

## Extension of Newton’s Planetary Theory Based Upon Generalized Gravitational Scalar Potential

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### Abstract

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*In a previous work, we obtained a generalized Newtonian dynamical planetary equation of motion for static homogeneous spherical massive bodies. In this paper the generalized Newtonian dynamical planetary equation of motion are applied to the solar system to obtain generalized dynamical planetary parameters (orbital angular frequency, eccentricity and amplitude). The results are that the generalized dynamical planetary parameters are augmented with correction terms of order  $c^{-2}$  which are not found in Newton’s dynamical planetary parameters or Einstein’s geometrical planetary parameters.*

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**Keywords:** Planetary equation of motion, Generalized Orbital Angular Frequency, Eccentricity and Amplitude.

### 1.0 Introduction

The Newtonian planetary parameters were derived based on Newton’s dynamical planetary equation of motion for static homogeneous spherical massive bodies [1-5] given by

$$\frac{d^2v}{d\phi^2} + v = \frac{k}{l^2} \tag{1}$$

The generalized Newtonian dynamical planetary equation of motion for static homogeneous spherical massive bodies [6, 7] is given as

$$\frac{d^2v}{d\phi^2} = \frac{k}{l^2} \left(1 - \frac{k}{c^2 R}\right) - \left(1 + \frac{2k^2}{c^2 l^2}\right) v \tag{2}$$

where  $c$  is the speed of light,  $l$  is the angular momentum per unit rest mass and  $R$  is the radius of the planets. In this article, we formulate generalized Newtonian dynamical planetary parameters (orbital angular frequency, eccentricity and amplitude) for static homogeneous spherical massive bodies. This study will help to determine climatic changes or effects.

### 2.0 Theoretical Analysis

Let us seek the complementary solution in (2) as

$$V(\phi) = A \cos(\omega\phi) \tag{3}$$

$$\omega = \pm \left(1 + \frac{2k^2}{c^2 l^2}\right)^{1/2} \tag{4}$$

where,  $A$  is arbitrary constant and  $\omega$  is the generalized Newtonian dynamical orbital angular frequency.

Let us seek the particular solution in (2) as

$$V_p(\phi) = B \tag{5}$$

$$B = \frac{k}{l^2} \left(1 - \frac{k}{c^2 R}\right) \left(1 + \frac{2k^2}{c^2 l^2}\right)^{-1} \tag{6}$$

$$V(\phi) = A \cos(\omega\phi) + \frac{k}{l^2} \left(1 - \frac{k}{c^2 R}\right) \left(1 + \frac{2k^2}{c^2 l^2}\right)^{-1} \tag{7}$$

Let the apsides of the motion occur at  $\phi_1 = 0$  and  $\phi_2 = \pi$

Then,

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$$\frac{1}{r_1} = A + \frac{k}{l^2} \left(1 - \frac{k}{c^2 R}\right) \left(1 + \frac{2k^2}{c^2 l^2}\right)^{-1} \quad (8)$$

$$V(\phi) = \left[ \frac{1}{r_1} - \frac{k}{l^2} \left(1 - \frac{k}{c^2 R}\right) \left(1 + \frac{2k^2}{c^2 l^2}\right)^{-1} \right] \cos(\omega\phi) + \frac{k}{l^2} \left(1 - \frac{k}{c^2 R}\right) \left(1 + \frac{2k^2}{c^2 l^2}\right)^{-1} \quad (9)$$

Solving equation (9) we obtain the following;

$$\varepsilon = \left[ 1 - \frac{l^2}{kr_1} \left(1 - \frac{k}{c^2 R}\right)^{-1} \left(1 + \frac{2k^2}{c^2 l^2}\right) \right] \quad (10)$$

as the generalized Newtonian dynamical orbital eccentricity of the orbit of the planet and

$$\varepsilon_o = 1 - \frac{l^2}{kr_1} \quad (11)$$

is the corresponding pure Newtonian orbital eccentricity.

Similarly,

$$A = \left[ \frac{l_o^2}{k} \left(1 - \frac{k}{c^2 R}\right)^{-1} \left(1 + \frac{2k^2}{c^2 l^2}\right) \right] \quad (12)$$

as the generalized dynamical Newtonian orbital amplitude of the orbit of the planet and

$$A_o = \frac{l_o^2}{k} \quad (13)$$

is the corresponding pure Newtonian constant of the motion.

All these results reduces to  $c^0$ , corresponding to pure Newtonian equation of motion and to the order of  $c^2$ , it contain additional correction terms which are not found in Newton’s dynamical and Einstein’s geometrical expressions.

### 3.0 Remarks and Conclusion

We have in this paper shown how to derive the generalized Newtonian dynamical planetary parameters by seeking the complementary and particular solution of the generalizedNewtonian dynamical planetary equation of motion for static homogeneous spherical massive bodies. The generalized dynamical orbital angular frequency, eccentricity and amplitude of the planets are found to be equations (4), (10) and (12) respectively. The leading terms on the right hand of these equation [(4), (10) and (12)] is the well known Newtonian terms while the additional terms are correction terms which are not found in Newton’s dynamical or Einstein’s geometrical planetary parameters. The door is henceforth open for the theoretical investigation and application of the post Newton and post Einstein correction terms obtained in the generalized dynamical planetary parameters.

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