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Modelling Mutualism and Its Stability: A Case Study of Two Dis-similar Carrying Capacities

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Abstract

The study of a mutualistic interaction with two dis-similar carrying capacities and the effect of this idea on the stability of its co-existence steady-state solution is primarily a crop science problem that requires a mathematical reasoning. In this paper, we propose to study the stability of a mutualistic interaction for three scenarios of the intrinsic growth rate parameter values for cowpea and groundnut while other model parameter values are fixed. The results which we have obtained have not been seen elsewhere, they are presented here and discussed.

Keywords: Co-existence steady-state solution, stability, interacting legumes.

1.0 Introduction

The concept of stability is well known and applied within mathematical literatures [1 – 6]. However, the numerical modelling of stability with respect to a mutualistic interaction when the intrinsic growth rates of interacting populations are considered does seem to be a popular approach. This present analysis is based on the data of Ekpo and Nkanang [7]. While the idea of the carrying capacities between two interacting populations has a long-standing in the biological sciences, environmental sciences and mathematical ecology, its impact on an analysed mutualistic system and its stability is a very rare contribution. Since the carrying capacity specifies the maximum population size or biomass which sustains the growth of a population in question, it is vital to study its variation on the stability of a mutualistic interaction between legumes. Our present study is clearly differentiated from our most recent analysis [8]. Our previous study concerns an investigation on the impact of the intra-species coefficients on the stability, instability and degeneracy of a co-existence steady-state solution between two competing yeast species while this study has focused on the simulation analysis of the impact of the carrying capacities on the stability of the mutualistic interaction between two types of legumes. Therefore, this present analysis has made its own contribution to knowledge which has indicated its distinct ecological characterization and mathematical tractability than our previous study.

2.0 Mathematical Formulation

Following Ekaka-a [5], we consider the following system of model equations of continuous nonlinear first order ordinary differential equations

$$\frac{dC(t)}{dt} = C(t)[a - bC(t) + cG(t)] \quad (1)$$

$$\frac{dG(t)}{dt} = G(t)[d - fG(t) + eC(t)] \quad (2)$$

Where C(0) > 0 and G(0) > 0 define the starting biomasses of cowpea and groundnut at the start of the growing season otherwise called the initial conditions when t = 0. The duration of growth is in the unit of days hereby denoted by the independent variable t. For the purpose of this simulation study, the best-fit model parameters such as aand d that define the intrinsic growth rates for cowpea and groundnut were selected using the data of Ekpo and Nkanang [7]. The next best-fit parameters such as b and f define the intra-species competition parameters which measure the inhibiting factors on the growth of cowpea and groundnut due to self-interaction whereas the parameters c and e define the inter-species competition parameters which also measure other inhibiting factors on the growth of cowpea and groundnut due to interspecific interaction between cowpea and groundnut. In this study, we have considered the following parameter values: a = 0.0225, d = 0.0446, b = 0.006902, f = 0.0133, c = 0.0005, e = 0.01.

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2.1 Method of Solution

Following the recent idea of Ekaka-a andAgwu [2013], we have used a computational method to determine each type of stability for a system of two interacting populations undergoing a mutualistic interaction. First of all, the two carrying capacities for the interacting legumes were defined and coded using a Matlab programming language. Secondly, the co-existence steady-state solution which was derived analytically by solving the two simultaneous linear equations in terms of C and Gwhich were obtained by equating the growth rate equations to zero was also coded. Thirdly, the four partial derivatives of the two interaction functions in terms of C and Gwith respect to C and G were derived and evaluated at the arbitrary co-existence steady-state solution or point (C, G). Fourthly, a Jacobian matrix of four elements were constructed and coded from which two eigenvalues were calculated computationally and tested to be consistent with their counterpart analytical calculations. From the theory of the sign method in the study of stability of a steady-state solution, the qualitative values of the eigenvalues were determined which form the basis for each type of stability of a co-existence steady-state solution. If upon the evaluation of the Jacobian matrix and we obtain either two positive eigenvalues or eigenvalues of opposite signs then the co-existence steady-state solution can be classified as being unstable. On the other hand, if two negative eigenvalues were obtained then the co-existence steady-state solution is said to be stable. It should also be noted that if any of the co-ordinates of the co-existence steady-state solution bears a negative sign, this observation has a counter-intuitive ecological meaning. In this scenario, such a steady-state solution can be classified as being degenerate. When a steady-state solution is degenerate, it should be considered as a quantitative indication in which one of the interacting legumes can go into the ecological risk of extinction escaping survival while the other legume can tend to survive.

3.0 Results and Discussion

The results which we have obtained and have not been seen elsewhere are presented and discussed here in the Tables below: the notation a stands for the intrinsic growth rate parameter value for cowpea, the notation css stands for the co-existence steady-state solution while the notations λ_1 and λ_2 stand for the two eigenvalues whose signs define the type of stability for the co-existence steady-state solution.

Table 1: Calculating the qualitative stability of a co-existence steady-state solution (css) due to a variation of the intrinsic growth rate **a** of cowpea and the intrinsic growth rate **d** of groundnut: summary of results 1

Example	a	d	css	λ_1	λ_2	Type of stability
1	0.0225	0.0445	3.71:6.14	-0.024	-0.084	stable
2	0.0022	0.0045	0.37:0.61	-0.0024	-0.0084	stable
3	0.0034	0.0067	0.56:0.92	-0.0035	-0.0125	stable
4	0.0045	0.0089	0.74:1.23	-0.0047	-0.0167	stable
5	0.0056	0.0112	0.93:1.54	-0.0059	-0.0209	stable
6	0.0067	0.0134	1.11:1.84	-0.0071	-0.0251	stable
7	0.0079	0.0156	1.30:2.15	-0.0083	-0.0293	stable
8	0.0090	0.0178	1.48:2.46	-0.0094	-0.0334	stable
9	0.0101	0.0201	1.67:2.76	-0.0106	-0.0376	stable
10	0.0113	0.0223	1.85:3.07	-0.0118	-0.0418	stable

The first row of the above Table shows that the co-existence steady-state solution (3.71, 6.14) is stable having two negative eigenvalues -0.024 and -0.084 when the intrinsic growth rates of cowpea and groundnut are 0.0225 and 0.0446 in the unit of grams. It is unanimously consistent that every other co-existence steady-state solution ranging from (0.37, 0.61) to (1.85, 3.07) is stable having two negative eigenvalues [Table 1, example 2 to example 10]. In the next series of examples, we will consider a scenario when the values of a andd are 0.0225 grams per area.

Table 2: Calculating the qualitative stability of a co-existence steady-state solution (css) due to a variation of the intrinsic growth rate **a** of cowpea and the intrinsic growth rate **d** of groundnut: summary of results 2

Example	a	d	css	λ_1	λ_2	Type of stability
11	0.0225	0.0225	3.58:4.38	-0.0225	-0.0605	stable
12	0.0022	0.0022	0.36:0.44	-0.0022	-0.0060	stable
13	0.0034	0.0034	0.54:0.66	-0.0034	-0.0091	stable
14	0.0045	0.0045	0.72:0.88	-0.0045	-0.0121	stable
15	0.0056	0.0056	0.89:1.09	-0.0056	-0.0151	stable
16	0.0067	0.0067	1.07:1.31	-0.0067	-0.018	stable
17	0.0079	0.0079	1.25:1.53	-0.0079	-0.0212	stable
18	0.0090	0.0090	1.43:1.75	-0.009	-0.024	stable
19	0.0101	0.0101	1.61:1.97	-0.0101	-0.0272	stable
20	0.0113	0.0113	1.78:2.19	-0.0112	-0.0302	stable

In this second scenario, Table 2 shows that the co-existence steady-state solution (3.58, 4.38) is stable having two negative eigenvalues -0.0225 and -0.0605 when the intrinsic growth rates of cowpea and groundnut are 0.0225 and 0.0225 in the unit of grams. We have also observed that every other co-existence steady-state solution ranging from (0.36, 0.44) to (1.78, 2.19) is stable having two negative eigenvalues in all the nine instances [Table 2, example 11 to example 20]. What if the two intrinsic growth rates are 0.0445 and 0.0445 with the appropriate units? So far in the present analysis we have not tackled this level of simulation analysis in order to determine the co-existence steady-state solution and its type of stability. Our next results in this context are presented in Table 3.

Table 3: Calculating the qualitative stability of a co-existence steady-state solution (css) due to a variation of the intrinsic growth rate **a** of cowpea and the intrinsic growth rate **d** of groundnut: summary of results 3

Example	**a**	**d**	**css**	λ_1	λ_2	**Type of stability**
21	0.0445	0.0445	7.08:8.67	-0.0445	-0.1196	stable
22	0.0045	0.0045	0.71:0.87	-0.0045	-0.012	stable
23	0.0067	0.0067	1.06:1.30	-0.0067	-0.0179	stable
24	0.0089	0.0089	1.42:1.73	-0.0089	-0.0239	stable
25	0.0111	0.0111	1.77:2.16	-0.011	-0.03	stable
26	0.0133	0.0133	2.12:2.60	-0.0133	-0.0359	stable
27	0.0156	0.0156	2.48:3.03	-0.0156	-0.0419	stable
28	0.0176	0.0176	2.83:3.47	-0.0178	-0.0478	stable
29	0.0200	0.0200	3.18:3.90	-0.0200	-0.0538	stable
30	0.0222	0.0222	4.55:5.10	-0.0222	-0.0770	stable

In this third scenario, Table 3 shows that the co-existence steady-state solution (7.08, 8.67) is stable having two negative eigenvalues -0.0445 and -0.1196 when the intrinsic growth rates of cowpea and groundnut are 0.0445 and 0.0445 in the unit of grams. We have also observed that every other co-existence steady-state solution ranging from (0.71, 0.87) to (4.55, 5.10) is stable having two negative eigenvalues in all the nine instances [Table 3, example 21 to example 30].

4.0 Conclusion

A dominant deduction from this present analysis shows that when the two intrinsic growth rates are varied, these two parameters at a time when other model parameters are fixed, stability of each co-existence steady-state solution is guaranteed. This key observation supports biodiversity gain in these thirty (30) illustrating examples. We would expect these results to provide some insights that can aid in the food production planning of the Niger Delta agricultural region.

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