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Generalized Dynamical Gravitational Scalar Potential for Static Homogeneous Spherical Distribution of Mass

L.W. Lumbi[1], S.X.K. Howusu[2], Tsaku Nuhu[3] and O. Nwagbara[1]

[1]**Department of Physics, Nasarawa State University, Keffi, P.M.B 1022, Keffi, Nigeria.**
[2]**Department of Physics, Kogi State University, Ayingba, Kogi State, Nigeria.**
[3]**Department of Mathematical Sciences, Nasarawa State University, Keffi, P.M.B 1022, Keffi, Nigeria.**

Abstract

The generalized dynamical gravitational field equation for static homogeneous spherical massive bodies is applied to obtain additional correction terms of all orders of cto Newton's dynamical gravitational scalar potential exterior and interior.

Keywords: Field Equation, Spherical Bodies, Gravitational Scalar Potential, Dynamical Theory.

1.0 Introduction

According to Newton's dynamical law of gravitation any two bodies in the universe attracts each other with a force that is directly proportional to the product of the masses and inversely proportional to the square of the distance separating the bodies [1, 2].The well knownNewton's dynamical gravitational scalar potentialexterior to a homogeneous spherical body Φ^+[1- 4] is given as

$$\Phi^+ = \quad -\frac{GM}{r} \tag{1}$$

and the gravitational scalar potential interior to the bodyΦ^- [1- 3] is given as

$$\Phi^- = \frac{-3GM}{2R} \tag{2}$$

where G is the universal gravitational constant, M is the mass of the spherical bodies and R is the radius of the spherical bodies.

In this article, the generalized dynamical gravitational scalar potential exterior and interiorto a spherical body is formulated using a new dynamical approach.

2.0 Theoretical Analysis

The generalized dynamical gravitational field equation for a static homogeneous spherical massive body [5-8] is given explicitly as

$$f'' + \frac{2}{c^2}ff'' + \frac{2}{r}f' + \frac{4}{c^2 r}ff' + \frac{2}{c^2}(f')^2 = \begin{cases} 0 \;\; ; \;\; r > R \\ 4\pi G\rho_{o\; ;\; r<R} \end{cases} \tag{3}$$

where f' is differentiation once $\frac{df}{dr}$and $f''$is differentiation twice$\frac{d^2f}{dr^2}$

Let us seek the solution of the exterior field equation (3) as

$$f^+(r) = \frac{A_1}{r} + \frac{A_2}{r^2} + \cdots \tag{4}$$

where $A_1$and $A_2$are arbitrary constants.

Substituting equation (4)into the generalized dynamical gravitational field equation (3) and equating coefficients of $\frac{1}{r^3}$ and $\frac{1}{r^4}$ on both sides, we obtain

Corresponding author: L.W. Lumbi E-mail: williamslucas44@gmail.com, Tel.: +2348030849994

$$A_1 = ArbitraryConstant \tag{5}$$

$$A_2 = -\frac{1}{c^2}A_1^2 \tag{6}$$

Hence, the general solution of the exterior field equation (3) is given by

$$f^+(r) = \frac{A_1}{r} - \frac{A_1^2}{c^2r^2} + \cdots \tag{7}$$

Let us also seek the solution to the interior field equation (3) as

$f^-(r) = f_c^-(r) + f_p^-(r)$ (8)

where, $f_c^-(r)$ is the complementary solution and $f_p^-(r)$ is the particular solution.

It is physically convenient to choose the complementary solution to be of the form:

$$f_c^-(r) = B_0 \tag{9}$$

where B_0 is an arbitrary constant.

Let us seek the particular solutionas

$$f_p^-(r) = D_2r^2 + D_4r^4 + \cdots \tag{10}$$

where D_2 and D_4 are arbitrary constants.

Substituting (10) into the generalized dynamical gravitational field equation (3) and equating coefficients of r^0 and r^2 on both sides, we obtain

$$D_2 = \frac{2}{3}\pi G\rho_o \tag{11}$$

$$D_4 = -\frac{4}{9c^2}\pi^2G^2\rho_o^2 \tag{12}$$

Therefore the general solution of the interior field equation can be written as

$$f^-(r) = B_0 + D_2r^2 + D_4r^4 + \cdots \tag{13}$$

Now, imposing the condition of continuityof gravitational scalar potential function across boundaries, ($r = R$), we obtain

$$B_0 + D_2R^2 + D_4R^4 + \cdots = \frac{A_1}{R} + \frac{A_2}{R^2} + \cdots \tag{14}$$

And by the condition of continuity of normal derivatives of gravitational scalar potential functionacross all boundaries,

$$\left(\frac{\partial F^+}{\partial r}\right)_{r=R} = \left(\frac{\partial F^-}{\partial r}\right)_{r=R}$$

$$2D_2R + 4D_4R^3 = -\frac{1}{R^2}A_1 - \frac{2A_1^2}{c^2R^3} + \cdots \tag{15}$$

Solving equation (14)for A_1 we obtain

$$A_1 = \frac{\gamma}{\beta} + \frac{\propto\gamma^2}{\beta^3} \ldots \tag{16}$$

or

$$A_1 = \frac{\beta}{\propto} - \frac{\gamma}{\beta} - \frac{\propto\gamma^2}{\beta^3} \ldots \tag{17}$$

where $\propto, \beta, and \gamma$ are arbitrary constants given by $\frac{2}{c^2R^3}$, $\frac{1}{R^2}$ and $2D_2R + 4D_4R^3$ respectively.

Substituting the values of $\propto, \beta, and \gamma$ into the first solution we obtain

$$A_1 = -2D_2R^3 - 4D_4R^5 + \cdots \tag{18}$$

Using the well known physical relationship between mass and density for a sphere of radius R, gives

$$A_1 = -GM + \frac{G^2M^2}{c^2R} + \cdots \tag{19}$$

$$A_2 = -\frac{G^2M^2}{c^2} + \tag{20}$$

$$D_2 = \frac{GM}{2R^3} \quad (21)$$

$$D_4 = -\frac{G^2M^2}{4c^2R^6} \quad (22)$$

Solving equation (14) for $B_0$to the order of c^4gives

$$B_0 = \frac{-3GM}{2R} + \frac{G^2M^2}{4c^2R^2} + \frac{2G^3M^3}{c^4R^3} + \cdots \quad (23)$$

Substituting(19) and (20) into(4)yields (24)

$$f^+(r) = -\frac{GM}{r}\left\{1 - \frac{GM}{c^2R}\right\} - \frac{G^2M^2}{c^2r^2} + \frac{2G^2M^2}{c^4r^2R} + \cdots \quad (24)$$

The interior field equation (13) becomes (25) after substituting (21), (22)and (23)

$$f^-(r) = B_0 + \frac{GM}{2R}r^2 - \frac{G^2M^2}{4c^2R^2}r^4 + \cdots \quad (25)$$

Equations (24) and (25)are the generalized dynamical gravitational scalar potentials exterior and interior for static homogeneous spherical massive bodies.The leading termon the right hand side of these equationsis the well known Newtonian dynamicalgravitational scalar potential exterior and interior to the body while the other additional terms are correction terms introduced by the generalized dynamical approach. It must be noted that these equations also contain correction terms of order c^4 not found in [6].

3.0 Remarks and Conclusion

We have in this paper shown how to derive a generalized dynamical gravitational scalar potential exterior and interior to static homogeneous spherical massive bodies.The immediate consequences of these results are:

(i) The generalized dynamical gravitational scalar potentials obtained can be applied to the motion of test particles in the gravitational fieldsunder study

(ii) The generalized dynamical gravitational scalar potentialcan also besubstituted intothe well known Newton's dynamical equations of motion, Einstein's geometrical equations of motion and General dynamical equations of motion to obtain correspondingrevisions to the planetary equation of motion and hence the planetary parameters such as the anomalous orbital precession in the solar system, eccentricity, angular frequency, amplitude, period, perihelion distance and the aphelion distance.

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