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Choosing Appropriate Membership Function for Adaptive Neuro-Fuzzy Inference System (Anfis) Models

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Abstract

Scientific solutions to problems begin with modelling and simulation of the structure and behaviour of the conceived or existing systems.Adaptive Neuro-Fuzzy Inference System (ANFIS), a synergy of Fuzzy Logic and Artificial Neural Networks, is one of the models used in the artificial intelligence parlance tomimmic human intelligencein solvingcomplex real-world problems. To use ANFIS, membership functions are chosen to measure the degree to which a fuzzy set element meets the specific properties, (i.e.to measure thedegree of belongingness)of an element in a specific fuzzy set. Many researchers and authors have used membership functionsfor their ANFIS models arbitrarily or because of the simplicity of the membership functions. In this study, it has been shown that the choice of a membership function for an ANFIS model should not be based on the simplicity or otherwise of the membership function, but on theleast "minimum checking error" value from the simulation of the model using experimental data.

Keywords:ANFIS,model,vague concepts,membership functions,minimum checking error.

1.0 Introduction

A model is some form of abstract representation of an existing or proposed system with a view to understand, change, manage, and/or to control the system it represents. According to [1], "modelling, in general sense, refers to the establishment of a description of a system (a plant, a process, etc) in mathematical terms, which characterises the input-output behaviour of the underlying system". In science and engineering,mathematical models in the form of formulae or equations,are oftenused to represent a system both qualitatively and quantitatively. Sometimes, however, mathematical modelling of systems is not possible due to uncertain and vague knowledge about the studied system, or the behaviour of the system is too complex to be represented with accurate and precise mathematical or physical models.Solutions to such complex problems of this nature require intelligentsystems modelling techniques that combine knowledge and methodologies from various sources. For these cases, it is often more advantageous to use hybrid intelligent systems rather than use a single technique. An example ofa hybrid of intelligentsystems is the neuro-fuzzy computing system which integrates the advantages of the Fuzzy Inference system (FIS), and Artificial Neural Network (ANN).

Fuzzy Logic is a rigorous mathematical field which provides an effective formulation for modelling the imprecise and qualitative knowledge of experts, as well as the transmission of the uncertainty in human reasoning [2]. It is based on three core concepts of fuzzy sets, linguistic variables and possibility distribution.

Artificial neural networks are adaptive biologically-inspired nervous system networks that are composed of simple elements operating in parallel. As in nature, the network function is determined largely by the weighted connections between the elements. Neural networks are adjusted or trained to learn the profile or pattern of a particular input set that leads to specific target output. They have the generic advantages of massive parallelism, robustness and strong learning capability in data-rich (i.e., numeric) environments.

Adaptive Neuro-Fuzzy Inference System (ANFIS), which is an example of aneuro-fuzzy computing system, is the implementation of fuzzy inference system to adaptive networks for developing fuzzy rules with suitable membership functions to obtain required inputs and outputs.

This paper presents a method for the determination of an appropriate membership functions for an ANFIS model since the shapes of the membership functions of an ANFIS model depend on the notion the set of trained data by the model is intended to describe and on the particular application involved real-world systems and/or processes is fraught with these features.

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2.0 Background Information

2.1 Fuzzy set

Introduced by Zadeh in 1965, fuzzy sets are a mathematical way to represent and deal with vagueness in everyday life [3].A fuzzy set is a simple extension of the definition of a classical set in which the characteristic function is permitted to have any values between 0 (zero) and 1 (unity) [4]. It is a generalization to classical set to allow objects to take membership values between zero and unity in vague concepts [3]. If the value of the membership function, called the membership degree (or grade), equals one, x belongs completely to the fuzzy set.x does not belong to the set ifthe membership function equals zero, butpartially belongs to it if the membership degree is between 0 and 1 as expressed in equation (1)

$$\mu_F(x)\begin{cases} = 1 & xisafullmemberofF \\ \in (0,1) & xisapartialmemberofF \\ = 0 & xisanotmemberofF \end{cases} \qquad (1)$$

$$\mu_F(x): X \rightarrow [0, 1] \qquad (2)$$

where $\mu_F(x)$ is called the membership function of x in F.

Hence the membership function of a fuzzy set is allowed to have values between0 and 1 that denote the degree of membership (degree of belongingness)of an element in the given set.

3.0 Membership Functions

The membership function(also called characteristic function, discrimination function, or indicator function) is the key idea introduced in fuzzy set theory to measure the degree to which the fuzzy set elements meet the specific properties, *i.e.* to measure the *degree of belongingness*of an element in a specific fuzzy set. It is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 (zero) and 1 (unity). Formally, if Xis the universe of discourse and its elements are denoted by x, then a fuzzy set Fin Xis defined as a set of ordered pairs, such that

$$F = \{x, \mu_F(x) | x \in X\} \qquad (3)$$

A membership function can take on any shape. A number of shapes that are commonly used include:

(i) Triangular membership function depends on three scalar parameters *a*, *b*, and *c* as shown in Figure 1.

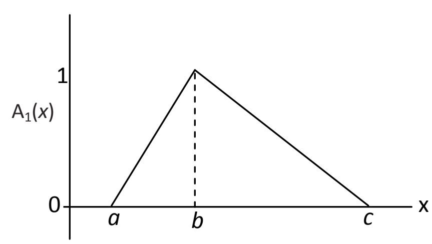

Figure 1: Triangular Membership Function

The parameters *a* and *c*respectively set the left and right "feet," or base points, of the triangle. The parameter *b* sets the location of the triangle peak. Mathematically, the function is given by:

$$\text{Triangle}(x;a,b,c) = max\left(min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right) \qquad (4)$$

(ii) The Trapezoidal Membership Function block implements a trapezoidal-shaped membership function, which depends on four scalar parameters *a*, *b*, *c*, and *d* as given in Figure 2.

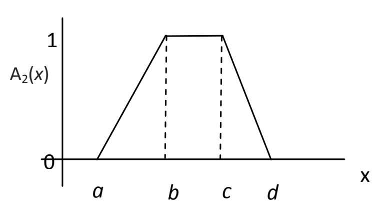

Figure 2: Trapezoidal Membership Function

The parameters *a* and *d* respectively set the left and right "feet," or base points, of the trapezoid. The parameters *b* and *c* set the "shoulders," or top of the trapezoid. The trapezoidal membership function can be expressed mathematically as given by equation (5).

$$\text{Trapezoidal}(x;a,b,c,d) = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) \qquad (5)$$

(iii) The Generalized Bell Membership Function implements a membership function based on a generalized bell-shaped curve.

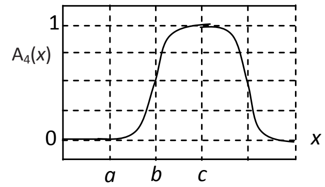

Figure 3: Generalized Bell Membership

The generalized bell-shaped curve depends on three parameters *a,b* and *c* as shown in Figure 3. The parameters *a* and *b* vary the width of the curve, while the parameter *c* locates the centre of the curve. The parameter *b* is usually positive. The Generalized Bell membership function is given by

$$f(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}} \quad . \quad . \quad . \quad (6)$$

(iv) The Difference between two sigmoidal functions membership function implements a membership function based on the difference between two sigmoids. The two sigmoid curves depend on two parameters *a*and *c* as shown in Figure 4, and the membership function is given by the mathematical expression of equation (7).

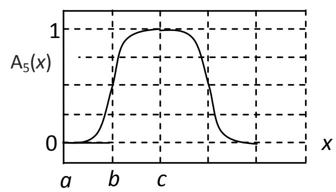

Figure 4: Difference Between Two Sigmoidal Functions Membership Function

$$f_k(x) = \frac{1}{1+e^{(-a_k((x-c_k))}} \quad . \quad . \quad . \quad . \quad (7)$$

where k = 1,2. While the parameters a_1 and a_2 control the slopes of the left and right curves, the parameters c_1 and c_2 control the points of inflection of the left and right curves. The parameters a_1 and a_2 should be positive.

4.0 Literature Review

Many authors have chosen to use a membership function arbitrarily either because of its simplicity or because it is has been used by others. Examples of works where reasons of such choices were made include the following.

(i) In their work on the "Identification level of security, usability and transparency effects on trust in B2C Commercial websites using adaptive neuro fuzzy inference system (ANFIS)"Nilashi et al [5] made use of Gauss membership function (Gaussmf) to build an expert ANFIS model without giving any reasons for their choice.

(ii) The triangular membership function (trimf) was selected and used for each input by Samandar [6] to develop an ANFIS model in his work to predict friction coeffitient in open channel flow. The author did not give any reason for his selection and use of trimf.

(iii) The decision in [7] to use the trapezoidal membership function (trapmf) was based on its simplicity, coherence with the the medical expert's intuition, and the fact that trapmf converges faster than Gaussmf.

(iv) Shirdar et al [8] adopted Gaussmf in their work without giving any reasons for their choice.

(v) In their study,Jloandan et al [9] stated that " trapezoidal MF is used. The selection of this MF is to some extent arbitrary".

5.0 Methodology for Selection of Appropriate Membership Functions for ANFIS Models

Choice of a membership function for an ANFIS modelshould be scientific; hence this study. AnANFIS is a multi-layered feed-forward network which uses neural network learning algorithm and fuzzy reasoning to map inputs into an output. It is an inference system implemented in the framework of adaptive of neural network. The architecture of a typical ANFIS with two input variables x_1 and x_2 is as shown in Figure 5.

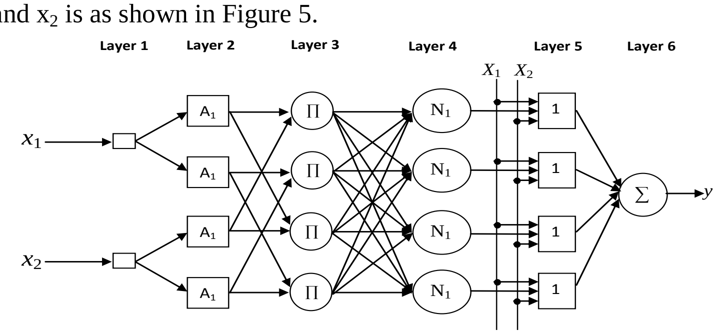

Figure 5: Architecture of an ANFIS

Layer 1: This is the input layer. Each input node in this layer corresponds to a specific input variable (x_1, $x_2 \ldots x_n$). These nodes only transmit input signals to the second layer.
Layer 2: This is the input membership function layer. Every node in this layer is adaptive, and performs a fuzzyfication procedure of converting the crisp input into a fuzzy domain through a membership function given by:

$$O_{2,i} = \mu_{A_i}(x), for\ i = 1,2 \quad . \quad . \quad . \quad . \quad (8)$$

$$O_{2,i} = \mu_{B_i}(y), for\ i = 1,2 \quad . \quad . \quad . \quad .(9)$$

where x is the input to node i, and A_i (or B_{i-2}) is a linguistic label (such as "low" or "high") associated with this node. In order words, $O_{1,i}$ is the membership grade of a fuzzy set A and it specifies the degree to which the given input x satisfies the quantifier A. The membership function can be any appropriate membership function, such as the Triangular or Gaussian. Parameters in this layer are referred to as the "premise" or "antecedence" parameters.
Layer 3: This is the Rule layer. Every node i in this layer is a fixed node labelled as Π. The firing strength of every node, due to all signals coming into the node, is generated in this layer through multiplication. The firing strength w_i, for each node is given by:

$$O_{3,i} = w_i = \prod_{i=1}^{n} \mu_{A_i}(x)\mu_{B_i}(y) \quad . \quad . \quad . \quad (10)$$

FromFigure 5,

$$O_{3,i} = w_{ii} = \mu_{A_i}(x)\mu_{B_i}(y), i = 1,2 \quad . \quad . \quad . \quad (11)$$

The nodes in this layer are called rule nodes.
Layer 4: is the Normalisation layer. Every node i in this layer is a fixed node labeled N to indicate the normalization of the firing levels. The node normalizes the firing strength of the incoming signal by calculating the ratio of the rule's firing strength to the sum of all the rule's firing strengths. It is given by:

$$O_{4,i} = \overline{w}_i = \frac{w_i}{\sum_{i=1}^{n} w_i} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

From the figure 5,

$$O_{4,i} = \overline{w}_i = \frac{w_1}{w_1 + w_2} i = 1,2 \quad . \quad . \quad . \quad . \quad (13)$$

Layer 5: is the Output membership function layer. Every node i in this layer is an adaptive node with a node function given by:

$$0_{5,i} = \overline{w}_i f_i = \overline{w}_I (p_i x + q_i y + r_i) \quad . \quad . \quad . \quad (14)$$

where $\overline{w}_i$ is a normalised firing strength from layer 4, and $\{p_i, q_i, r_i\}$ is the parameter set of this node. Parameters in this layer are called "conclusion" or "consequent" parameters.
Layer 6: This is a single-node layer that acts as a defuzzyfier. The single node is denoted by Σ and computes the overall output by summing up of all incoming signals. Then a selected defuzzyfication strategy is carried out producing the overall output of the model. The process is given by:

$$O_{6,i} = \sum_{i=1}^{n} \overline{w}_i f_i = \frac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} w_i} \quad . \quad . \quad . \quad (15)$$

To show how an appropriate membership function can be selected for an ANFIS model, the grid partition technique from the simulation engine of the MATLAB fuzzy toolbox was used to genetrate a Fuzzy Inference System (FIS) shown in Figure 6.The ANFIS normally adopts grid partition techniques to simplify and evenly partition the whole data space in the design of covering mode of membership function (curve).

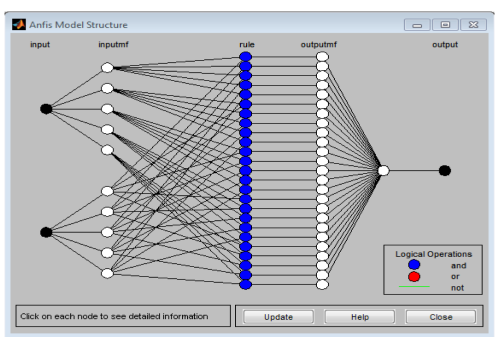

Figure 6: Fuzzy Inference System Structure

The FIS consists of a five-layered neural network that simulates the working principles of the model.The nodes in the first layer represent the input linguistic variables. Nodes in the second layer are term-nodes that act as the membership functions for the input variables. It is the condition elements layer. The third layer is a layer of neurons where each neuron represents a fuzzy rule. Input connection to each neuron represents the consequences of the rule. The action elements are represented by thenodes in the fourth layer. The output layer is the fifth layer. It aggregates the outputs from the fourth layer to give one single output.

For this study,input variables tothe FIS of Figure 6 arevibration signal frequency and power. To obtain values for these variables, an experiment was conducted on a steel pipe to acquire its vibration signal data when an attempt was made to drill a hole on it using a drilling machine. The frequency (in Hertz) and power (in Decibel) values were extracted from the power spectrumof the vibration signal. Simulation of the FIS using these values produced the membership function curves of for the vibration frequency and poweras shown in Figures 7 and 8 respectively.

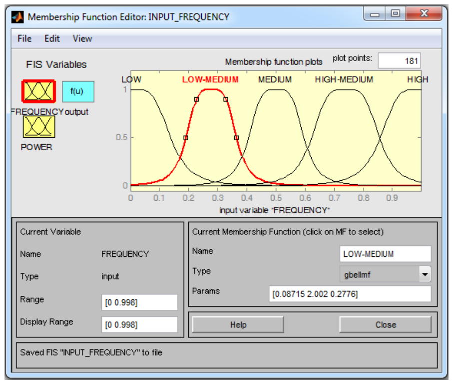

Figure 7: Vibration Signal Frequency Input Membership Function

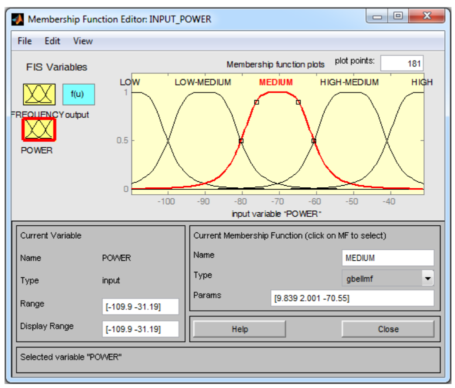

Figure 8: Vibration Signal Power Input Membership Function

The choice of the Generalised Bell membership function was chosen as the idealmembership function for this ANFIS. The choice was based on the fact that it gave the least "minimum checking error" value of 3.61954E-5 (highlighted in Table 1)in comparison with all other error values produced by the simulation when other membership functions were used.

Table 1: Training and Checking/Testing Errors from various Membership Function Types Using 1024 Sample DataPairs for Simulation

S/N	Membership Function Type	Membership Function Matrix	Number of Epochs	Converging Errors from the Use of the Command Line of the Fuzzy Logic Toolbox			
				Training Errors		Checking Errors	
				Minimum	Maximum	Minimum	Maximum
1.	Triangular	[5 5]	100	1.08387e-05	2.28431e-05	3.30112 e-03	3.38963 e-03
2.	Trapezoidal	[5 5]	100	1.43228e-05	2.60181e-05	3.87493e-05	8.35349e-03
3.	Generalised Bell	[5 5]	100	2.31918e-05	3.72829e-05	3.61954e-05	1.63614 e-03
4.	Gauss	[5 5]	100	2.81126e-05	4.07641e-05	1.35083 e-04	1.1085e-03
5.	Gauss2	[5 5]	100	1.76318e-05	2.9488e-05	4.44395 e-04	5.38993e-03
6.	Pi	[5 5]	100	9.42594e-06	2.45458e-06	7.79432e-05	1.04904e-02
7.	DSigmoid	[5 5]	100	1.60926e-05	3.64899e-05	5.87956e-05	1.13597e-03
8.	PSigm-oid	[5 5]	100	1.60344e-05	3.64968e-05	5.96654e-05	1.13636e-03

Thechecking data set is very useful for validating ANFIS model because after a certain point in the training, the model begins to overfit the training data set. The model error for the checking data set decreases as the training takes place, up to the point that overfitting begins. Overfitting sets in just after the minimum checking error; and this causes the model error for the checking data to suddenlybeginto increase until the end of the number of epochs set for the training of the model.

Overfitting is accounted for by testing the trained ANFIS against the checking data. The "Generalized Bell" membership function associated with the minimum checking error (highlighted in Table 1) is thus chosen for the ANFIS.

6.0 Conclusion

Complexity of real-world problems has led to the development of artificial intelligent systems that mimmic human intelligencein solving problems. Scientific solutions to problems begin with modelling and simulation of the structure and behaviour of the conceived or existing systems to avoid the risk, difficulty and high cost associated with direct experimentation with real systems. ANFIS is one of the models used in the artificial intelligence parlance. In this study, it has been shown that the choice of a membership function for an ANFIS model should not be based on the simplicity or otherwise of the membership function, but on theleast "minimum checking error" value from the simulation of the model using experimental data.

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