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On the Solution of Transportation Problems Using S_N Algorithm

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Abstract

In this study, the S-N algorithm has been used tosolved the transportation problem. In this algorithm, it has been attempted to find the optimal solution directly by using mathematical suffix average, without finding initial basic feasible solution. Even for all types of transportation problems it gives best optimal solution. It gives overall feasibility and optimality with less steps.

Keywords: Source, demand, supply, degeneracy, loops, maximization, row, column, minimization and optimal solution.

1.0 Introduction

The conventional objective transportation problem consists of finding an optimal solution using one of the classical methods the basic feasible solution can be optimized by using optimization techniques like u-v method and stepping stone method. It has been observed that the classical methods reach the optimal solution with more steps but this S_N algorithm seeks to optimize the transportation problem with less steps and best optimal solution. In this study, we have discussed about the mathematical model of S_N algorithm to solve transportation and also the relevant examples are presented.

2.0 Theoretical Development

The transportation problem deals with the transportation of a single product from several sources (origins or supply or capacity centers) to several sinks (destinations or demand or requirements centers). In general, let there be in sources S_1, S_2, S_3……S_m with a_i (i = 1, 2,……m) available supplies or capacity at each source I, to be allocated among n destinations D_1, D_2,…… D_n with b_j (j = 1, 2,…..n) specified requirements at each destination j. Here, we state already existing mathematical model introduced and studied by Kuhn [1] for such a problem, which is

$$\text{Min}Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\sum^{n} x_{ii} = b_i \,;\, j = 1, 2, \ldots.. n$$

$$\sum_{i=1}^{m} x_{ij} = a_i \,;\, i = 1, 2, \ldots.. m \qquad \sum_{i=1}^{n} a_i = \sum_{j=1}^{n} b_j \text{ (Rim condition)}$$

$$x_{ij} \geq 0, \text{ For i and j}$$

Where,

$\mathbf{c_{ij}}$ = The cost transporting goods from the ith supply point to the jthdestination.

x_{ij} = The amount transporting goods from theith supply point to the jthdestination.

Motivated by earlier works [2 - 8]. In the following study we define new algorithm to solve the transportation problems.

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2.1 S_N Algorithm for Getting Optimal Solution of Transportation Problem

This S_N algorithm gives optimal solution directly without finding initial basic feasible solution. Not only that, there is no need to bother about looping as well as degeneracy. It gives best optimal solution, when compared to the existing method. Also, it works for unbalanced and maximization cases. In maximization case usually, the problem is to be converted to minimization and that can be solved in the regular method. We shall in this study itemize the following steps.

Step 1: Initially check if the given transportation has equal demand and supply. If not appropriate demand and supply to be adjusted with zero cost.
To construct a reduced cost matrix for the given transportation problem we required the following Step 1 which was stated by Kuhn [1].

- Find the row minimum for each row of the original cost matrix

  $$\min_j \langle \text{ith row cost}; c_{ij} \rangle = \theta, \quad \text{where } i = 1, 2, \ldots m$$

  After obtaining the minimum θ, we can construct a new cost matrix. Each new cost in the ith row

- Find the column minimum for each column of the new cost matrix

  $$\min_i \langle j^{th} \text{row cost}; c'_{ij} \rangle = \sigma_j, \text{ where } j = 1, 2, \ldots n$$

- Construct a reduced cost matrix for each new cost in the jth column (c^n_{ij})

  $$c''_{ij} = \text{old}(c'_{ij}) - \sigma_j, \text{ where } j = 1, 2, \ldots n$$

There are many zeros in the reduced cost matrix, at least one should be in each row and column.
With help of the above step now we define next step of the S_N algorithm.
Step 2: Let

(1) $$s = \frac{1}{4}\left\{c''_{i,j-1} + c''_{i,j+1} + c''_{i-1,j} + c''_{i+1,j}\right\}, \text{where } \begin{Bmatrix} j = 1, 2, ..n \\ i = 1, 2, ..m \end{Bmatrix}$$

In the reduced cost matrix by applying the above Eq. 1 in each zero ($c''_{ij}= 0$) from the resultant matrix, write all values of s to the suffix of corresponding c''_{ij}.
Step 3: Select maximum of s, if it has one maximum value then first supply to that demand corresponding to the c''_{ij}.If it has more equal values then select $\{a_i, b_i\}$ and supply to that demand maximum possible. After supplied, the exhausted demand (column) or supply (row) to be trimmed.
Further, the resultant matrix must possess at least one c''_{ij}in each row and column, else repeat the step 1. Repeat the above process until the optimal cost is obtained.

Maximization transportation problems: In maximization transportation problem replace each element of the transportation table by its difference from the maximum element of the table. Then by applying the steps of minimization transportation problem on the revised transportation table.

Numerical examples
Example 1: (Balanced Problem), i.e. the supply equals to the total demand.
A company has three production facilities S_1, S_2 and S_3 with production capacities of 6, 1 and 10 units per week of a product, respectively. These units are to be shipped to four warehouses D_1, D_2, D_3 and D_4 with requirements of 7, 5,3 and 2 per week respectively. The transportation costs (in dollar) per unit between factories to warehouses are given in the table below; for this transportation problem we shall use the S_N Algorithm to obtain the optimal destination for this company in order to minimize its total shipping costs.

Table 1a: Production Facilities and Capacities
Destination Centers

	D_1	D_2	D_3	D_4	SUPPLY
S_1	2	3	11	7	6
S_2	1	0	6	1	1
S_3	5	8	15	9	10
DEMAND	7	5	3	2	

SOLUTION:

We shall seek the use of S-N Algorithm steps stated above. We then construct a new cost matrix for the row and column,thedirect optimal solution is table 1b.

Table 1b: Optimal Solution of Production Facilities and Capacities

2	3 5	11 1	7
1	0	6	1 1
5 7	8	15 2	9 1

(3x5)+(11x1)+(1x1)+(5x7)+(15x2)+(9x2)= USD 101.

The Total optimal cost of transportation is USD. 101.
Example 2: (Unbalanced problem). I.e. The supply is not equal to the total demand.
The table below shows unbalanced transportation problem using the S_N Algorithm to obtain the optimal distribution in order to minimize its total shipping cost.

Table 2a: Un-Balanced Transportation
DESTINATION CENTERS

	D_1	D_2	D_3	D_4	SUPPLY
S_1	6	1	9	3	70
S_2	11	5	2	8	55
S_3	10	12	4	7	70
DEMAND	85	35	50	45	

Solution:
We shall seek the use of S-N Algorithm steps stated above. We then construct a new cost matrix for the row and column, thedirect optimal solution is table 2b

Table 2b: Optimal Solution Of Un-Balanced Transportation

6 **40**	**1** **30**	**9**	**3**
11	**5** **5**	**2** **50**	**8**
10 **25**	**12**	**4**	**7** **45**

(6x40)+(1x30)+(5x5)+(2x50)+(10x25)+(7x45)= USD 960.

The total optimal cost of transportation is USD. 960.

3.0 Conclusion

In the present study, we have devised a method to find the optimal solution for the transportation problem by using the above algorithm. The propose algorithm is simple and its execution time arefast i.e. its optimal solution are reached/obtained earlier than some existing methods.It is errorfree;this method works effectively to provide the desired solution.

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