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Computation of the Fraction of Energy Transfer to the Plasma during Neutral Beam Injection using the Test-Particle Approach

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Abstract

The slowing down process of fast ions in plasma due to dynamical friction and diffusion in velocity space between the fast ions and the plasma during Neutral Beam Injection (NBI) is studied using the Test-Particle approach. The instantaneous Slowing down as well as the whole slowing down from the fast ion birth velocity to its thermalisation is described. The fraction of the fast ion energy that is transferred to the plasma electrons F_e, and to the ions F_i, after complete slow down where obtained. A computer program FRACT in Fortran was written to obtain the relationship between F_e, F_i, and the injection energy (W_{b0}), as well as the critical energy (W_c). It was found that as the fast ion slows down the fraction of energy going to the electrons $F_e$gradually decreases, while the fraction that goes to the ions F_i increases, consistent with literature. The effect of varying the temperature of the plasma was studied, it was found that the beam energy W_b at which F_e=$F_i = 0.5$ increases with temperature, and is given by $W_b = 2.55W_c$ for the range of temperature considered. This is consistent with $W_b = 2.41W_c$ found in literature.

Keywords: Birth velocity, Critical energy, Test-Particle, Birth velocity.

1.0 Introduction

The Slowing down process of fast injected neutral beams can be studied through two complimentary approaches, the Test-Particle approach and the Fokker-Planck ones, where the latter allows for the computation of the fast ion distribution function, while the former allows one to compute the fraction of energy going to the plasma components from first principle[1]. Based on the experimental results on the energetic particle heating in the present tokamaks, it is expected that the energetic fusion produced,alpha-particles in the International Experimental Thermonuclear Reactor (ITER) type plasmas will be well confined to heatplasma efficiently and their slowing down and energy transfer to the main plasma will be definedby classical Coulomb collisions with the background electrons and ions [2]. Current work in test particle theory described the instantaneous slowdown of an energetic ion in a plasma, analytically. In this work a simple program FRACT is written and used to compute the fraction of energy given to the plasma ions (F_i)and electrons (F_e)at the end of complete slowdown. Section (1.0) gives the introduction. In section (2.0), the test-particle theory which describes the time evolution of the instantaneous and complete fast ion energy due to slowdown in plasma is given. Section (3.0) contains the methodology. In section (4.0) we have the results, and finally section (5.0) comprises of the discussions and conclusion.

2.0 Test-Particle Theory

Starting from the classical theory of binary Coulomb collision by Spitzer [3], and Sivukhin [4], the slowing down or energy decrease of a test particle due to background species plasma, with Maxwellian distribution was given by Stix [5] as

$$\frac{dW_b}{dt} = -\frac{2W_b}{\tau_s}\left[1 + \left(\frac{W_c}{W_b}\right)^{3/2}\right] \qquad (1)$$

Where W_b is the instantaneous particle beam energy, W_c is the critical energy, and τ_s the Spitzer slowing down time for the electrons. These are given by:

$$W_c = \left(\frac{3Z\sqrt{\pi}}{4}\right)^{2/3}\left(\frac{M}{m_e}\right)^{1/3}\left(\frac{M_b}{M}\right)T_e = 14.8T_eM_b\left(\frac{1}{n_e}\frac{\sum_i n_iZ_i^2}{M_i}\right)^{2/3} \qquad (2)$$

$$\tau_s = 0.012\,\frac{(T_e[keV])^{3/2}M_b}{n_e[10^{20}m^{-3}]Z_b^2} \qquad (3)$$

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Where T_e is the plasma temperature, M_b the mass of the injected beam, M_i is the mass of the plasma ion, and n_e the plasma density. The first term in the square bracket of equation (1), corresponds to energy transfer to the electrons and the second one to the background ions. When $W_c = W_b$, an equal amount of power is transferred to electrons and ions. When$W_b \gg W_c$, Eq. (1) becomes much simpler

$$\frac{dW_b}{dt} = -\frac{2W_b}{\tau_s} \quad (4)$$

In this case all the energy is transferred to the electrons. Eqns. (1 and 4) describe the instantaneous slowing down of an ion in the plasma.

In order to describe completely the whole slowing down process from birth velocity to thermal velocity, the fraction of the total energy that goes on to the electrons F_e and to the ions F_i after complete slowing-down are evaluated. The instantaneous power transferred to the ions is given by[1]

$$P_i = -\frac{2W_b}{\tau_s}\left(\frac{W_c}{W_b}\right)^{3/2} \quad (5)$$

while the energy transferred to the ions during the whole slowing down process is

$$W_i = \int_0^\infty P_i \mathrm{dt} \quad (6)$$

Eq. (1) is re-written as [1]

$$-\frac{2}{\tau_s}\, dt = \frac{dW_b}{W_b\left[1+\left(\frac{W_c}{W_b}\right)^{3/2}\right]} = \frac{dy}{y\left(1+y^{-\frac{3}{2}}\right)}, \quad (7)$$

$where y = \frac{W_b}{W_c}, and\ dy = \frac{dW_b}{W_c}$

From Eqns. (5,6, and 7), the fraction of energy going to the plasma ions is given by

$$F_i = \frac{W_i}{W_{b0}} \quad (8)$$

where W_{b0} is the beam injection energy.

$$F_i = \frac{2}{W_{b0}}\int_0^\infty W_b\left(\frac{W_c}{W_b}\right)^{3/2}\frac{dt}{\tau_s} = -\frac{1}{W_{b0}}\int_0^\infty W_b\left(\frac{W_c}{W_b}\right)^{3/2}\left(\frac{-2}{\tau_s}\right)dt (9)$$

Substituting eqn. (7) into (9) we have

$$F_i = -\frac{1}{W_{b0}}\int_0^{\frac{W_{b0}}{W_c}} W_b\left(\frac{W_c}{W_b}\right)^{3/2}\frac{dy}{y\left(1+y^{-\frac{3}{2}}\right)} \quad (10)$$

Using $= \frac{W_b}{W_c}$, eqn. (10) becomes

$$F_i = -\frac{W_c}{W_{b0}}\int_0^{\frac{W_{b0}}{W_c}} y\,(y)^{-3/2}\frac{dy}{y\left(1+y^{-\frac{3}{2}}\right)} \quad (11a)$$

$$F_i = -\frac{W_c}{W_{b0}}\int_{\frac{W_{b0}}{W_c}}^{0}\frac{dy}{\left(1+y^{\frac{3}{2}}\right)} \quad (11b)$$

$$F_i = \frac{W_c}{W_{b0}}\int_0^{\frac{W_{b0}}{W_c}}\frac{dy}{\left(1+y^{\frac{3}{2}}\right)} \quad (11c)$$

and the fraction of energy going to the electrons $(F_e)$is given by

$$F_e = 1 - F_i \quad (12)$$

The total energy transfer to plasma ions and electrons are equal for an injection energy of $W = 2.41\, W_c$ [6]. For a higher injection energy, electrons are heated predominantly and vice versa.

For computing the total energy that goes to the ions and electrons in a plasma, and the fraction of energy that goes to them during the slow-down process, a simple program named FRACT is written in Fortran, that solves Eqns. (11 and 12).(see the appendix)

3.0 Methodology

A program named FRACT was written in Fortran to solve Eqns. (11 and 12). For given injection energy $W_{b0} = 3.5\, MeV$, and a plasma at given temperatures,$T_e = 5, 10, 15\ and\ 20\ keV$ and density $n_e = 10^{20}m^{-3}$, the critical energy W_c was calculated using eqn. (2). The program was run for different values of plasma temperatureT_e, with fixed injection energyW_{b0}. The procedure was then repeated for different injection energies $W_{b0} = 180 keV$, . The output was plotted using a computer software known as Origin 5.0.

Table 1.0 Variation of Critical energy W_c with temperature $T_e W_{b0} = 3.5\ MeV$,

S/N	T_e/ keV	W_c/ keV
1	5	165.21
2	10	330.42
3	15	495.64
4	20	660.86

Table 2.0Variation of Critical energy W_c with temperature T_e, $W_{b0} = 180\ KeV$

S/N	T_e	W_c
1	5	71.20
2	10	142.30
3	15	213.45
4	20	284.60

4.0 Results

The output from the program FRACT for the fraction of total energy going to the ions and electrons when an energetic beam of fast ions with energy $W_{b0} = 3.5\ MeV$ slows down in a plasma consisting of ions and electrons of varying temperatures is as shown in Figs. 1-4 below. The graphs are seen to compare very well with that obtained from literature that is shown in fig. 5. A similar graph is shown for a 180 keV fast ion decaying in a 5 keV plasma in Fig. 6.

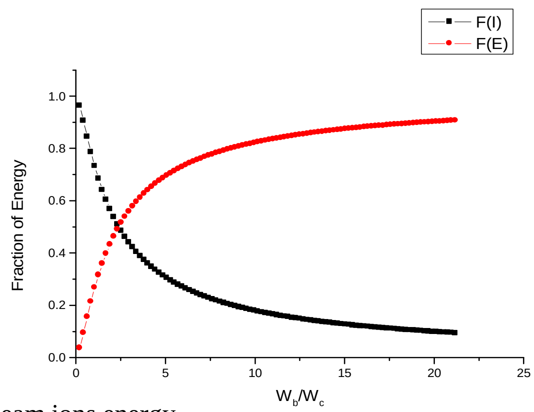

Fig.1: Fractions Fe and Fi of the beam ions energy going respectively to electrons and to ions during the slowing down process.$T_e = 5\ keV, W_c = 165.21 keV, W_{b0} = 3.5\ MeV$ beam.

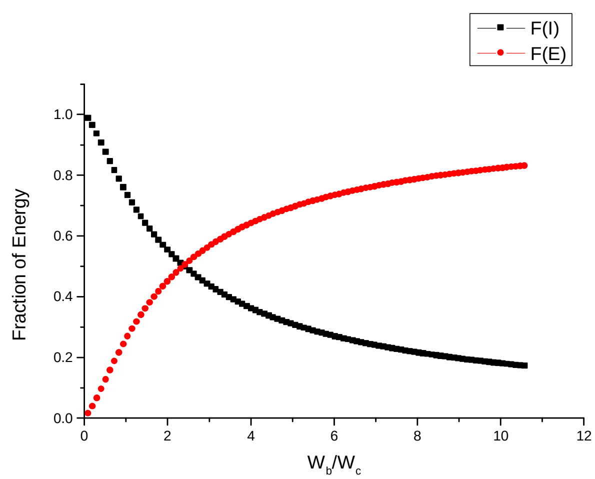

Fig.2: Fractions Fe and Fi of the beam ions energy going respectively to electrons and to ions during the slowing down process. $T_e = 10\ keV, W_c = 330.42 keV, W_{b0}$ =3.5 MeV beam.

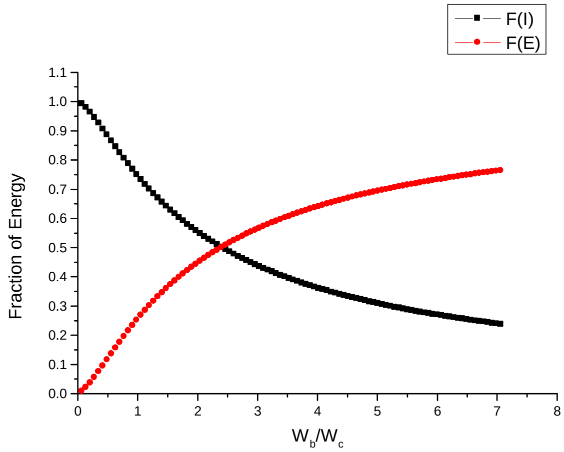

Fig.3. Fractions Fe and Fi of the beam ions energy going
respectively to electrons and to ions during the slowing down process.
$T_e = 15\ keV, W_c = 495.64keV, W_{b0}$ =3.5 MeV beam.

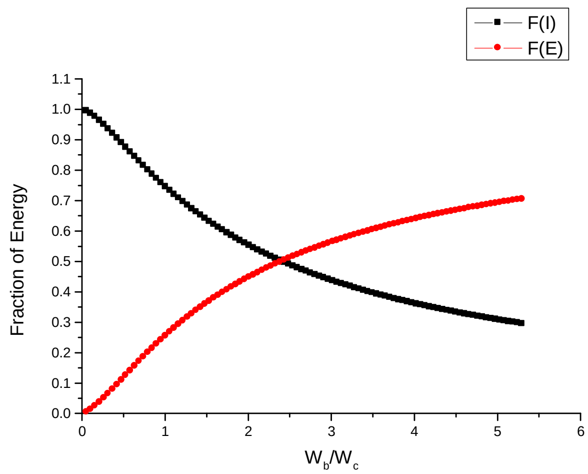

Fig.4. Fractions Fe and Fi of the beam ions energy going
respectively to electrons and to ions during the slowing down process.
$T_e = 20\ keV, W_c = 660.86keV, W_{b0} = 3.5\ MeV$ beam.

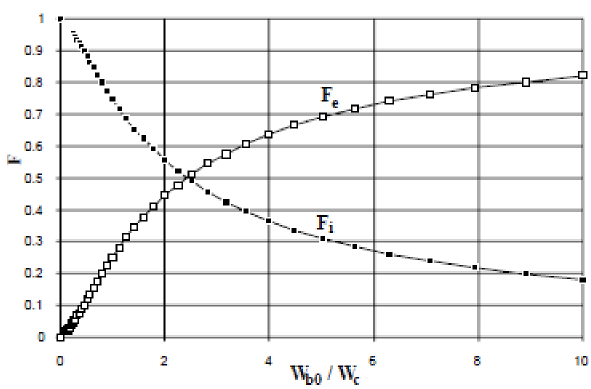

Fig.5. Fractions Fe and Fi of the beam ions energy going
respectively to electrons and to ions during the slowing down process.[1]

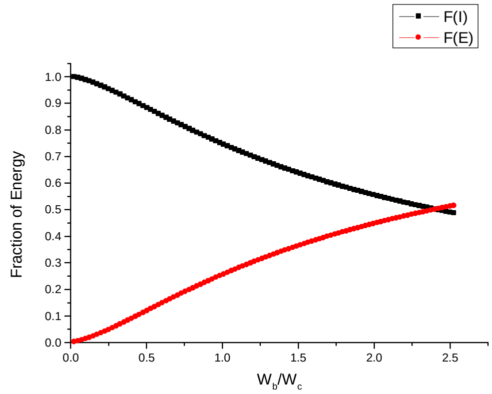

Fig.6. Fractions Fe and Fi of the beam ions energy going
respectively to electrons and to ions during the slowing down process.
$T_e = 5\ keV, W_c = 71.20keV,$ for a 180 keV beam.

5.1 Discussions

In Fig. 1, it can be seen that equal amount or fraction of energy is transferred to both ions and electrons when $W_b = 2.5W_c$. In Fig. 2 where $T_e = 10\ keV$, $W_b = 2.8W_c$for equal fraction of energy to be transferred to both ions and electron. In Fig. 3, where $T_e = 15\ keV$, $W_b = 2.4W_c$for equal fraction of energy to be transferred to both ions and electron. In Fig. 4 where $T_e = 20\ keV$, $W_b = 2.5W_c$for equal fraction of energy to be transferred to both ions and electron. Finally in Fig. 6 where $T_e = 5\ keV$, $W_b = 2.5W_c$for equal fraction of energy to be transferred to both ions and electron in a plasma injected with 180 keV beam. These values of $W_b$arein close agreement with $W_b = 2.41W_c$ found in literature[6].

5.2 Conclusion

The program FRACT was used to compute the fractions of energy going to both ions , and electrons. From the graphs obtained it was found that when the total injected energy is greater than the critical energy, more energy goes to the electrons than the ions, and as the injected energy decreases, the fraction going to the electrons decreases while the fraction going to the ions increases. The fraction of energy going to both the ions and the electrons was found to be equal when the total injected energy was found to be approximately $W_b = 2.55W_c$, which is close to the theoretical value of $W_b = 2.41W_c$, found in literature. The difference between our results and that from literature may be due the numerical integration used in FRACT.A better quadrature method may give better result.

Appendix

The progam FRACT

```
c This is the program FRACT for computing the fraction of injected beam energy going
c to the plasma ions Fi and electrons Fe
c  The ratio of the injection energy wb to the critical energy wc is dimensioned as y(2000)
c  wb0 is the injected energy of the beam at the start in keV
c  wbo is the array representing the injected beam as it slows down in keV
c  wc is the critical energy in keV
      real y(2000),wbo(100)

c  the choice of wb0, and wc are user supplied
      wb0=3500
      wc=165.21
      do 1 j=1,100
      wbo(j)=j/100.*wb0
      dy=wbo(j)/wc/1000
c numerical integration of dy/(1 + y**1.5) from 0 to wb0/wc
      y(1)=0.
      sumf=dy
      do 2 i=2,1000
      y(i)=i*dy
      sumf= sumf+dy/(1.+y(i)**1.5)
```

```
  2       continue
c Fi is the fraction of energy going to the plasma ions
          fi=wc/wbo(j)*sumf
c Fe is the fraction of energy going to the plasma electrons
          fe=1.-fi
          open(2,file='fract.dat',status='unknown')
c a file fract is open for output fi, and fe
          write(2,200)fi,fe
 200      format(1x,2f16.5)
  1       continue
c the variation of injected energy with slowdown is written to the output file fract
          write(2,300)wbo
 300      format(1x,f16.5)
          stop
          end
```

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