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Numerical Approximation of Zeros Using Modified Midpoint Method

Abdullahi Salisu

Department of Computer Science, Niger State Polytechnic,Zungeru, Niger State, Nigeria.

Abstract

There are no any general formulas for solving or finding the roots of Polynomial equations of higher degrees (higher than 4), some polynomial equations can be solvable in some straight forward manners but most polynomials (in real life) aren't solvable. Instead we find the numerical approximation to some degree of accuracy. In this paper,we propose a modified Approach of iterative method which is based on the idea of midpoint method and false position method. The proposed methodhas been illustrated with Numerical example. The Numerical results obtained indicate that this proposed Approach provide the good performance of iterations by reducing the number of iterations when compare with the Conventional midpoint method.

1.0 Introduction

we can always find the exact zero of a quadratic equation, because we have a formula (quadratic formula) there are formulas for cubic and quatic equations, but they are so complicated that we probably never see them, let alone use them. For polynomial equations of higher degrees higher than 4, there are no general formulas at all i.e. find the zero(the x-intercepts,solutions or root) of a polynomial equation, some polynomial equations can be solvable in some straight forward manners but most polynomials (in real life) aren't solvable[1]. Instead, we find a numerical approximation to some degree of accuracy, to achieve this approximation we present a modified approach of one of the most intuitive methods (iterative method) for finding roots, which is the Midpoint Method [2].
In mathematics, a zero also sometimes called a root, of a real-complex or generally vector-valued function f is a member x of the domain of f such that$f(x)$ vanishes at x. a root of polynomial is a zero of the associated function [3]. The zeros of a function are found by noting where the graph of a function crosses the x-axis (the x-intercepts). This relationship between zeros of the function and the x-intercepts of the graph suggest that we can find the approximation of the zeros of a function by interpreting the graph [4].
Several researchers have presented different Numerical techniques based on false position method (Regular Falsi Method) to solve for the roots of non-linear equations (see [6-10]). But in this paper, we introduce a new improvement to midpoint method by introducing false position method. The false position method requires picking (Guess) a clever value at the interval defined by the midpoint method.

2.0 Description of Midpoint Method

let f assume tobe continuous function defined on the interval $[a,b]$ and $f(a)$&$f(b)$ have opposite signs at both edges of the interval, i.e. $f(a)f(b)< 0$ [5].
then we say $f(x)$ has atleast one zero in $[a,b]$ which implies by intermediate value theorem[2]. At each step the method divide the interval into two equal subintervals by computing the midpoint, $c = \frac{(b+a)}{2}$ .This generate two subintervals $[a,c]$ and $[c,b]$ of equal length. We can keep the subinterval that is guaranteed to contain zero [2]. The value of the function $f(c)$ at that point , there are two possibilities either $f(a)$ and $f(c)$ have opposite signs and bracket the root or $f(c)$ and $f(b)$ have opposite signs and bracket a root. The method selects the subinterval that is a bracket as a new interval to be used in the next step or iteration. In this way the interval that contains a zero of f is reduced in width by 50%. The process continued until the interval is sufficiently small. If $f(a)$ and $f(c)$ are opposite signs,then the method sets c as the new values for b, $f(b)$ and $f(c)$ are opposite signs then the method sets c as the new value for a. Examine the sign of $f(c)$ and replace either $(a,f(c))$ or$(b,f(c))$ with $(c, f(c))$ so that there is a zero crossing within the interval[5].

Corresponding author:***Abdullahi Salisu***E-mail:saddullah2k3@yahoo.com, Tel.: +2348035925783

3.0 Description of False Position Method

Like the midpoint method starts with two points *a* and *b* such that *f (a)* and *f(b)* are of opposite signs, assuming continuity of the function *f*.

In our new approach, we pick another point known as (false position value or clever value) "d" within the set of new intervals defined by the midpoint method, [a,c] and [c,b],to produce another sets of intervals [a,d] or [d,c] and [c,d] or [d,b] respectively. The approach selects the subinterval that guaranteed to contain zero as a new interval to be used in the next step or iteration. The method proceeds by producing a sequence of shrinking intervals $[a_{k}\ ,\ b_{k}]$ that all contain a root of *f*.

4.0 Numerical Example

Find the rootof the polynomial function in eqn (1.0), accurate to three decimal places

$$y = x^5 + x^3 - 3x - 2 \qquad (1.0)$$

to solve this polynomial, we substitute the values of x = 1 and x = 2 into eqn (1.0), we have $at\ x = 1, the\ value\ of\ y = -3\ also\ \ at\ x = 2,\ the\ value\ of\ y = 32$), then we observed the polynomial is negative at *x=1* and positive at *x= 2*, so the polynomial must be zero somewhere in between *x=1* and *x=2,* to find the zero, we must look inside the interval *x=1*&*x=2*.

We apply the midpoint method; we consider the interval between the twox-intercepts

$$\frac{2+1}{2} = 1.5$$

Now the polynomial equation (1.0) $at\ x = 1.5, the\ value\ of\ \ y = 4.46875,$ the polynomial is positive at *X=1.5*and since the polynomial at*x = 1.5* and *x=2* are positive, but negative at *x=1*. Now the zero has to be between x =1 and x=1.5, as ournew interval.

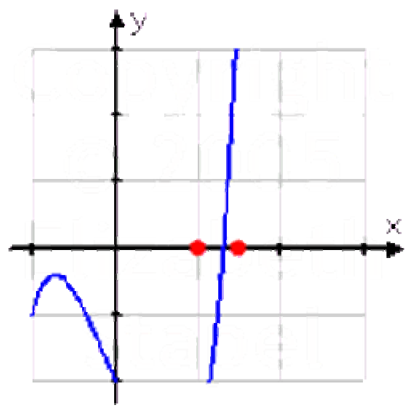

Fig. 1: Shows the graph at interval [1, 1.5]

We now apply the new approach, by picking a clever value (CV) within the new interval [1, 1.5] defined by midpoint method. The clever value (CV) is 1.3

We substitute for x= 1.3 into eqn (1.0), we have y= 0.00993

Since the f(1.5) > f(1.3), we then consider the new interval [1,1.3] for the next iteration.

Theresults of other computationsare presented in table 1.

Table 1: Numerical Results, using the Modified Midpoint Method

No of Iterations	Lower Endpoint with the corresponding y-value	Upper Endpoint with the corresponding y-value	Midpoint with the corresponding y- value	Clever Value (CV) with the corresponding y- value
1	1 Y= -3	2 Y= 32	1.5 Y= 4.46875	1.3 Y= 0.00993
2	1 Y= -3	1.3 Y= 0.00993	1.15 Y= -1.9177678125	1.25 Y= -0.7451171875
3	1.25 Y= -0.7451171875	1.3 Y= 0.00993	1.275 Y= -0.382940908203	1.29 Y= -0.1510058351
4	1.29 Y= -0.1510058351	1.3 Y= 0.00993	1.295 Y= -0.071177983441	1.299 Y= -0.006394647894
5	1.299 Y= -0.006394647894	1.3 Y= 0.00993	1.2995 Y= 0.0017612152629	1.2998 Y= 0.006609346568

Table 2: Numerical Results, using the Conventional Midpoint Method

No of Iterations	Lower endpoint with the corresponding y- value	Upper Endpoint with the corresponding y- value	Midpoint with the corresponding y- value
1	1 y = -3	2 y = 32	1.5 y = 4.46875
2	1 y = -3	1.5 y = 4.46875	1.25 y = -0.745117
3	1.25 y = -0.745117	1.5 y = 4.46875	1.375 y = 1.38950
4	1.25 y = -0.745117	1.375 y = 1.38950	1.3125 y = 0.218389
5	1.25 y = -0.745117	1.3125 y = 0.218389	1.28125 y = -0.287664
6	1.28125 y = -0.287664	1.3125 y = 0.218389	1.296875 y =-0.0409132
7	1.296875 y = -0.0409132	1.3125 y = 0.218389	1.3046875 y = 0.87143
8	1.296875 y = -0.0409132	1.3046875 y = 0.87143	1.30078125 y = 0.0227196
9	1.296875 y = -0.0409132	1.30078125 y = 0.0227196	1.299804688 y = 0.0067375297
10	1.296875 y = -0.0409132	1.299804688 y = 0.0067375297	1.299316407 y = -0.0012350122

5.0 Conclusion

Table 2 and 3 present the Numerical results from Modified Midpoint Method and Conventional Midpoint Method respectively. In table 1, the last Midpoint becomes the new upper boundary of the interval, and we have (bracketed) the polynomial's zero between $x = 1.299$ and $x= 1.2998$, since these two x-values agree in the first three decimal places. So we can conclude, that the polynomial's zero to three decimal places of accuracy is at $x=1.299$.We also observed in table 1 and 2, that the Modified Midpoint Method require 5 iterations to satisfy the condition of the solution, while the conventional Midpoint Method require 10 iterations. It becomes apparent that there is a convergence to about *1.299*in both methods but the Modified Midpoint Method guarantee convergence at low computational cost than the Conventional Midpoint Method, thus it converges faster than the Midpoint method.

We are still in the process of exploring how we could make our root finding methods better and more efficient, it is an unending quest; we recommend more research to be carry out in this direction.

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