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A Generalized Adomian Decomposition Method for Convective Heat Transfer Through a Porous Medium Over a Stretching Surface

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Abstract

We construct approximate analytical solutions for problems of fluid flow and heat transfer in a porous medium over a stretching sheet by using a combined transformation and Adomian decomposition method (ADM). The results for the Nusselt number compares favourably with those obtained for the numerical solutions in literature. It is observed that over shooting which is inherent in the numerical solutions are eliminated by the ADM. The advantage of the transformation is the avoidance of the Hermite-Padé approximation for boundary condition at infinity, hence greatly reducing the computational complexity.

Keywords: Adomian Decomposition Method, Transformation, porous medium, Heat Transfer.

1.0 Introduction

There are many important *non-linear* equations arising from applied mathematical physics and engineering problems for which it is not possible to find an analytic solution. There are however, techniques where you can find approximate analytic solutions that are close to the true solution, at least within a certain range, e.g. the perturbation method. In such cases, the advantage over a numerical solution is that you wind up with an equation, instead of just a long list of numbers, from where we can gain some insight.

The literature in the search for closed-form approximate solution which are often analytical in nature or close to exact is very vast. The ADM which was first introduced by George Adomian is one of such closed-form approximate method of solution with several applications to heat transfer problems. Adanhounme and Codo [1] solved the Blasius problem using the Adomian decomposition method. Ebaid and Al-Armani [2] applied a transformation and the ADM to solve the Blasius problem also. Their results compared favorably with those obtained in literatures. Siddiqui *et al* [3] used the Adomian decomposition method in the study of parallel plate flow of a third grade fluid. Nadeen and Akbar [4] studied the effect of heat transfer on the peristaltic transport of MHD Newtonian fluid with variable viscosity using the ADM.

In this research work, we follow the work of Ebaid and Al-Armani [2] to solve the problem of convective heat transfer through a porous medium over a stretching sheet. Cortell [5] and Liu [6] analyzed the problem using Runge-Kutta shooting technique. Cortell [7] recorded over shooting for certain values of the thermo physical parameters which are not observable in the combined transformation and modified ADM. The transformation reduces the infinite domain into a finite interval. The modified decomposition naturally computes the Adomian polynomials.

2.0 Mathematical Formulation

Consider the laminar two-dimensional flow of a viscous incompressible fluid over a vertical plate embedded in a homogeneous porous media. The basic governing boundary layer equations using the Darcy and Boussinesq approximations [5].

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Continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

Momentum

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\mu_0}{\rho k}u \quad (2)$$

Energy

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{q}{\rho C}\left(T-T_\infty\right) \quad (3)$$

Subject to the following boundary conditions

$$\left.\begin{array}{l} v(x,0)=V(x), u(x,0)=u_0 x,\ T(x,0)=T_W \\ u(x,\infty)=0,\ \ T(x,\infty)=T_\infty \end{array}\right\} \quad (4)$$

The u and v are the velocity components in x and y directions respectively, T is the temperature, α is the thermal diffusivity, q_r is the internal heat rate, T_W is wall surface temperature, T_∞ is the free stream temperature, while, u_w and u_e are the surface and free stream velocity respectively , c is the specific heat capacity and ρ is the density of the fluid, k is the permeability of the porous media and (x, y) are the Cartesian coordinates along the surface normal to it.

Let the following be a set of dimensionless variables which we shall introduce into (1) – (4).

$$\left.\begin{array}{l} \eta=y\sqrt{\dfrac{U(x)}{\gamma x}}, \quad \psi(x,y)=\sqrt{\gamma x U(x)}\,f(\eta), \\ u=\dfrac{\partial\psi}{\partial y},\ v=-\dfrac{\partial\psi}{\partial x},\ \theta=\dfrac{T-T_\infty}{T_W-T_\infty},\ k_1=\dfrac{\mu_0}{k u_0\rho},\ f_w=-\dfrac{u_w}{\sqrt{u_0 V(x)}},\ \lambda=\dfrac{q}{\rho c u_0} \end{array}\right\} \quad 4b$$

We obtain the following set of coupled nonlinear ordinary differential equations.

$$f'''+ff''-(f')^2-k_1 f'=0 \quad (5)$$

$$\theta''+f\theta'+\lambda\theta=0 \quad (6)$$

subject to

$$f(0)=f_w,\ f'(0)=1,\ f'(\infty)=0, \theta(0)=1, \theta(\infty)=0 \quad (7)$$

Where $f'(\eta)$ is the dimensionless velocity function, k_1 is the inverse Darcy parameter, $\theta(\eta)$ is the dimensionless temperature profile and λ is the internal heat generation parameter.

Cortell [5] obtained an analytical solution of the form:

$$f(\eta)=f_w+\frac{1-\exp(-\alpha\eta)}{\alpha} \quad (8)$$

Using (8) in (6) we have;

$$\theta''+\left(f_w+\frac{1-\exp(-\alpha\eta)}{\alpha}\right)\theta'+\lambda\theta=0 \quad (9)$$

Equation (9) is a variable coefficient second order linear ordinary differential equation which was solved numerically by Cortell [5].

Method of solution:
Following Ebaid and Al-Armani [2], we assume that

$$t = 1 - e^{-\alpha\eta} \tag{10}$$

So that equation (9) becomes

$$\frac{d^2\theta}{dt^2} - \frac{1}{1-t}\frac{d\theta}{dt} + \frac{\sigma}{1-t}\left\{f_w + \frac{t}{\alpha}\right\}\frac{d\theta}{dt} + \frac{\lambda\theta(t)}{(1-t)^2} = 0 \tag{11}$$

Based on the modified Adomian decomposition, the solution of (11) is given by:

$$\theta(t) = \sum_{n=0}^{\infty}\theta_n(t), \quad \frac{1}{1-t} = \sum_{n=0}^{\infty} t^n \tag{11b}$$

Inserting (11b) into (11) and $\sigma = 1$ we have

$$\theta(t) = \theta(0) + \theta'(0)t + \sum_{n=0}^{n}\iint\left\{\sum_{j=0}^{n} t^{n-j}\theta_j{}'(t) - f_w\sum_{j=0}^{n} t^{n-j}\theta_j'(t) - \frac{1}{\alpha}\sum_{j=0}^{n} t^{n-j+1}\theta_j'(t) - \lambda\sum_{j=0}^{n}(n-j+1)t^{n-j}\theta_j(t)\right\}dtdt$$

given that

$$\theta_{n+1}(t) = \iint\left\{\sum_{j=0}^{\infty} t^{n-j}\theta_j{}'(t) - f_w\sum_{j=0}^{\infty} t^{n-j}\theta_j'(t) - \frac{1}{\alpha}\sum_{j=0}^{\infty} t^{n-j+1}\theta_j'(t) - \lambda\sum_{j=0}^{n}(n-j+1)t^{n-j}\theta_j(t)\right\}dtdt \tag{12}$$

And

$$\theta_0(t) = \theta(0) + \theta'(0)t \tag{13}$$

Since

$$\theta(0) = 1 \ and \quad \theta'(0) = A$$

We have

$$\theta_0(t) = 1 + At \tag{14}$$

3.0 Numerical Application

In order to obtain simple series solutions in terms of η and t we choose the following numerical values for the thermo-physical parameters.

I. $\sigma = 1,\ f_w = 1.5, \alpha = 2, \lambda = 0.3$

$$\frac{-84299}{169140} + \frac{7014}{4681}e^{-2\eta} + \frac{15733}{84035}\left(1-e^{-2\eta}\right)^3 + \frac{30391}{135312}\left(1-e^{-2\eta}\right)^2 + \frac{4150}{89669}\left(1-e^{-2\eta}\right)^5 + \frac{2061}{35330}\left(1-e^{-2\eta}\right)^4 + \frac{254}{211893}\left(1-e^{-2\eta}\right)^7 - \frac{3911}{203306}\left(1-e^{-2\eta}\right)^6$$

II. $\sigma = 1,\ f_w = 2.0, \alpha = 2, \lambda = 0.3$

$$\frac{28508}{25349} + \frac{53857}{25349}e^{-2\eta} + \frac{11059}{47406}\left(1-e^{-2\eta}\right)^3 + \frac{45111}{49447}\left(1-e^{-2\eta}\right)^2 + \frac{11765}{1627193}\left(1-e^{-2\eta}\right)^5 - \frac{71}{5727}\left(1-e^{-2\eta}\right)^4 + \frac{3951}{2324533}\left(1-e^{-2\eta}\right)^7 - \frac{11765}{1627193}\left(1-e^{-2\eta}\right)^6$$

III. $\sigma = 1,\ f_w = 2.5, \alpha = 2, \lambda = 0.3$

$$-\frac{50923}{30598} + \frac{35043}{13121}e^{-2\eta} - \frac{141}{50396}\left(1-e^{-2\eta}\right)^3 + \frac{37206}{20131}\left(1-e^{-2\eta}\right)^2 - \frac{3584}{351151}\left(1-e^{-2\eta}\right)^5 - \frac{8190}{51521}\left(1-e^{-2\eta}\right)^4 + \frac{1469}{689216}\left(1-e^{-2\eta}\right)^7 - \frac{6683}{666023}\left(1-e^{-2\eta}\right)^6$$

4.0 Numerical Results and Discussion

The temperature, $\theta(\eta)$ and Nusselt number, $\theta'(0)$ were computed for various values of the thermo-physical parameters. Figure 1 shows the temperature profile for various values of α which is a function of the suction and inverse Darcy parameters, $f_w > 0$ to suction while $f_w < 0$ corresponds to blowing and $f_w = 0$ corresponds to impermeable surface. α was varied in the interval $[2.0, 5.27]$ and it was observed that as it increases the temperature profile decreases. While figure 2 shows the effect of the internal heat generation parameter, λ. It is observed that an increase in the internal heat generation parameter leads to an increase in the temperature profile. Lastly, Figure 3 shows the impact of the suction parameter f_w. From the profile it is observed that as f_w increases, the boundary layer thickness decreases.

In Table 1, we compared the values of temperature with the numerical results obtained and that obtained from the Runge-Kutta shooting techniques (NMR). We see that the results of ADM converges faster than NMR. Table 2 shows the values of the Nusselt numbers which is in good agreement between the values obtained in this work and what is obtained in literature.

Table.1: The Results of ADM and Numerical Results

σ	f_w	α	λ	η	**Present**	**Numerical**
1	1.5	2	0.3	0	1.0000	1.0000
				0.2	0.5380	0.7206
				0.4	0.2814	0.5127
				0.6	0.1460	0.3616
				0.8	0.0763	0.2535
				1.0	0.0407	0.1771
				1.2	0.0224	0.1233
				1.4	0.0171	0.0857
				1.6	0.0075	0.0595
				1.8	0.0045	0.0413
				2.0	0.0028	0.0286
				2.2	0.0018	0.0198
				2.4	0.0012	0.0137
				2.6	0.0008	0.0095
				2.8	0.0005	0.0066
				3.0	0.0003	0.0046

Table.2: Nusselt Numbers

σ	f_w	α	λ	Present $-\theta(0)$	Cortell[2005] $-\theta(0)$	Numerical $-\theta(0)$
1	1.5	2	0.3	1.498400	1.563600	1.597700
		2.350781		1.462800	1.544600	1.571800
		5.274917		1.351800	-	1.445500

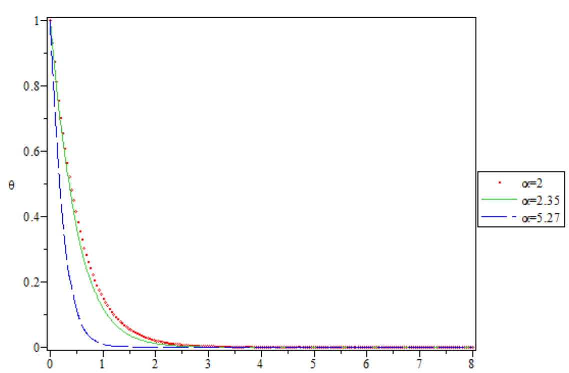

Fig 1: Temperature profile for various values of α for $f_0 = 1.5$, $\lambda = 0.3$

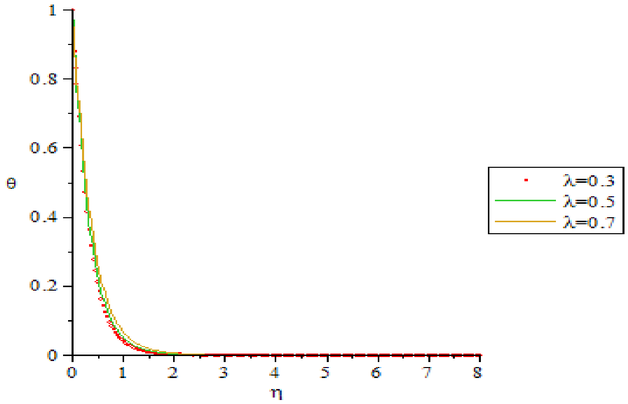

Fig.2: Temperature profile for various values of λ, with $f_0 = 1.5,\ \alpha = 2$

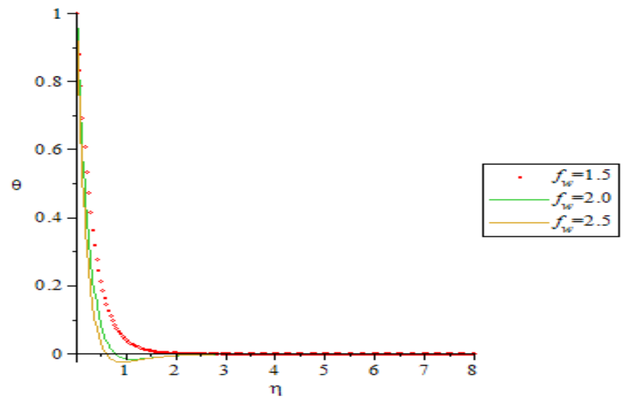

Fig.3: Temperature profile for various values of f_w, for $\lambda = 0.3,\ \alpha = 2$

5.0 Conclusion

In this work we provide a modified Adomian Decomposition method to the problem of heat transfer in porous medium over a stretching sheet. Results obtained are compared with that obtained in literature and an excellent agreement was observed between them.

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