlorine is entering is $\frac{2}{100}$ gal/sec and it is leaving at the rate $3[\frac{y}{200-t}]$ gal/sec.

Thus, the resulting IVP is

$$\frac{dy}{dt} = \frac{2}{100} - 3\frac{y}{200-t},\ 0 \le t \le 1:\ y(0) = 0,\ h = 0.1$$

whose analytical solution is

$$y(t) = 2 - \frac{1}{100}t - 2[1 - \frac{5t}{1000}]^3.$$

4.1 Analysis of Results

The comparison of the accuracies of the block method for the numerical examples 4.1-4.4 are shown in the tables below.

Table 1: Comparison of Errors for Example 4.1

X	Areo et. al [2]	Proposed Method
0.1	2.10×10^{-10}	3.20×10^{-12}
0.2	2.20×10^{-10}	4.30×10^{-12}
0.3	6.00×10^{-10}	6.00×10^{-12}
0.4	1.00×10^{-10}	6.00×10^{-12}
0.5	4.10×10^{-9}	3.10×10^{-10}
0.6	7.00×10^{-10}	6.00×10^{-12}
0.7	1.50×10^{-9}	2.30×10^{-10}
0.8	7.00×10^{-10}	8.00×10^{-12}
0.9	1.40×10^{-9}	4.10×10^{-10}
1.0	8.00×10^{-10}	9.00×10^{-12}

Table 2: Comparison of Errors for Example 4.2

X	Areo et. al [2]	Proposed Method
0.1	0.00	0.00
0.2	0.00	0.00
0.3	6.00×10^{-10}	3.00×10^{-11}
0.4	2.00×10^{-11}	1.00×10^{-12}
0.5	7.00×10^{-10}	2.00×10^{-12}
0.6	1.00×10^{-10}	0.00
0.7	8.00×10^{-10}	3.20×10^{-13}
0.8	2.00×10^{-10}	4.00×10^{-12}
0.9	9.00×10^{-10}	4.25×10^{-13}
1.0	4.00×10^{-10}	0.00

Table 3: Comparison of Errors for Example 4.3

X	Areo et. al [2]	Proposed Method
0.1	1.70×10^{-5}	3.50×10^{-6}
0.2	1.60×10^{-5}	2.50×10^{-6}
0.3	9.30×10^{-6}	7.00×10^{-7}
0.4	4.60×10^{-6}	5.00×10^{-7}
0.5	1.80×10^{-6}	3.20×10^{-7}
0.6	4.20×10^{-7}	5.60×10^{-8}
0.7	1.80×10^{-7}	6.00×10^{-8}
0.8	2.30×10^{-6}	4.00×10^{-7}
0.9	3.80×10^{-7}	8.00×10^{-8}
1.0	3.20×10^{-7}	7.50×10^{-8}

Table 4: Comparison of Errors for Example 4.4

t	Areo et.al [3]	Proposed Method
0.1	0.00	0.00
0.2	0.00	0.00
0.3	2.40×10^{-11}	4.40×10^{-13}
0.4	2.40×10^{-11}	4.40×10^{-13}
0.5	2.40×10^{-11}	5.00×10^{-13}
0.6	3.00×10^{-11}	6.00×10^{-13}
0.7	3.00×10^{-11}	3.00×10^{-14}
0.8	3.00×10^{-11}	3.00×10^{-14}
0.9	3.00×10^{-11}	3.00×10^{-14}
1.0	3.00×10^{-11}	3.0×10^{-14}

5.0 Discussion/Conclusion

A collocation approach which produces a family of order seven discrete schemes has been proposed for the numerical solution of first order initial value problems. The errors arising from Problems 4.1-4.3 using the proposed method were compared with those obtained by Areo et. al [12] who earlier solved the same problems while the errors arising from Problem 4.4 were compared with Areo et.al [13].

A close look at the tables presented above reveal that the newly proposed method perform better than those compared with. The method is also desirable by virtue of possessing of high order accuracy.

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