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Numerical Modelling of Stability of a Mutualistic Interaction with Changes In the Intrinsic Growth Rate

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Abstract

The study of the stability with respect to a mutualistic interaction between two legumes remains to be a challenging crop science and mathematical modelling problem. We propose to tackle this problem using the approach of a numerical simulation. In this paper, we propose to study the stability of a mutualistic interaction when the factor of the intrinsic growth rate parameter value of cowpea is varied while other precise model parameter values are fixed. The results which we have obtained have not been seen elsewhere, they are presented here and discussed.

Keywords: Co-existence steady-state solution, stability, interacting legumes.

1.0 Introduction

The concept of stability is well known and applied within the mathematical literature [1 – 6]. However, the numerical modelling of stability with respect to a mutualistic interaction when the intrinsic growth rates of interacting populations are changing is not a popular approach. This present analysis is based on the data of Ekpo and Nkanang [7]. A unanimous observation is the fact it is the intrinsic growth rate that can be severely affected by any environmental factor on the ecosystem than any other model parameter which defines the dynamics of interacting populations. It is on the basis of this idea that we have proposed to test the variation of the intrinsic growth rate on the stability of a mutualistic interaction between two types of legumes such as cowpea and groundnut. This sophisticated numerical simulation study tackles a distinct long-standing crop science problem using a sound mathematical reasoning that is clearly differentiated from our earlier analysis as reported by Ekaka-a and Agwu [8]. While our most recent analysis [8] concerns the impact of the changes in the intra-species competition on its stability, instability and degeneracy of a co-existence steady-state solution, our present analysis has focused on the impact of the intrinsic growth rate of the cowpea species in a mutualistic interaction which is characteristic of legumes in terms of its stability behaviour. The present numerical study clearly supports a strong evidence for a whole biodiversity gain and can be very attractive as a decision-policy capacity-building initiative in crop science data mining and food production with an excellent sustainable development application in Nigeria.

2.0 Mathematical Formulation

Following Ekaka-a [5], we consider the following system of model equations of continuous nonlinear first order ordinary differential equations

$$\frac{dC(t)}{dt} = C(t)[a - bC(t) + cG(t)] \qquad (1)$$

$$\frac{dG(t)}{dt} = G(t)[d - fG(t) + eC(t)] \qquad (2)$$

Where C(0) > 0 and G(0) > 0 define the starting biomasses of cowpea and groundnut at the start of the growing season otherwise called the initial conditions when t = 0. The duration of growth is in the unit of days hereby denoted by the independent variable t. For the purpose of this simulation study, the best-fit model parameters such as aand d that define the intrinsic growth rates for cowpea and groundnut were selected using the data of Ekpo and Nkanang [7]. The next best-fit parameters such as b and f define the intra-species competition parameters which measure the inhibiting factors on the growth of cowpea and groundnut due to self-interaction whereas the parameters c and e define the inter-species competition parameters which also measure other inhibiting factors on the growth of cowpea and groundnut due to interspecific interaction between cowpea and groundnut. In this study, we have considered the following parameter values: a = 0.0225, d = 0.0446, b = 0.006902, f = 0.0133, c = 0.0005, e = 0.01.

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2.1 Method of Solution

In this present study, we have used a computational method to determine each type of stability for a system of two interacting populations in the absence of a time delay [Ekaka-a andAgwu, 2013]. First of all, the two carrying capacities for the interacting legumes were defined and coded using a Matlab programming language. Secondly, the co-existence steady-state solution which was derived analytically by solving the two simultaneous linear equations in terms of C and Gwhich were obtained by equating the growth rate equations to zero was also coded. Thirdly, the four partial derivatives of the two interaction functions in terms of C and Gwith respect to C and G were derived and evaluated at the arbitrary co-existence steady-state solution or point (C, G). Fourthly, a Jacobian matrix of four elements were constructed and coded from which two eigenvalues were calculated computationally and tested to be consistent with their counterpart analytical calculations. From the theory of the sign method in the study of stability of a steady-state solution, the qualitative values of the eigenvalues were determined which form the basis for each type of stability of a co-existence steady-state solution. If upon the evaluation of the Jacobian matrix and we obtain either two positive eigenvalues or eigenvalues of opposite signs then the co-existence steady-state solution can be classified as being unstable. On the other hand, if two negative eigenvalues were obtained then the co-existence steady-state solution is said to be stable. It should also be noted that if any of the co-ordinates of the co-existence steady-state solution bears a negative sign, this observation has a counter-intuitive ecological meaning. In this scenario, such a steady-state solution can be classified as being degenerate. When a steady-state solution is degenerate, it should be considered as a quantitative indication in which one of the interacting legumes can go into the ecological risk of extinction escaping survival while the other legume can tend to survive.

3.0 Results and Discussion

The results which we have obtained and have not been seen elsewhere are presented and discussed here in the Tables below: the notation a stands for the intrinsic growth rate parameter value for cowpea, the notation ss stands for the co-existence steady-state solution while the notations λ_1 and λ_2 stand for the two eigenvalues whose signs define the type of stability for the co-existence steady-state solution.

Table 1: Calculating the qualitative stability of a co-existence steady-state solution (ss) due to a variation of the intrinsic growth rate ‘a’ of cowpea: summary of results 1

Example	**a**	**ss**	λ_1	λ_2	**Type of stability**
1	0.0225	3.71: 6.14	-0.0236	-0.0836	stable
2	0.0011	0.43:3.68	-0.0028	-0.0491	stable
3	0.0023	0.60:3.81	-0.0039	-0.0509	stable
4	0.0034	0.77:3.94	-0.0050	-0.0527	stable
5	0.0045	0.95:4.07	-0.0061	-0.0545	stable
6	0.0056	1.12:4.20	-0.0072	-0.0563	stable
7	0.0067	1.29:4.32	-0.0083	-0.0581	stable
8	0.0079	1.46:4.45	-0.0094	-0.0599	stable
9	0.0090	1.64:4.58	-0.0105	-0.0617	stable
10	0.0101	1.81:4.71	-0.0116	-0.0635	stable

The first row of Table 1 shows that the co-existence steady-state solution (3.71, 6.14) is stable having two negative eigenvalues -0.0236 and -0.0836 when the intrinsic growth rate of cowpea is 0.0225 in the unit of grams. It is unanimously consistent that every other co-existence steady-state solution ranging from (0.43, 3.68) to (1.81, 4.71) is stable having two negative eigenvalues [Table 1, example 2 to example 10] while the other co-existence steady-state solutions ranging from (1.98, 4.84) to (3.53, 6.01) are similarly said to be stable [Table 2, example 11 to example 20]. For the close intervals of stable co-existence steady-state solutions that range from (3.74, 6.17) to (5.77, 7.69), our results are also displayed in Table 3 for example 21 to example 33. If each varied value of the model parameter a is represented by an independent variable x and the estimated biomasses of cowpea and groundnut are represented by the dependent variable y, we can quantify statistically a percentage of the variation in y which can be explained by x in future analysis. There is also a relationship between the carrying capacity of cowpea and each component of the co-existence steady-state solution which is yet to be empirically verified.

Table 2: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate 'a' of cowpea: summary of results 2

Example	**a**	**ss**	λ_1	λ_2	**Type of stability**
11	0.0113	1.98:4.84	-0.0127	-0.0653	stable
12	0.0124	2.15:4.97	-0.0138	-0.0672	stable
13	0.0135	2.33:5.10	-0.0149	-0.0690	stable
14	0.0146	2.50:5.23	-0.0160	-0.0708	stable
15	0.0158	2.67:5.36	-0.0171	-0.0726	stable
16	0.0169	2.84:5.49	-0.0182	-0.0745	stable
17	0.0180	3.02:5.62	-0.0193	-0.0763	stable
18	0.0191	3.188:5.75	-0.0204	-0.0781	stable
19	0.0203	3.36:5.88	-0.0214	-0.0799	stable
20	0.0214	3.53:6.01	-0.0225	-0.0818	stable

Table 3: Calculating the qualitative stability of a co-existence steady-state solution due to a variation of the intrinsic growth rate 'a' of cowpea: summary of results 3

Example	**a**	**ss**	λ_1	λ_2	**Type of stability**
21	0.0227	3.74:6.17	-0.0238	-0.0840	stable
22	0.0236	3.88:6.27	-0.0247	-0.0854	stable
23	0.0248	4.05:6.40	-0.0258	-0.0873	stable
24	0.0259	4.22:6.53	-0.0268	-0.0891	stable
25	0.0270	4.39:6.66	-0.0279	-0.0910	stable
26	0.0281	4.57:6.80	-0.0290	-0.0928	stable
27	0.0293	4.74:6.92	-0.0301	-0.0946	stable
28	0.0304	4.91:7.05	-0.0311	-0.0965	stable
29	0.0315	5.08:7.18	-0.0322	-0.0983	stable
30	0.0326	5.26:7.31	-0.0333	-0.1002	stable
31	0.0338	5.43:7.44	-0.0343	-0.1020	stable
32	0.0349	5.60:7.57	-0.0354	-0.1039	stable
33	0.0360	5.77:7.69	-0.0365	-0.1057	stable

4.0 Conclusion

It is systematically very clear in this important inter-disciplinary study that we have successfully used the technique of a numerical simulation to model the type of co-existence steady-state solution and its stability when the intrinsic growth rate parameter 'a' is varied in the context of a mutualistic interaction between cowpea and groundnut. On the basis of stability theory, our present analysis shows that the two negative values of the eigenvalues contribute to the decaying behaviour of the solution trajectories.

In comparison with a competition interaction in which degeneracy of a few steady-state solutions were observed, our present analysis in the scenario of a mutualistic interaction shows the entire loss of the phenomenon of degeneracy of a co-existence steady-state solution which does not have any a sound ecological meaning since either of the biomass of cowpea or groundnut is only expected to have a positive value and not a negative value. Therefore, for a tropical ecosystem such as that of the Niger Delta Region of Nigeria, a mutualistic interaction otherwise called mutualism sustains a dominant co-existence steady-state solution which is unanimously said to be stable. In this context, the interaction between cowpea and groundnut brings mutual benefits to these two legumes.

It is interesting to mention that in the event of a climate change which is more likely to affect the intrinsic growth rate parameter more than any of the model parameters, it is more likely to have a stable co-existence steady-state solution. These numerical results are capable of providing insight to farmers and agriculturists in terms of the planning for effective crop production and the management of food production which have clear sustainable development implications in Nigerian agricultural sector. The idea which we have proposed to tackle an agricultural problem in the tropical world zone can be extended to tackle a similar problem in the harsh climate like the arctic region where growth patterns and other intrinsic scientific factors can take a longer time to achieve. In this present analysis, biodiversity gain is unanimously restored.

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