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Measuring the Impact of Environmental Random Noise on the Carrying Capacities of a Mathematical Model of Survival of Species Dependent on a Resource in a Polluted Environment

[1]James I. Galadima, [2]Enu-Obari.N. Ekaka-a, [3]I.A. Agwu and [4]Obioma R. Nwaogbe

[1]Maritime Institute, Department of Mathematics and Computer Science, IBB University, Lapai, Niger State, Nigeria.
[2]Department of Mathematics and Computer Science, Rivers State University of Science and Technology, Port Harcourt, Nigeria.
[3]Department of Mathematics, Abia State Polytechnic, Aba, Nigeria.
[4]Department of Transport Management Technology, Federal University of Technology, Minna, Niger State, Nigeria.

Abstract

While the mathematical modelling which describes the survival of species dependent on a limited resource in a polluted environment is a not new, we have utilised the technique of a numerical simulation to study the impact of environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment. Since the ecosystem is characteristically stochastic, random noise intensity on the maximum population size (otherwise called the carrying capacity) that supports the growth of a population is inevitable. On the basis of this study, it is a best-fit scenario to select the low random noise intensity of 0.8 which attracts lower error values of 4.9880, 1.6725 and 0.7904 using the three popular p-mathematical norms. This is one of the typical results which we have obtained in this study. These observations are in contrast with a high random intensity of 2.4 which attracts higher error values of 16.5637, 5.8055 and 2.3294 by using the 1-norm, 2-norm and infinity-norm measures. The policy implication of these novel contributions is briefly mentioned. This present study was also conducted for an instance when the random noise intensity is 1.6. The key contribution in this scenario is the fact that the errors between the K_{1new} and K_{1old} differentiated data sets are 9.6252, 3.3565 and 1.5568 by using the 1-norm, 2-norm and infinity-norm measures whereas the errors between the K_{2new} and K_{2old} differentiated data sets are 7.7413, 2.8141 and 1.3205 by using the 1-norm, 2-norm and infinity-norm measures. The results which we have obtained have not been seen elsewhere; they are presented here and discussed.

Keywords: Co-existence steady-state solution, stability, interacting legumes.

1.0 Introduction

The severe uncertainties which characterize most polluted ecosystems in the developing countries necessitate the effect of a few environmental factors that are capable in contributing to some sort of random noise in such ecosystems [1 – 10]. While all these cited authors have made substantial contributions to solve a few environmental problems using the tool of a mathematical modelling, the application of a computational approach to measure the impact of random noise on the carrying capacities of interacting populations remains to be an un-resolved open environmental problem. One of the methods of studying the impact of the random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment is called numerical simulation using a Matlab programming scheme. Since the ecosystem is characteristically stochastic on one hand and the precise values of model parameters are estimated based on an approximated model formulation, it will be an interesting challenge to decide on how to measure the impact of environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment such as the Ogoni polluted ecosystem which has turned out to be a typical polluted

Corresponding author: James I. Galadima,E-mail: drgaladima@gmail.com,Tel.: +2348055959199

environment in the world. One of the typical examples of induced random noise on the carrying capacities can be attributed to the oil spillage. The impact of this extrinsic factor on the carrying capacities which may take a longer period of time and huge funding for experimental scientists to quantify will be cost-effective and beneficial when a computational approach is successfully implemented. It is this approach we propose to utilise in a bid to mitigate this challenging environmental problem on the basis of a sound mathematical reasoning.

2.0 Mathematical Formulation

Following Dubey et al. [1], this model formulation under some simplifying assumptions is

$$\frac{dN}{dt} = r(B)N - \frac{r_0 N^2}{K(B,T)} \quad (1)$$

$$\frac{dB}{dt} = r_B(U,N)B - \frac{r_{B0} B^2}{K_B(T)} \quad (2)$$

$$\frac{dT}{dt} = Q(t) - \delta_0 T - \alpha BT + \theta_1 \delta_1 U + \pi\ \upsilon BU \quad (3)$$

$$\frac{dU}{dt} = \beta B + \theta_0 \delta_0 T - \delta_1 U + \alpha BT - rBU \quad (4)$$

Here, the initial conditions are N (0) > 0, B (0) > 0, T(0) > 0, U(0) > 0 when the independent variable time t is equal to zero. For the purpose of this simulation study, the two carrying capacities are defined by $K(B,T) = K_0 + K_1B - K_2T$ and $K_B(T) = K_{B0} - K_{B1}T$. Following Dubey et al. [1], we have considered the following precise parameter values: $K_0 = 60$, $K_1 = 0.02$, $K_2 = 0.03$, $B = 1.46$, $T = 9$, $K_{B0} = 3.0$, $K_{B1} = 0.05$.

3.0 Method of Solution

The posed problem demands the application of a simple calculation scheme without a resort to the technique of a solution trajectory over a longer period of time which does not yield the expected results in this context. By the application of the two carrying capacities formulae as specified in the last section of this paper, the detailed method of analysis is defined next.

According to Dubey and Hussain [1], the function K(B, T)specifies the maximum density of the species population that the environment can support in the presence of the resource biomass and the environmental pollutant. Following these authors, this function is said to increase as the density of the resource biomass increases while it is said to decrease as the environmental concentration of the pollutant increases. In contrast, the function $K_B(T)$ specifies the maximum density of the resource biomass that the environment can support in the presence of the pollutant and it is said to decrease as the environmental concentration of the pollutant increases. In the event of an extrinsic factor such as the extreme climate change, it is highly probable that the model parameters which can be impacted are the K_0 and K_{B0} having the precise values of 60 and 3 respectively. For the purpose of this study, we have assumed the value of K_{B0} to be 50.

In this pioneering study, the random noise intensity of 0.8 on these two parameters was numerically simulated. Other values of the random noise intensities which we have used are 1.6 and 2.4. Since the variation on the chosen parameters are random, the simulated data have to be repeated a couple of times to characterise the random nature of the generated data so as to differentiate these data from deterministic data. The impact of the random noise intensity on the two carrying capacities was determined over ten (10) repeated simulations. The difference between the carrying capacity data sets without random noise and those carrying capacity data sets with random noise was defined by the notations of D_1 and D_2. The numerical method of error analysis using the 1-norm, 2-norm and infinity-norm was utilized to determine the three measures of error between the data sets. The results which we have obtained are presented in the next section of this paper.

4.0 Results and Discussion

The proposed method of analysis has produced the following results as presented in the following list of Tables.

Table 1: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 0.8.

Example	K_{1new}	K_{1old}	Differences	K_{2new}	K_{2old}	Differences
1	60.5023	59.7592	0.7431	3.1343	2.55	0.5843
2	60.1501	59.7592	0.3909	3.0128	2.55	0.4628
3	59.9490	59.7592	0.1898	2.9171	2.55	0.3671
4	60.5297	59.7592	0.7705	2.9874	2.55	0.4374
5	60.1761	59.7592	0.4169	2.7353	2.55	0.1853
6	60.1503	59.7592	0.3911	3.0492	2.55	0.4992
7	60.3025	59.7592	0.5433	2.8664	2.55	0.3164
8	60.0531	59.7592	0.2939	3.3404	2.55	0.7904
9	59.7894	59.7592	0.0302	3.2581	2.55	0.7081
10	60.4898	59.7592	0.7306	3.1869	2.55	0.6369

How do we measure the size of the error of the data sets D_1 and D_2? Let $D_1 = K_{1new} - K_{1old}$ and $D_2 = K_{2new} - K_{2old}$. Here, we calculated the 1-norm of D_1, the 2-norm of D_1 and the infinity-norm of D_1 to obtain 4.5004, 1.6055 and 0.7705 while the 1-norm of D_2, the 2-norm of D_2 and the infinity-norm of D_2 are calculated to be 4.9880, 1.6725 and 0.7904.

Table 2: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 0.8 (another 10 repeated simulations).

Example	K_{1new}	K_{1old}	Differences	K_{2new}	K_{2old}	Differences
1	60.0398	59.7592	0.2806	3.3012	2.55	0.7512
2	60.4600	59.7592	0.7008	2.9901	2.55	0.4401
3	60.2572	59.7592	0.4980	3.0196	2.55	0.4696
4	59.9254	59.7592	0.1662	2.7910	2.55	0.2410
5	60.1359	59.7592	0.3767	2.7344	2.55	0.1844
6	60.4346	59.7592	0.6754	2.7058	2.55	0.1558
7	59.9399	59.7592	0.1807	2.6866	2.55	0.1366
8	59.9413	59.7592	0.1821	2.8986	2.55	0.3486
9	60.0081	59.7592	0.2489	3.2887	2.55	0.7387
10	60.1034	59.7592	0.3442	2.6979	2.55	0.1479

In order to measure the size of the error between the new carrying capacity value and the old carrying capacity value for interacting populations, we have considered the difference between the second and third columns as well as the difference between the fifth and sixth columns. Upon the application of these formulae, we calculated the 1-norm of D_1, the 2-norm of D_1 and the infinity-norm of D_1 to obtain 3.6536, 1.2999 and 0.7008 while the calculated 1-norm of D_2, the 2-norm of D_2 and the infinity-norm of D_2 are 3.6138, 1.3426 and 0.7512.

Table 3: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 0.8 (another 10 repeated simulations).

Example	K_{1new}	K_{1old}	Differences	K_{2new}	K_{2old}	Differences
1	60.4831	59.7592	0.7239	3.3338	2.55	0.7838
2	60.1103	59.7592	0.3511	2.6389	2.55	0.0889
3	59.9657	59.7592	0.2065	2.8770	2.55	0.3270
4	60.2351	59.7592	0.4759	2.7598	2.55	0.2098
5	60.2415	59.7592	0.4823	3.1190	2.55	0.5690
6	59.9366	59.7592	0.1774	2.6439	2.55	0.0939
7	59.9965	59.7592	0.2373	2.8050	2.55	0.2550
8	60.0985	59.7592	0.3393	2.9563	2.55	0.4063
9	59.8276	59.7592	0.0684	2.7600	2.55	0.2100
10	60.4000	59.7592	0.6408	2.5734	2.55	0.0234

In this scenario, the calculated 1-norm, 2-norm and infinity-norm of D_1 are 3.7029, 1.3294 and 0.7239 while the calculated 1-norm, 2-norm and infinity-norm of D_2 are 2.9670, 1.1749 and 0.7838.

Table 4: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 0.8 (another 10 repeated simulations).

Example	K_{1new}	K_{1old}	Differences	K_{2new}	K_{2old}	Differences
1	59.8382	59.7592	0.0790	2.7595	2.55	0.2095
2	60.0275	59.7592	0.2683	3.0938	2.55	0.5438
3	59.8684	59.7592	0.1092	3.1270	2.55	0.5770
4	59.8446	59.7592	0.0854	3.0730	2.55	0.5230
5	60.1545	59.7592	0.3953	3.1732	2.55	0.6232
6	60.3312	59.7592	0.5720	3.2730	2.55	0.7230
7	60.4719	59.7592	0.7127	2.8173	2.55	0.2673
8	60.3182	59.7592	0.5590	2.7082	2.55	0.1582
9	59.7836	59.7592	0.0244	3.1453	2.55	0.5953
10	60.1592	59.7592	0.4000	2.9339	2.55	0.3839

In this scenario, the calculated 1-norm, 2-norm and infinity-norm of D_1 are 3.2055, 1.2498 and 0.7127 while the calculated 1-norm, 2-norm and infinity-norm of D_2 are 4.6043, 1.5670 and 0.7230.

Table 5: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 0.8 (another 10 repeated simulations).

Example	K_{1new}	K_{1old}	Differences	K_{2new}	K_{2old}	Differences
1	60.4830	59.7592	0.7238	3.0379	2.55	0.4879
2	60.2533	59.7592	0.4941	3.2376	2.55	0.6876
3	60.4036	59.7592	0.6444	3.0114	2.55	0.4614
4	59.9055	59.7592	0.1463	2.7419	2.55	0.1919
5	60.4684	59.7592	0.7092	2.5729	2.55	0.0229
6	60.1511	59.7592	0.3919	2.6843	2.55	0.1343
7	60.5421	59.7592	0.7829	3.1202	2.55	0.5702
8	60.1596	59.7592	0.4004	2.9269	2.55	0.3769
9	59.8069	59.7592	0.0477	3.0956	2.55	0.5456
10	59.7931	59.7592	0.0339	2.6072	2.55	0.0572

Here, the calculated 1-norm, 2-norm and infinity-norm of D_1 are 4.3747, 1.6242 and 0.7829 whereas the calculated 1-norm, 2-norm and infinity-norm of D_2 are 3.5358, 1.3218 and 0.6876.
What if the intensity of the random noise tends to increase? Consider a situation when the intensity of random noise is 1.6. The following results were obtained:

Table 6: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 1.6 (another 10 repeated simulations).

Example	K_{1new}	K_{1old}	Differences	K_{2new}	K_{2old}	Differences
1	60.5938	59.7592	0.8346	2.7048	2.55	0.1548
2	61.0682	59.7592	1.3090	3.8581	2.55	1.3081
3	60.9151	59.7592	1.1559	2.7898	2.55	0.2398
4	60.8146	59.7592	1.0554	3.3798	2.55	0.8298
5	61.3160	59.7592	1.5568	3.5884	2.55	1.0384
6	61.0397	59.7592	1.2805	3.2761	2.55	0.7261
7	60.4510	59.7592	0.6918	3.8705	2.55	1.3205
8	59.8928	59.7592	0.1336	2.7631	2.55	0.2131
9	60.0366	59.7592	0.2774	3.1755	2.55	0.6255
10	61.0894	59.7592	1.3302	3.8354	2.55	1.2854

The errors between the K_{1new} and K_{1old} differentiated data sets are 9.6252, 3.3565 and 1.5568 by using the 1-norm, 2-norm and infinity-norm measures whereas the errors between the K_{2new} and K_{2old} differentiated data sets are 7.7413, 2.8141 and 1.3205 by using the 1-norm, 2-norm and infinity-norm measures. Next, we have considered another set of ten repeated simulations, of which the corresponding results are presented as follows:

Table 7: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 1.6 (another 10 repeated simulations).

Example	**K_{1new}**	**K_{1old}**	**Differences**	**K_{2new}**	**K_{2old}**	**Differences**
1	59.8560	59.7592	0.0968	3.1888	2.55	0.6388
2	60.6022	59.7592	0.8430	3.2169	2.55	0.6669
3	60.8102	59.7592	1.0510	3.5548	2.55	1.0048
4	60.2264	59.7592	0.4672	3.2406	2.55	0.6906
5	59.7840	59.7592	0.0248	4.1245	2.55	1.5745
6	60.0267	59.7592	0.2675	2.7199	2.55	0.1699
7	60.3551	59.7592	0.5959	2.8670	2.55	0.3170
8	60.5427	59.7592	0.7835	3.0932	2.55	0.5432
9	61.2818	59.7592	1.5226	4.0225	2.55	1.4725
10	59.8435	59.7592	0.0843	3.7306	2.55	1.1806

In this scenario, the errors between the K_{1new} and K_{1old} differentiated data sets are 5.7364, 2.3258 and 1.5226 by using the 1-norm, 2-norm and infinity-norm measures whereas the errors between the K_{2new} and K_{2old} differentiated data sets are 8.2588, 2.9673 and 1.5745 by using the 1-norm, 2-norm and infinity-norm measures.
Over ten repeated simulations when the noise intensity is 1.6, the calculated impact of random noise is presented as follows:

Table 8: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 1.6 (another 10 repeated simulations).

Example	**K_{1new}**	**K_{1old}**	**Differences**	**K_{2new}**	**K_{2old}**	**Differences**
1	60.1898	59.7592	0.4306	3.2265	2.55	0.6765
2	60.6358	59.7592	0.8766	4.0584	2.55	1.5084
3	60.4276	59.7592	0.6684	4.1229	2.55	1.5729
4	60.2415	59.7592	0.4823	3.6718	2.55	1.1218
5	60.8253	59.7592	1.0661	3.4126	2.55	0.8626
6	60.8762	59.7592	1.1170	3.6164	2.55	1.0664
7	60.0442	59.7592	0.2850	2.7548	2.55	0.2048
8	61.3577	59.7592	1.5985	2.8238	2.55	0.2738
9	59.8114	59.7592	0.0522	3.4479	2.55	0.8979
10	61.1702	59.7592	1.4110	3.6207	2.55	1.0707

In the same manner, the errors between the K_{1new} and K_{1old} differentiated data sets are 7.9877, 2.9407 and 1.5985 by using the 1-norm, 2-norm and infinity-norm measures whereas the errors between the K_{2new} and K_{2old} differentiated data sets are 9.2558, 3.2274 and 1.572 by using the 1-norm, 2-norm and infinity-norm measures.
Next, we have considered another ten repeated simulations when the noise intensity is 1.6, the calculated impact of random noise is presented as follows:

Table 9: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 1.6 (another 10 repeated simulations).

Example	K_{1new}	K_{1old}	Differences	K_{2new}	K_{2old}	Differences
1	60.0639	59.7592	0.3047	3.1403	2.55	0.5903
2	60.4964	59.7592	0.7372	4.1206	2.55	1.5706
3	60.0094	59.7592	0.2502	3.9188	2.55	1.3688
4	60.7908	59.7592	1.0316	3.1520	2.55	0.6020
5	60.0647	59.7592	0.3055	3.2352	2.55	0.6852
6	60.5304	59.7592	0.7712	2.7430	2.55	0.1930
7	60.7024	59.7592	0.9432	2.9119	2.55	0.3619
8	60.3746	59.7592	0.6154	3.4828	2.55	0.9328
9	60.1621	59.7592	0.4029	3.0147	2.55	0.4647
10	60.7465	59.7592	0.9873	2.9744	2.55	0.4244

In the same manner, the errors between the K_{1new} and K_{1old} differentiated data sets are 6.3493, 2.2038 and 1.0316 by using the 1-norm, 2-norm and infinity-norm measures whereas the errors between the K_{2new} and K_{2old} differentiated data sets are 7.1938, 2.6373 and 1.5706 by using the 1-norm, 2-norm and infinity-norm measures. The final consideration of this extended simulation analysis is presented next and discussed.

Table 10: Measuring the impact of the environmental random noise on the carrying capacities of a mathematical model of survival of species dependent on a resource in a polluted environment with the noise intensity of 2.4 (another 10 repeated simulations).

Example	K_{1new}	K_{1old}	Differences	K_{2new}	K_{2old}	Differences
1	61.7145	59.7592	1.9553	4.7239	2.55	2.1739
2	60.0640	59.7592	0.3048	4.7421	2.55	2.1921
3	61.2769	59.7592	1.5177	2.7841	2.55	0.2341
4	60.4276	59.7592	0.6684	3.8625	2.55	1.3125
5	62.0572	59.7592	2.2980	4.8657	2.55	2.3157
6	60.1375	59.7592	0.3783	4.8794	2.55	2.3294
7	62.0564	59.7592	2.2972	3.7149	2.55	1.1649
8	61.6799	59.7592	1.9207	2.8905	2.55	0.3405
9	60.7714	59.7592	1.0122	4.7478	2.55	2.1978
10	61.6605	59.7592	1.9013	4.8528	2.55	2.3028

In the same manner, the errors between the K_{1new} and K_{1old} differentiated data sets are 14.2538, 5.0691 and 2.2980 by using the 1-norm, 2-norm and infinity-norm measures whereas the errors between the K_{2new} and K_{2old} differentiated data sets are 16.5637, 5.8055 and 2.3294 by using the 1-norm, 2-norm and infinity-norm measures.

5.0 Conclusion

The carrying capacity of interacting populations in a polluted environment which supports the growth of such populations over time can suffer from some sort of a random noise variation. Being able to measure it numerically is a challenging mathematical problem which this present study has achieved on the application of a Matlab computational efficient method. The error analysis of the differentiated data sets without and with random environmental noise was fully implemented. The precise values of the three popular mathematical norms indicate that the errors between the data sets can be said to be best-fit in this context. Therefore, a further sophisticated capacity building data mining initiative that is presently lacking in most developing countries should be encouraged and funded so as to take this level of simulation analysis beyond this present research report. If the oil spillage which can create some sort of random noise intensity cannot be completely avoided, this simulation study makes a strong case for the minimization of oil spillage induced-random noise which has a long-standing sustainable development insight in Nigeria.

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