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On the Generalized forms of Exact Solutions to a modified Liouville Equation using a Direct Approach

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Abstract

Using a direct approach, two generalized forms of exact solutions to the modified Liouville equation is presented. This approach is in contrastsimpler than other approaches available in the literature which involves the use of sophisticated and rather cumbersome tools. One of the solutions obtained is new and does not seem to have been reported previously in the literature.

1.0 Introduction

In this paper, we study a modified Liouville equation of the form

$$\omega_{tt} = a^2\omega_{xx} + be^{\beta\omega} \qquad (1.1)$$

Where a, b and β are arbitrary constants and subscripts represent partial derivatives with respect to the variables.

The importance of the modified Liouville equation as a field theoretic model has been established in a number of studies such as those stated in [1]. Several analytic methods have been proposed for finding the explicit travelling wave solutions to non linear partial differential equation. This includes the tanh-function method and its various extensions [2], the Jacobi elliptic function expansion method [3], the homogenous balance method, the F-expansion method [4], the variational iteration method [5], the modified simple equation method [6] , $\left(G'/G\right)$-expansion method as cited in[7] to mention but a few. In this paper, however, we adopt a direct approach. This approach is in contrast simpler than other approaches available in the literature which involves the use of sophisticated and rather cumbersome tools. Not only do we obtain the same solutions in [7], using this simpler method, a new solutionis also obtained which has not been reported previously in the literature.

2.0 Method of Solution

We would like to obtain new and more general exact travelling wave solution to the modified Liouville equation (1.1). Suppose

$$e^{\beta\omega} = u(x,t) \qquad (2.1a)$$

Introducing a new variable

$$\xi = Ax + Bt \qquad (2.1b)$$

so that $u = u(\xi)$. This being the travelling wave transformation of u Equation (1.1) therefore becomes

$$uu_{\varepsilon\varepsilon} - u_\varepsilon^2 + ku^3 = 0 \qquad (2.2)$$

where

$$k = \frac{b\beta}{a^2A^2 - B^2} \qquad (2.3)$$

Using the substitution

$$u_\xi = p(u) \qquad (2.4)$$

Equation (2.2) becomes

$$pp_u + f(u)p^2 + g(u) = 0 \qquad (2.5)$$

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where

$$f(u) = -\frac{1}{u} \text{ and } g(u) = ku^2 \qquad (2.6)$$

The solution to equation (2.5) can be written as

$$p^2 e^{2\varphi} = c - 2\int g(u)e^{2\varphi}\, du \qquad (2.7)$$

where

$$\varphi(u) = \int f(u)du \qquad (2.8)$$

and$c$is a constant of integration. Using the expressions for $f(u)$ and $g(u)$ and equation (2.8), equation (2.7) can be expressed as

$$p^2 = u^2(c + 2ku) \qquad (2.9)$$

There are two classes of solutions to equation (2.9) depending on the value of c.

Case (i) $c = 0$ In this case

$$u(\xi) = \frac{2k}{(\xi + \xi_0)^2} \qquad (2.10)$$

where ξ_0 is a constant of integration. Therefore

$$e^{\beta\omega} = \frac{2k}{(\xi + \xi_0)^2} \qquad (2.11)$$

This gives the travelling wave solution to (1.1) to be

$$\omega = \frac{1}{\beta}\ln\left(\frac{1}{a^2A^2 - B^2}\left(\frac{2b\beta}{(Ax + Bt + C)^2}\right)\right) \qquad (2.12)$$

Case (ii) $c \neq 0$

In this case $u(\xi)$ satisfies

$$u(\xi) = -\frac{c}{2k}\sec h^2\left(-\frac{\sqrt{c}}{2k}(\xi + \xi_0)\right) \qquad (2.13)$$

So that another travelling wave solution to (1.1) can be written as

$$\omega = \frac{1}{\beta}\ln\left(-\frac{c(a^2A^2 - B^2)}{2b\beta}\sec h^2\left(-\frac{\sqrt{c}(a^2A^2 - B^2)}{2b\beta}(Ax + Bt + C)\right)\right) \qquad (2.14)$$

This solution to the modified Liouville equation is new and does not seem to have been reported previously in the literature to the best of our knowledge.

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